P wave charmonium spectrum in a relativistic quark model

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- Confined One Gluon Exchange Potential (COGEP)
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Introduction

- The quarkonium spectroscopy has undergone a great resurgence in recent years
- Experimental facilities like CLEO BELLE BABAR BES DELPHI etc. have been providing new information about hadrons
- Quarkonia $(Q\bar{Q})$ consists of a heavy quark and an antiquark: $c\bar{c}$, $b\bar{b}$, $c\bar{b}$ or $b\bar{c}$

Introduction

- Investigation of the properties of hadronic states gives very important information about the quark-antiquark dynamics
- Heavy quarkonium decays provide useful information on understanding the nature of interquark forces and decay mechanisms [1]
- The annihilation of a heavy quark-antiquark pair into final states consisting of leptons, photons and light quarks can provide useful information on α_s [2]

Introduction

- $Q\bar{Q}$ potential cannot be derived starting from the first principles of QCD
- Hence there is a theoretical uncertainity in the $Q\bar{Q}$ potential at large and intermediate distances
- Therefore one has to go for phenemenological models to explain the hadron properties

Introduction

- Existing potential models: Buchmuller and Tye, Martin, Log, Cornell, Coulomb plus power law, etc.
- This also implies that there are different potentials that reproduce the same $Q\bar{Q}$ spectra
- In the present work we investigate the S and P wave charmonium spectrum using the RHM along with COGEP

Introduction

- The experimentally observed states are [3]

Quantum numbers				Name	Mass	Width
n	L	J^{PC}	$n^{2S+1}L_J$		MeV	MeV
1	0	0^{-+}	1^1S_0	$\eta_c(1S)$	2980.4 ± 1.2	25.5 ± 3.4
1	0	1^{--}	1^3S_1	J/ψ	3096.916 ± 0.011	93.4 ± 2.1 keV
1	1	0^{++}	1^3P_0	$\chi_{c0}(1P)$	3414.76 ± 0.35	10.4 ± 0.7
1	1	1^{++}	1^3P_1	$\chi_{c1}(1P)$	3510.66 ± 0.07	0.89 ± 0.05
1	1	2^{++}	1^3P_2	$\chi_{c2}(1P)$	3556.20 ± 0.09	2.06 ± 0.12
1	1	1^{+-}	1^1P_1	$h_c(1P)$	3525.93 ± 0.27	< 1
1	2	1^{--}	1^3D_1	$\psi(3770)$	3771.1 ± 2.4	23.0 ± 2.7
2	0	0^{-+}	2^1S_0	$\eta_c(2S)$	3638 ± 4	14 ± 7
2	0	1^{--}	2^3S_1	$\psi(2S)$	3686.093 ± 0.034	337 ± 13 keV
2	1	2^{++}	2^3P_2	$\chi_{c2}(2P)$	3929 ± 5	29 ± 10

Introduction

- The states are characterised by the orbital angular momentum L, the total spin S of the $Q\bar{Q}$ pair and the total angular momentum J, which is the spin of the state viewed as a particle
- $\vec{J} = \vec{L} + \vec{S}$, where $S = 0, 1$
- The excitation of the radial motion of the $Q\bar{Q}$ pair results in a spectrum with same L, S and J, and differing by the radial quantum number n
- States are represented by the symbol n $^{(2S+1)}L_J$
- The states can also be represented by the J^{PC} values, where $P = (-1)^{L+1}$ is the parity and $C = (-1)^{L+S}$ is the charge conjugation parity

Relativistic Harmonic Model

- For the confinement of quarks we use the relativistic harmonic model (RHM) [4]
- RHM was highly successful in explaining various properties of hadrons: light meson spectrum, N-N scattering phase-shifts, properties of glue balls, etc. [5, 6, 7]
- In RHM quarks are Dirac particles subjected to Lorentz scalar plus vector potentials

Relativistic Harmonic Model

- The Dirac equation with a general potential is

  $$[\boldsymbol{\alpha} \cdot \mathbf{P} + \beta(m + S(r)) + V(r)]\psi = E\psi$$

  where S(r) is the scalar potential and V(r) is the time component of the vector potential
- A pure vector potential would produce only $Q\bar{Q}$ bound states, whereas a scalar potential provides an attractive force for both $Q\bar{Q}$ and QQ states [8]
- Thus, for the confinement of quarks, a scalar plus vector potential is a more appropriate choice

Relativistic Harmonic Model

- In the RHM [4], quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic oscillator potential
$$V_C(r) = \frac{1}{2}(1+\gamma_0)A^2r^2$$
- In RHM, the single quark wave function ψ is given by
$$\psi = N \begin{pmatrix} \phi \\ \frac{\sigma.\mathbf{P}}{E+M}\phi \end{pmatrix}$$
where E is an eigenvalue of the single particle Dirac equation
- The lower component of ψ can be eliminated by a similarity transformation $U\psi = \phi$, where U is given by
$$U = \frac{1}{N[1+\frac{P^2}{(E+M)^2}]} \begin{pmatrix} 1 & \phi \\ -\frac{\sigma.\mathbf{P}}{E+M} & 1 \end{pmatrix}$$

Relativistic Harmonic Model

- With this transformation the upper component of ϕ satisfies the equation

$$\left[\frac{\mathbf{P}^2}{E+M} + A^2 r^2\right]\phi = (E-M)\phi$$

- The eigenvalue E is then given by

$$E_n^2 = M^2 + (2n+1)\,\Omega_n^2$$

- Here Ω_n is an energy dependent parameter

$$\Omega_n^2 = A\,(E_n + M)^{1/2}$$

- The total energy of the hadron is obtained by adding the individual contributions of the quarks

COGEP

- The quark-antiquark interaction potential is given by the confined one gluon exchange potential (COGEP) [8]
- In the phenomenological quark models the effect of confinement of gluons on mesonic states is not been taken into account
- The essential new ingredient in our investigation of the mesonic states is to take into account the confinement of gluons

COGEP

- The gluons which are the quanta of the color field carry color charges which interact among themselves
- The confinement schemes for quarks and gluons have to be more closely connected to each other in QCD and the confinement of gluons has to be taken into account
- For the confinement of gluons we have made use of the current confinement model (CCM) which was developed in the spirit of the RHM [9, 10]

COGEP

- The CCM has been quite successful in describing the glue-ball spectra
- The confined gluon propagators (CGP) are derived in CCM
- Using CGP the confined one gluon exchange potential (COGEP) was obtained [8]

COGEP

- $$V_{COGEP} = V_{COGEP}^{cent} + V_{COGEP}^{L.S} + V_{COGEP}^{Ten}$$
- The central part of COGEP is:

$$\begin{aligned} V_{COGEP}^{cent} &= \frac{\alpha_s}{4} N^4 \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j [D_0(r) + \frac{1}{(E+M)^2}[4\pi\delta^3(r) \\ &\quad -c^4 r^2 D_1(r)][1 - \frac{2}{3}\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j]] \end{aligned}$$

- Here the first term is the residual Coulomb energy and the second and the third are the chromomagnetic interaction leading to the hyperfine splittings
- Here $D_0(r)$ and $D_1(r)$ are propagators of the CCM

COGEP

- The spin-orbit part of COGEP is given by

$$\begin{aligned} V^{L.S}_{COGEP} &= -\frac{\alpha_s}{4}\frac{N^4}{(E+M)^2}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\frac{1}{2r} \\ &\quad [\{(r\times(p_i-p_j)\cdot(\sigma_i+\sigma_j))(D_0'(r)+2D_1'(r))\} \\ &\quad +\{(r\times(p_i-p_j)\cdot(\sigma_i-\sigma_j))(D_0'(r)-D_1'(r))\}] \end{aligned}$$

- The tensor part of COGEP is given by

$$V^{Ten}_{COGEP} = -\frac{\alpha_s}{4}\frac{N^4}{(E+M)^2}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\left[\frac{D_1''}{3}-\frac{D_1'}{3r}\right]\hat{S}_{ij},$$

  where $\hat{S}_{ij} = [3(\sigma_i\cdot\hat{r})(\sigma_j\cdot\hat{r})-(\sigma_i\cdot\sigma_j)]$

- Both spin-orbit and tensor forces affect states with $L > 0$
- The spin-orbit and the tensor terms describe the fine structure of the states while the spin-spin term gives the spin singlet-triplet splittings

Procedure

- The $Q\bar{Q}$ wave function for each meson is expressed in terms of harmonic oscillator wave functions
- The Hilbert space of relative wave functions is truncated at radial quantum number $n = 5$
- Further increase of n doesn't effect the spectrum appreciably

Procedure

- The Hamiltonian matrix is constructed in the basis of harmonic oscillator wave functions and diagonalised
- The diagonal values give the masses of the ground and the radially excited states
- The model parameters and the radial wave functions used for the prediction of the mass spectrum are used to calculate the two-photon and two-gluon decay widths of η_c, χ_{C0} and χ_{C2} and also the leptonic decay width of J/ψ

Decay Widths

- The two-photon decay width is given by [11]:

$$\Gamma_{\gamma\gamma} = \frac{12\pi e_q^4 \alpha^2}{m_q^2} |\psi(0)|^2 \left(1 - \frac{3.4\alpha_s}{\pi}\right),$$

- The decay width for two-gluon decay is given by [11]:

$$\Gamma_{gg} = \frac{8\pi\alpha_s^2}{3m_q^2} |\psi(0)|^2 \times \begin{cases} (1 + 4.8\alpha_s/\pi), & \text{for } \eta_c \\ (1 + 4.4\alpha_s/\pi), & \text{for } \eta_b \end{cases}$$

- The leptonic decay width of J/ψ is calculated using [11]

$$\Gamma_{l^+l^-} = 16\pi\alpha^2 e_q^2 \frac{|\psi(0)|^2}{m_V^2} \left(1 - \frac{16\alpha_s}{3\pi}\right)$$

Decay Widths

- The two-photon widths are calculated using [11]:

$$\begin{aligned}\Gamma_{\chi_{c0}\to\gamma\gamma} &= \frac{2^4 3^3 e_Q^4 \alpha^2}{M^4}|R'_P(0)|^2[1+\frac{\alpha_s}{3\pi}(\pi^2-28/3)] \\ \Gamma_{\chi_{c2}\to\gamma\gamma} &= \frac{2^6 3^2 e_Q^4 \alpha^2}{5M^4}|R'_P(0)|^2[1-\frac{16\alpha_s}{3\pi}]\end{aligned}$$

- The two-gluon widths are calculated using [11]:

$$\begin{aligned}\Gamma_{\chi_{c0}\to gg} &= \frac{6\alpha_s}{m_Q^4}|R'_P(0)|^2[1+\frac{9.5\alpha_s}{\pi}] \\ \Gamma_{\chi_{c2}\to gg} &= \frac{8\alpha_s^2}{5m_Q^4}|R'_P(0)|^2[1-\frac{2.2\alpha_s}{\pi}]\end{aligned}$$

Decay Widths

- The two-photon widths are calculated using [11]:

$$
\begin{aligned}
\Gamma_{\chi_{c0}\to\gamma\gamma} &= \frac{2^4 3^3 e_Q^4 \alpha^2}{M^4} |R_P'(0)|^2 [1 + \frac{\alpha_s}{3\pi}(\pi^2 - 28/3)] \\
\Gamma_{\chi_{c2}\to\gamma\gamma} &= \frac{2^6 3^2 e_Q^4 \alpha^2}{5 M^4} |R_P'(0)|^2 [1 - \frac{16\alpha_s}{3\pi}]
\end{aligned}
$$

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\Gamma_{\chi_{c2}\to gg} &= \frac{8\alpha_s^2}{5 m_Q^4} |R_P'(0)|^2 [1 - \frac{2.2\alpha_s}{\pi}]
\end{aligned}
$$

Results and Discussions

- S wave charmonium spectrum (vector)

Meson	**Present**	**PDG[3]**	**[12]**
J/ψ	3097	3097	3100
$\psi(2S)$	3666	3686	3730
$\psi(3S)$	4102		4180
$\psi(4S)$	4687		4560
$\psi(5S)$	4892		

Results and Discussions

- S wave (pseudoscalar states)

Meson	**Present**	**PDG[3]**	**[12]**
$\eta_c(1S)$	2980	2980	3000
$\eta_c(2S)$	3591	3637	3670
$\eta_c(3S)$	3725		4130
$\eta_c(4S)$	4169		

Results and Discussions

- Partial decay widths for two-photon and two-gluon decays are shown in table below

Table: Decay Widths

Decay	**Present**	**Exp.**	**[13]**
$\Gamma_{e^+e^-}$	4.04 keV	5.55 keV	5.41 keV[**?**]
$\eta_c \rightarrow \gamma\gamma$	5.61 keV	7.2 keV	3.48 keV
$\eta_c \rightarrow gg$	15.06 MeV	26.7 MeV	10.57 MeV

Results and Discussions

- The obtained mass spectra of P wave charmonia are listed in the table below in comparison with the PDG [3] values

Table: Mass Spectrum (in MeV)

Meson	**Present**	**PDG**	**[12]**
$\chi_{c0}(1P)$	3415	3415	3440
$\chi_{c0}(2P)$	3821		3940
$\chi_{c1}(1P)$	3502	3510	3500
$\chi_{c1}(2P)$	3925		3990
$\chi_{c2}(1P)$	3566	3556	3540
$\chi_{c2}(2P)$	4006	3929	4020
$h_c(1P)$	3530	3526	3510
$h_c(2P)$	3957		3990

Results and Discussions

- The spin averaged mass for1P state is defined as:

$$\bar{M}(1P) = \frac{\chi_{c0}(1P) + 3\chi_{c1}(1P) + 5\chi_{c2}(1P)}{9}$$

- $$\bar{M}(1P)_{Exp.} = 3525 \text{ MeV}$$

- $$\bar{M}(1P)_{Present} = 3528 \text{ MeV}$$

- The hyperfine splitting for the 1P state is $\Delta_{hf} = |\bar{M} - h_{c1}|$

- $$\Delta_{hf}^{Exp.} = 1 \text{ MeV}$$

- $$\Delta_{hf}^{Present} = 2 \text{ MeV}$$

Results and Discussions

- Partial decay widths for two-photon and two-gluon decays are shown in table below

Table: DecayWidths

Decay	**Present**	**Exp.**	**[13]**
$\chi_{c0} \rightarrow \gamma\gamma$	2.7 keV	2.53 keV	5.35 keV
$\chi_{c2} \rightarrow \gamma\gamma$	1.0 keV	0.60 keV	1.55 keV
$\chi_{c0} \rightarrow gg$	9.71 MeV	10.2 MeV	4.88 MeV
$\chi_{c2} \rightarrow gg$	2.10 MeV	2.03 MeV	0.69 MeV

Conclusions

- The S and P wave charmonium spectrum is calculated in the framework of RHM along with COGEP and are compared with the experimental results
- Calculated partial widths (two-photon and two-gluon) are comparable with the experimental values
- RHM along with COGEP give a good description of hadronic properties

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THANK YOU

$D_0(r)$ and $D_1(r)$ appearing in the COGEP are given by:

$$\begin{aligned} D_0(r) &= \left(\frac{\alpha_1}{r} + \alpha_2\right) exp\left(\frac{-r^2 c_0^2}{2}\right) \\ D_1(r) &= \frac{\gamma}{r} exp\left(\frac{-r^2 c_1^2}{2}\right) \end{aligned}$$

where $\alpha_1 = 1.036$, $\alpha_2 = 2.016$, $c_0 = 1.732\ fm^{-1}$, $c_1 = 2.090\ fm^{-1}$
and $\gamma = 0.864$
$c = 1.74\ fm^{-1}$

Harmonic Oscillator wave function

$$\psi_{nlm}(r,\theta,\phi) = N\left(\frac{r}{b}\right)^{l} L_n^{l+\frac{1}{2}}\left(\frac{r^2}{b^2}\right)\exp[-\frac{r^2}{2b^2}]\; Y_{lm}(\theta,\phi), \qquad (1)$$

where N is the normalization constant given by

$$|N|^2 = \frac{2\ n!}{b^3\pi^{\frac{1}{2}}}\frac{2^{(2(n+l)+1)}}{(2n+2l+1)!}(n+l)!, \qquad (2)$$

and $L_n^{l+\frac{1}{2}}(x)$ are the associated Laguerre polynomials.

Matrix elements of angular part of tensor term of COGEP

$$\begin{aligned}
\langle {}^3P_0|S_{ij}|{}^3P_0\rangle &= -4\\
\langle {}^3P_1|S_{ij}|{}^3P_1\rangle &= 2\\
\langle {}^3P_2|S_{ij}|{}^3P_2\rangle &= \frac{-2}{5}\\
\langle {}^3P_2|S_{ij}|{}^3F_2\rangle &= \frac{6\sqrt{6}}{5}
\end{aligned}$$

Matrix elements of angular part of spin-orbit term of COGEP

$$\begin{aligned}
\langle {}^3P_0|\mathbf{L.S}|{}^3P_0\rangle &= -4\\
\langle {}^3P_1|\mathbf{L.S}|{}^3P_1\rangle &= -2\\
\langle {}^3P_2|\mathbf{L.S}|{}^3P_2\rangle &= 2
\end{aligned}$$