Forward-backward asymmetry of top quark at Tevatron using four fermions operators

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Forward backward asymmetry of top quark at Tevatron

- $\mathbf{A_{FB}}$ of top quark has been measured at Tevatron in process $\mathbf{p\bar{p} \longrightarrow t\bar{t}}$,
- $\mathbf{A_{FB}}$ can be defined in center of mass frame of t-$\bar{\mathbf{t}}$ pair as :

$$\mathbf{A_{FB}} \equiv \frac{\sigma(\cos\theta_t > 0) - \sigma(\cos\theta_t < 0)}{\sigma(\cos\theta_t > 0) + \sigma(\cos\theta_t < 0)}$$

- At tree level in QCD, $\mathbf{A_{FB}}$ vanishes,
- From **EW** process, contribution to $\mathbf{A_{FB}}$ is negligible $\sim \mathbf{10^{-4}}$,
- Next-to-leading order(**NLO**) in **QCD**, it is predicted to be $\mathbf{5.8 \pm 0.9\%}$ by MCFM and $\mathbf{1^{+2}_{-1}\%}$ by MC@NLO,
- At NLO non-zero $\mathbf{A_{FB}}$ arises due to the interference with 2-gluon intermediate state (Box-diagram) as well as gluon bremsstrahlung,

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- **CDF** collaboration with $\mathbf{3.2}$ fb^{-1} data measured $\mathbf{A_{FB} = (0.24 \pm 0.13 \pm 0.04)}$,
- Hence, there has been more than 2σ deviation between the measured and predicted value for last few years,
- With most recent data, the new number is somewhat lower viz. $\mathbf{A_{FB} = 0.158 \pm 0.072 \pm 0.019}$
- Most recent results by combined analysis of **CDF** and **D0** on cross section of $\mathbf{t\text{-}\bar{t}}$ pair is $\boldsymbol{\sigma_{t\bar{t}} = 7.50 \pm 0.48}$ pb,
- Also, they produce a invariant $\mathbf{t\text{-}\bar{t}}$ mass distribution,
- All these measurements guide us to look for new physics in $\boldsymbol{\sigma_{t\bar{t}}}$, $\mathbf{d\sigma/dm_{t\bar{t}}}$ and $\mathbf{A_{FB}}$,

Our Procedure

- We assume new scale of physics large enough beyond the reach of Tevatron,
- So we can use effective operators approach in order to study new physics in $\boldsymbol{\sigma_{t\bar{t}}}$ and $\mathbf{A_{FB}}$,
- For $\mathbf{t\text{-}\bar{t}}$ pair production in Tevatron, new physics effects can be well described by dimension-6 effective Lagrangian,
- We employ Markov Chain Monte Carlo (MCMC) analysis
- Using MCMC, we constrain the parameter space of the Wilson coefficients of effective operators at $\boldsymbol{\Lambda^4}$ order,
- We consider two scenarios for parameter estimation :
  - Fitting only $\mathbf{A_{FB}}$ and $\boldsymbol{\sigma_{t\bar{t}}}$,
  - Included Fitting of $\mathbf{d}\boldsymbol{\sigma_{t\bar{t}}}/\mathbf{m_{t\bar{t}}}$ bins,

Four quark contact interactions

Following Jung[1] et al, we consider following dimension **6** operators:

$$\mathcal{L}_6 = \frac{g_s^2}{\Lambda^2} \sum_{A,B} \left[C_{1q}^{AB} (\bar{q}_A \gamma_\mu q_A)(\bar{t}_B \gamma^\mu t_B) + C_{8q}^{AB} (\bar{q}_A T^a \gamma_\mu q_A)(\bar{t}_B T^a \gamma^\mu t_B) \right]$$

where $T^a = \lambda^a/2$, $\{A, B\} = \{L, R\}$, $q = (u, d)^T \,/\, (s, c)^T$ and the suffix **1q(8q)** denotes color singlet (octet) interaction.

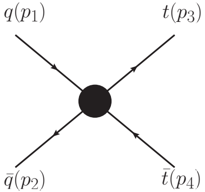

Figure: Four quark contact interaction

[1] arXiv:hep-ph/0912.1105

Some comments

- There can be many more operators involving $\mathbf{t}$, $\bar{\mathbf{t}}$ and gluon field strength tensor $\mathbf{G}^{\mathbf{a}}_{\mu\nu}$,
- Unlike the operators we choose, these are generated at loop order,
- Hence their effects would be suppressed by loop factor and coupling $\mathbf{g_s}$,
- Our choice of operators are enough to study $\mathbf{t}$-$\bar{\mathbf{t}}$ pair production at Tevatron,

Order $1/\Lambda^2$ contribution to the squared amplitude

$$
\begin{aligned}
\overline{|\mathcal{M}|^2}\Big|_{\mathcal{O}(1/\Lambda^2)} &= 2\mathcal{C}_f \frac{g_s^4}{\hat{s}^2}\left[2m_t^2\hat{s}\left\{\left[1+\frac{\hat{s}}{2\Lambda^2}(C_1+C_2)\right]s_\theta^2\right\}\right. \\
&+ \frac{\hat{s}^2}{2}\left\{\left(1+\frac{\hat{s}}{2\Lambda^2}(C_1+C_2)\right)(1+c_\theta^2)\right\} \\
&+ \left.\beta_t\frac{\hat{s}^2}{2}\left\{\frac{\hat{s}}{\Lambda^2}(C_1-C_2)\right\}c_\theta\right],
\end{aligned}
$$

- $\beta_t^2 = 1 - 4m_t^2/\hat{s}$,
- $\hat{s}$ is the centre of mass energy in the parton frame,
- $\mathcal{C}_f = 2/9$ is the color factor
- $C_1 = C_{8q}^{LL} + C_{8q}^{RR}; \;\; C_2 = C_{8q}^{LR} + C_{8q}^{RL}$.

Order $1/\Lambda^2$ contribution to the squared amplitude

- We look at the interference of dimension six operators with the SM amplitude,
- At Λ^2 order, singlet operators do not interfere with the SM,
- Only contribution come from color octet operators,
- A_{FB} mainly arises from CP violating term in the $|M|^2$ which here is $\cos\theta$ and $\Delta A_{FB} = A_{FB} - A_{FB}^{SM} \propto (C_1 - C_2)$
- While $\Delta\sigma_{t\bar{t}} = \sigma_{t\bar{t}} - \sigma_{t\bar{t}}^{SM} \propto C_1 + C_2$,
- Hence, contributions of new physics to A_{FB} and $\sigma_{t\bar{t}}$ are orthogonal,
- New physics can change A_{FB} considerably without changing $\sigma_{t\bar{t}}$

Constraints on parameters at $1/\Lambda^2$ order

- For non-zero contributions to A_{FB}, we need $C_1 - C_2 \neq 0$,
- Hence, to maximize A_{FB} and keeping $\sigma_{t\bar{t}}$ within desired range, C_1 and C_2 should be of opposite in sign,
- For parity conservation in light quark sector, $C_{8q}^{LL} = C_{8q}^{RL}$ and $C_{8q}^{LR} = C_{8q}^{RR}$,
- For parity conservation in top quark sector, $C_{8q}^{LL} = C_{8q}^{LR}$ and $C_{8q}^{RL} = C_{8q}^{RR}$,
- In both cases, $C_1 - C_2 = 0$,
- Hence, to have non-zero A_{FB}, we need parity violation both in light quark and top sector,

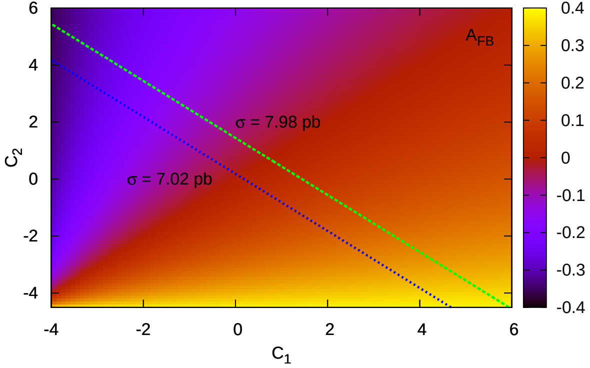

Figure: The region in $(\mathbf{C_1}, \mathbf{C_2})$ plane consistent with the Tevatron

Order $1/\Lambda^4$ contribution to the squared amplitude

$$\begin{aligned}
\overline{|\mathcal{M}|^2}\Big|_{\mathcal{O}(1/\Lambda^4)} &= 2\mathcal{C}_f \frac{g_s^4}{\hat{s}^2}\left[2m_t^2\hat{s}\left\{\frac{\hat{s}^2}{4\Lambda^4}\left[2\ C_3 - (C_{12}+C_{22})c_\theta^2\right]\right\}\right. \\
&+ \frac{\hat{s}^2}{2}\left\{\left(\frac{\hat{s}^2}{4\Lambda^4}(C_{12}+C_{22})\right)(1+c_\theta^2)\right\} \\
&+ \left.\beta_t\frac{\hat{s}^2}{2}\left\{\frac{\hat{s}^2}{2\Lambda^4}(C_{12}-C_{22})\right\}c_\theta\right] + \overline{|\mathcal{M}|^2}\Big|_{\mathcal{O}(1/\Lambda^2)},
\end{aligned}$$

where we have defined

$$\begin{aligned}
C_{12} &= \left([C_{8q}^{LL}]^2 + \frac{1}{\mathcal{C}_f}[C_{1q}^{LL}]^2\right) + \left([C_{8q}^{RR}]^2 + \frac{1}{\mathcal{C}_f}[C_{1q}^{RR}]^2\right) \\
C_{22} &= \left([C_{8q}^{LR}]^2 + \frac{1}{\mathcal{C}_f}[C_{1q}^{LR}]^2\right) + \left([C_{8q}^{RL}]^2 + \frac{1}{\mathcal{C}_f}[C_{1q}^{RL}]^2\right) \\
C_3 &= \left(C_{8q}^{LL}C_{8q}^{LR} + \frac{1}{\mathcal{C}_f}C_{1q}^{LL}C_{1q}^{LR}\right) + \left(C_{8q}^{RR}C_{8q}^{RL} + \frac{1}{\mathcal{C}_f}C_{1q}^{RR}C_{1q}^{RL}\right).
\end{aligned}$$

Markov Chain MonteCarlo (MCMC) Algorithm

1. Start : $\mathbf{X_0}$ is a randomly chosen starting point in the parameter space, $\mathbf{f_0 = 1}$ is its starting frequency and $\chi^2(\mathbf{X_0})$ is its chi-square (calculate it).

2. Proposal : Around $\mathbf{X_0}$ randomly chose another point $\mathbf{X_1}$. One example is $\mathbf{x_1^i = \texttt{Gaussian}(x_0^i, \delta x^i)}$, i.e., the components of $\mathbf{X_1}$ are normally distributed around components of $\mathbf{X_0}$ with variances $\{\delta \mathbf{x^i}\}$.

3. Evaluate: Evaluate all the observables for new point $\mathbf{X_1}$ and also the chi-square.

1. Decision: Metropolis-Hastings algorithm

$$\text{If } \mathtt{Random}() < \frac{\exp[-\chi^2(\mathbf{X_1})/2]}{\exp[-\chi^2(\mathbf{X_0})/2]}$$

Where Random() is a uniformly distributed random number between 0 and 1.

- TRUE:
  1.1 Print $\mathbf{f_0}$, $\chi^2(\mathbf{X_0})/2$, $\mathbf{X_0}$, $\mathcal{O}_\mathbf{j}(\mathbf{X_0})$ in that order.
  1.2 Set $\mathbf{f_0 = 1}$, $\chi^2(\mathbf{X_0}) = \chi^2(\mathbf{X_1})$, $\mathbf{X_0 = X_1}$.
  1.3 Go to Proposal or Stop if the number of iterations are over.
- FALSE:
  1.1 Print $\mathbf{f_1 = 0}$, $\chi^2(\mathbf{X_1})/2$, $\mathbf{X_1}$, $\mathcal{O}_\mathbf{j}(\mathbf{X_1})$ in that order [optional, usually omitted to save disk space]
  1.2 Set $\mathbf{f_0 = f_0 + 1}$
  1.3 Go to Proposal or Stop if the number of iterations are over.

MCMC Analysis

Cross-sections, in forward and backward directions and total cross-sections expressed as $\boldsymbol{\sigma_i = \sigma_i^{SM} + \sum_j c_{ij}\sigma_{ij}}$,

- $\boldsymbol{\sigma_i}$ is one of the above mentioned cross-sections,
- $\mathbf{c_{ij}}$ are some combinations of the coefficients of the 4-fermion operators
- $\boldsymbol{\sigma_{ij}}$ are the numerically calculated coefficients of $\mathbf{c_{ij}}$,
- We have taken new physics scale $\boldsymbol{\Lambda = 1}$ TeV.
- We used them in the MCMC code and obtained a fit for two cases:
  1. fitting to $\boldsymbol{\sigma_{total}}$ and $\mathbf{A_{FB}^t}$, the best fit point is called "F1". For F1 we have $\boldsymbol{\chi^2_{min} = 9 \times 10^{-7} \approx 0}$, $\boldsymbol{\sigma_{t\bar{t}} = 7.327}$ pb and $\mathbf{A_{FB}^t = 0.193}$.
  2. fitting to $\boldsymbol{\sigma_{total}}$, $\mathbf{A_{FB}^t}$ and $\mathbf{d\boldsymbol{\sigma}/dm_{t\bar{t}}}$, the best fit point is called "F2". For F2 we have $\boldsymbol{\chi^2_{min} = 1.176}$, $\boldsymbol{\sigma_{t\bar{t}} = 7.132}$ pb and $\mathbf{A_{FB}^t = 0.143}$.

Couplings	SM	F1	Bins (GeV)	σ_i^{SM} (pb)	σ_i^{F1} (pb)
C_{LL}^8	0.00	2.79	350 – 400	2.42	1.62
C_{RR}^8	0.00	−2.31	400 – 450	1.84	1.29
C_{LR}^8	0.00	−4.57	450 – 500	1.15	0.91
C_{RL}^8	0.00	−4.01	500 – 550	0.69	0.68
C_{LL}^1	0.00	0.90	550 – 600	0.40	0.53
C_{RR}^1	0.00	1.16	600 – 700	0.38	0.78
C_{LR}^1	0.00	0.29	700 – 800	0.13	0.50
C_{RL}^1	0.00	1.82	800 – 1400	0.06	0.57

Table: Best fit points without (F1) $\mathbf{m_{t\bar{t}}}$ distribution observables in the fit.

Case1

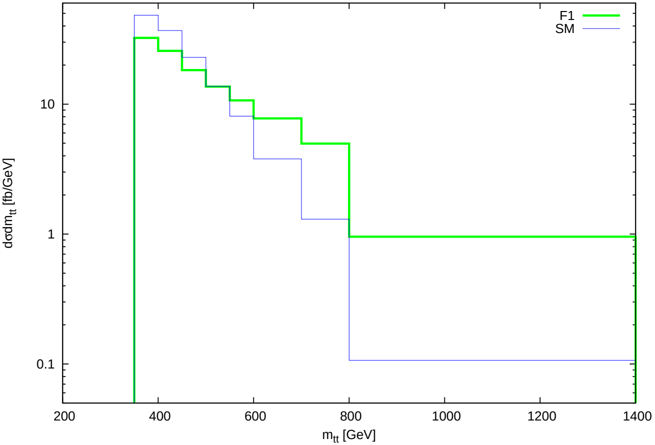

Figure: $\mathbf{m_{t\bar{t}}}$ distribution with points $\mathbf{F_1}$

Case2

Couplings	SM	F2	Bins (GeV)	σ_i^{SM} (pb)	σ_i^{F2} (pb)
C_{LL}^8	0.00	0.97	350 – 400	2.42	2.35
C_{RR}^8	0.00	0.48	400 – 450	1.84	1.79
C_{LR}^8	0.00	−0.89	450 – 500	1.15	1.11
C_{RL}^8	0.00	−0.97	500 – 550	0.69	0.67
C_{LL}^1	0.00	0.06	550 – 600	0.40	0.40
C_{RR}^1	0.00	−0.07	600 – 700	0.38	0.38
C_{LR}^1	0.00	−0.03	700 – 800	0.13	0.14
C_{RL}^1	0.00	−0.01	800 – 1400	0.06	0.08

Table: Best fit points with (F2) $\mathbf{m_{t\bar{t}}}$ distribution observables in the fit.

Case2

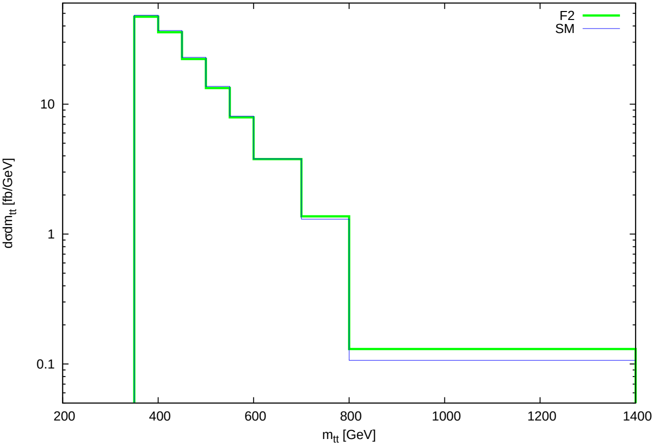

Figure: $\mathbf{m_{t\bar{t}}}$ distribution with points $\mathbf{F_2}$

Important results and conclusions from this study :

- We have performed Markov chain Monte Carlo (**MCMC**) analysis to fit the 8 parameters coming from 8 fermion operators,
- We use all constraints to top pair production i.e., cross section and invariant mass distribution of t-$\bar{t}$ pair from Tevatron,
- Including square of the four quark operators further constrains the parameter space compared to bounds coming from just looking at interference of these operators with the **SM**,
- Bounds on the Wilson's coefficients of these operators are stronger when constraint from invariant mass of top-antitop pair is also included into the fitting of **MCMC** procedure,

THANK YOU FOR YOUR ATTENTION