The Euclidean Yang-Mills theory from Gribov-Zwanziger theory

Sudhaker Upadhyay and B. P. Mandal

Banaras Hindu University, Varanasi, India

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Outline

- The Yang-Mills (YM) theory: introduction
- The Gribov-Zwanziger (GZ) theory
- The nilpotent BRST symmetry transformation for GZ theory
- Generalized BRST transformation
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The Yang-Mills (YM) theory: introduction

- The generating functional for YM theory (in the manifestly covariant gauge)

$$Z_{YM} = \int [D\phi] \; e^{-S_{YM}},$$

where,

$$S_{YM} = \int d^4x[-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + B^a\partial_\mu A^a_\mu + \bar{c}^a\mathcal{M}^{ab}c^b]$$

the field strength tensor $F^a_{\mu\nu}$ and Faddeev-Popov operator $\mathcal{M}^{ab}$ are defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu$$

and

$$\mathcal{M}^{ab} = -\partial_\mu \mathcal{D}^{ab}_\mu$$

Nilpotent BRST symmetry for YM theory

- The above YM action is invariant under following BRST transformation:

$$
\begin{aligned}
\delta_b A_\mu^a &= -\mathcal{D}_\mu^{ab} c^b \ \Lambda, \\
\delta_b c^a &= \frac{1}{2} g f^{abc} c^b c^c \ \Lambda, \\
\delta_b \bar{c}^a &= B^a \ \Lambda, \\
\delta_b B^a &= 0,
\end{aligned}
$$

where, Λ is usual infinitesimal and anticommuting BRST parameter.

The Gribov-Zwanziger theory

- The gauge fixing condition $\partial_\mu A^a_\mu = 0$ can not fix the gauge uniquely. Each gauge orbit intersects the gauge fixing hypersurface $\Gamma = \{A^a_\mu; \partial_\mu A^a_\mu = 0\}$ many times. Therefore the theory has *Gribov-copies*.
- Such Gribov copies can be neglected in the ultraviolet (perturbative) regime. It plays an important role in the infrared (non-perturbative) regime, which is closely related to the gluon confinement.
- To avoid the Gribov copies, Gribov (1978) proposed to restrict the domain of integration in the functional integral to the Gribov region, whose boundary is the first *Gribov horizon*.

The Gribov-Zwanziger theory

- Such a restriction can be achieved by adding a nonlocal term in the YM action[1] as :

$$S_h = \int d^4x\; h(x),$$

$$h(x) \;=\; \gamma^4 \int d^4y\; g^2 f^{abc} A^b_\mu(x) (\mathcal{M}^{-1})^{ce}(x,y) f^{ade} A^d_\mu(y)$$

- The nonlocal term S_h can be localized[2] by introducing a quartet of auxiliary fields $(\bar{\varphi}^{ac}_\mu, \varphi^{ac}_\mu, \bar{\omega}^{ac}_\mu, \omega^{ac}_\mu)$ as

$$\begin{aligned} S_{local} \;&=\; \int d^4x \left[\bar{\varphi}^a_i \partial_\mu \mathcal{D}^{ab}_\mu \varphi^b_i - \bar{\omega}^a_i \partial_\mu \mathcal{D}^{ab}_\mu \omega^b_i - 4(N^2-1)\gamma^4 \right. \\ &- \left. \gamma^2 g f^{abc} A^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) - g f^{abc} \partial_\mu \bar{\omega}^a_i \mathcal{D}^{bd}_\mu c^d \varphi^c_i \right]. \end{aligned}$$

[1] D.Zwanziger, Nucl.Phys.B **323**, 513 (1989).

[2] D. Dudal, S. P. Sorella and N. Vandersickel, Eur. Phys. J. C **68**, 283 (2010).

The Gribov-Zwanziger theory

- The complete GZ action S_{GZ} is given as

$$S_{GZ} = S_{YM} + S_{local}.$$

Which further can be recast as

$$S_{GZ} = S_{exact} + S_{\gamma},$$

where

$$\begin{aligned} S_{exact} &= S_{YM} + \int d^4x \left[\bar{\varphi}_i^{ac}\partial_\mu \mathcal{D}_\mu^{ab}\varphi_i^{bc} - \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab}\omega_i^b \right. \\ &- \left. gf^{abc}\partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c \right], \end{aligned}$$

and

$$S_\gamma = -\gamma^2 \int d^4x \left[gf^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) + 4(N^2-1)\gamma^2 \right].$$

The exact symmetry for GZ theory

- The BRST transformation for the fields :

$$
\begin{aligned}
\delta_b A_\mu^a &= -\mathcal{D}_\mu^{ab} c^b \, \Lambda, & \delta_b c^a = \frac{1}{2} g f^{abc} c^b c^c \, \Lambda, \\
\delta_b \bar{c}^a &= B^a \, \Lambda, & \delta_b B^a = 0, \\
\delta_b \varphi_i^a &= -\omega_i^a \, \Lambda, & \delta_b \omega_i^a = 0, \\
\delta_b \bar{\omega}_i^a &= \bar{\varphi}_i^a \, \Lambda, & \delta_b \varphi_i^a = 0.
\end{aligned}
$$

- The S_γ is not invariant under the above BRST transformation. Thus the presence of γ dependent term breaks BRST symmetry of GZ theory *softly*.

The extended GZ action

- Further, S_γ is extended by introducing 3 doublets of sources $(U^{ai}_\mu, M^{ai}_\mu), (V^{ai}_\mu, N^{ai}_\mu)$ and (T^{ai}_μ, R^{ai}_μ) as

$$\begin{aligned}\Sigma_\gamma &= \int d^4x \left(-M^{ai}_\mu \mathcal{D}^{ab}_\mu \varphi^b_i - gf^{abc} U^{ai}_\mu \mathcal{D}^{bd}_\mu c^d \varphi^c_i + U^{ai}_\mu \mathcal{D}^{ab}_\mu \omega^b_i \right.\\ &- N^{ai}_\mu \mathcal{D}^{ab}_\mu \bar{\omega}^b_i - V^{ai}_\mu \mathcal{D}^{ab}_\mu \bar{\varphi}^b_i + gf^{abc} V^{ai}_\mu \mathcal{D}^{bd}_\mu c^d \bar{\omega}^c_i - M^{ai}_\mu V^{ai}_\mu \\ &+ \left. U^{ai}_\mu N^{ai}_\mu - gf^{abc} R^{ai}_\mu \mathcal{D}^{bd}_\mu c^d \bar{\omega}^c_i + gf^{abc} T^{ai}_\mu \mathcal{D}^{bd}_\mu c^d \bar{\varphi}^c_i \right).\end{aligned}$$

But we want our original theory. Therefore at the end, we have to set the sources equal to the following values:

$$\begin{aligned} U^{ai}_\mu|_{phys} &= N^{ai}_\mu|_{phys} = T^{ai}_\mu|_{phys} = 0 \\ M^{ab}_{\mu\nu}|_{phys} &= V^{ab}_{\mu\nu}|_{phys} = R^{ab}_{\mu\nu}|_{phys} = \gamma^2 \delta^{ab} \delta_{\mu\nu}. \end{aligned}$$

The extended GZ action

- Such that $\Sigma_\gamma|_{phys} = S_\gamma$.
- Σ_γ is invariant under following BRST transformation:

$$\begin{aligned} \delta_b U_\mu^{ai} &= M_\mu^{ai}\ \Lambda, \qquad \delta_b M_\mu^{ai} = 0, \\ \delta_b V_\mu^{ai} &= -N_\mu^{ai}\ \Lambda, \qquad \delta_b N_\mu^{ai} = 0, \\ \delta_b T_\mu^{ai} &= -R_\mu^{ai}\ \Lambda, \qquad \delta_b R_\mu^{ai} = 0. \end{aligned}$$

- Thus the nilpotent BRST symmetry is recovered for GZ action.

The Generalized BRST transformation

- We are using the generalized BRST formulation[3] in this case by allowing the usual BRST parameter to be finite and field dependent

$$
\begin{aligned}
\delta_b A_\mu^a &= -\mathcal{D}_\mu^{ab} c^b\ \Theta[\phi], & \delta_b c^a &= \frac{1}{2} g f^{abc} c^b c^c\ \Theta[\phi],\\
\delta_b \bar{c}^a &= B^a\ \Theta[\phi], & \delta_b B^a &= 0,\\
\delta_b \varphi_i^a &= -\omega_i^a\ \Theta[\phi], & \delta_b \omega_i^a &= 0,\\
\delta_b \bar{\omega}_i^a &= \bar{\varphi}_i^a\ \Theta[\phi], & \delta_b \varphi_i^a &= 0\\
\delta_b U_\mu^{ai} &= M_\mu^{ai}\ \Theta[\phi], & \delta_b M_\mu^{ai} &= 0,\\
\delta_b V_\mu^{ai} &= -N_\mu^{ai}\ \Theta[\phi], & \delta_b N_\mu^{ai} &= 0,\\
\delta_b T_\mu^{ai} &= -R_\mu^{ai}\ \Theta[\phi], & \delta_b R_\mu^{ai} &= 0,
\end{aligned}
$$

where $\Theta[\phi]$ is finite, field dependent BRST parameter.

[3] S. D. Joglekar and B. P. Mandal, Phys. Rev. **D 51**, 1919 (1995).

The Generalized BRST transformation

- For such a generalized BRST transformation we construct the transformation

$$\phi(x) \rightarrow \phi'(x),$$

by a continuous interpolation of κ in fields $\phi(x, \kappa), (0 \leq \kappa \leq 1)$ such that

$$\begin{aligned} \phi(x, \kappa = 0) &= \phi(x) \\ \phi(x, \kappa = 1) &= \phi'(x) \end{aligned}$$

- Such a generalized BRST is also symmetry of the action. But the path integral measure of generating functional is not invariant under such a transformation.

The Generalized BRST transformation

- The Jacobian J for path integral measure can be written as $e^{-S_1[\phi]}$ for some local functional $S_1[\phi]$ iff

$$\int [D\phi]\left[\frac{1}{J}\frac{dJ}{d\kappa}+\frac{dS_1[\phi(x,\kappa)]}{d\kappa}\right]\exp\left[-(S_{GZ}+S_1)\right]=0 \tag{0.1}$$

where $\phi(x,\kappa)$ is all fields involved in GZ action.

The Generalized BRST transformation

- The infinitesimal change in the $J(\kappa)$ can be written as

$$\frac{1}{J}\frac{dJ}{d\kappa} = -\int d^4x \left[\pm\delta_b\phi(x,\kappa)\frac{\partial\Theta'[\phi(x,\kappa)]}{\partial\phi(x,\kappa)}\right],$$

where $\pm$ sign refers to whether ϕ is a bosonic or a fermionic field.

- The infinitesimal change in fields with respect κ is calculated as

$$\frac{d\phi(x,\kappa)}{d\kappa} = \delta_b[\phi(x,\kappa)]\Theta'[\phi(x,\kappa)],$$

Relating the GZ theory and YM theory

- The choice for finite BRST parameter is

$$\Theta'[\phi] = \int d^4x \left[\bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b - U_\mu^a \mathcal{D}_\mu^{ab} \varphi_i^b - V_\mu^a \mathcal{D}_\mu^{ab} \bar{\omega}_i^b - U_\mu^{ai} V_\mu^{ai} + T_\mu^{ai} g f^{abc} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \right].$$

- The change in Jacobian for this case

$$\begin{aligned}\frac{1}{J}\frac{dJ}{d\kappa} &= -\int d^4x \left[-\bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b + \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b \right.\\ &+ M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b - U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b + g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c \\ &+ N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b + V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b - g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\ &+ M_\mu^{ai} V_\mu^{ai} - U_\mu^{ai} N_\mu^{ai} + g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\ &- \left. g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c + g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c \right].\end{aligned}$$

Relating the GZ theory and YM theory

- We consider an ansatz for S_1 as

$$
\begin{aligned}
S_1 &= \int d^4x \left[\chi_1 \bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b + \chi_2 \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b \right. \\
&+ \chi_3 M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b + \chi_4 U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b + \chi_5 g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c \\
&+ \chi_6 N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b + \chi_7 V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b + \chi_8 g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\
&+ \chi_9 M_\mu^{ai} V_\mu^{ai} + \chi_{10} U_\mu^{ai} N_\mu^{ai} + \chi_{11} g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\
&+ \left. \chi_{12} g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c + \chi_{13} g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c \right].
\end{aligned}
$$

Relating the GZ theory and YM theory

- The constants χ's can be evaluated using the necessary condition given in Eq. (0.1). The correct expression for S_1 after calculating χ's

$$
\begin{aligned}
S_1 &= \int d^4x \left[-\kappa\, \bar{\varphi}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \varphi_i^b + \kappa\, \bar{\omega}_i^a \partial_\mu \mathcal{D}_\mu^{ab} \omega_i^b \right.\\
&+ \kappa\, M_\mu^{ai} \mathcal{D}_\mu^{ab} \varphi_i^b + \kappa\, U_\mu^{ai} \mathcal{D}_\mu^{ab} \omega_i^b - \kappa\, g f^{abc} U_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \varphi_i^c \\
&+ \kappa\, N_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\omega}_i^b + \kappa\, V_\mu^{ai} \mathcal{D}_\mu^{ab} \bar{\varphi}_i^b - \kappa\, g f^{abc} V_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\
&+ \kappa\, M_\mu^{ai} V_\mu^{ai} - \kappa\, U_\mu^{ai} N_\mu^{ai} + \kappa\, g f^{abc} R_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\omega}_i^c \\
&- \left. \kappa\, g f^{abc} T_\mu^{ai} \mathcal{D}_\mu^{bd} c^d \bar{\varphi}_i^c + \kappa\, g f^{abc} \partial_\mu \bar{\omega}_i^a \mathcal{D}_\mu^{bd} c^d \varphi_i^c \right].
\end{aligned}
$$

Relating the GZ theory and YM theory

- The YM action is obtained by adding $S_1(\kappa = 1)$ to S_{GZ} as

$$
\begin{aligned}
S_{GZ} + S_1(\kappa = 1) &= \int d^4x [\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + B^a \partial_\mu A^a_\mu \\
&+ i\bar{c}^a \partial_\mu \mathcal{D}^{ab}_\mu c^b] \\
&\equiv S_{YM}
\end{aligned}
$$

Thus

$$
Z_{GZ} \left(= \int [D\phi] e^{-S_{GZ}} \right) \xrightarrow{\textit{Finite BRST}} Z_{YM} \left(= \int [D\phi] e^{-S_{YM}} \right),
$$

Conclusion

- We construct a generalized BRST transformation which connects the generating functional of GZ theory to the generating functional of YM theory.
- The Jacobian for path integral measure in the generating functional of GZ theory cancels the extra term in the GZ theory.
- Thus the theory which are free from Gribov copies (GZ theory) can be related to the theory with Gribov copies (YM theory).

THANK YOU