Finite Nilpotent Symmetry in Field/Antifield Formulation

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Plan of the Talk

- **Finite BRST Transformations.**
- **Field/Antifield formulation.**
- **Finite BRST in Field/Antifield formulation.**
- **Conclusions.**

BRST Transformation

BRST transformation for pure Yang-Mills theory:

$$\begin{aligned} \delta A_\mu^\alpha &= D_\mu^{\alpha\beta} c^\beta \delta\Lambda \\ \delta c^\alpha &= -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \delta\Lambda \\ \delta \bar{c}^\alpha &= \frac{F[A]}{\lambda} \delta\Lambda \end{aligned} \quad (1)$$

Properties

- S_{eff}^{FP} is symmetric under these transformations.
- The PI measure $\mathcal{D}A_\mu^\alpha \mathcal{D}c^\alpha \mathcal{D}\bar{c}^\alpha$ is invariant.
- Nilpotent, i.e. $\delta^2 A_\mu^\alpha = 0; \ \delta^2 c^\alpha = 0$

FFBRST Transformation

If we observe the usual BRST transformation and its properties carefully, we notice that the properties of infinitesimal BRST do not depend on whether,

- The parameter is finite or infinitesimal
- The parameter is field dependent or not

as long as the parameter is anti-commuting and explicit space-time independent. Therefore we have liberty to choose parameter finite and field dependent. We integrate the infinitesimal BRST to construct finite field dependent BRST [FFBRST]

FFBRST ...

(2)
$$\begin{aligned}\delta A_\mu^\alpha &= D_\mu^{\alpha\beta} c^\beta \Theta[A_\mu^\alpha, c^\alpha, \bar{c}^\alpha] \\ \delta c^\alpha &= -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \Theta[A_\mu^\alpha, c^\alpha, \bar{c}^\alpha] \\ \delta \bar{c}^\alpha &= \frac{F[A]}{\lambda} \Theta[A_\mu^\alpha, c^\alpha, \bar{c}^\alpha]\end{aligned}$$

Where $\Theta[A_\mu^\alpha, c^\alpha, \bar{c}^\alpha]$ is finite field dependent, anti-commuting and explicit x_μ independent parameter.

What do we mean by finite anti-commuting quantity?

Let us consider an example of $\Theta[A, c, \bar{c}]$,

(3)
$$\Theta[A, c, \bar{c}] = \int d^4y \bar{c}^\alpha(y) \partial \cdot A^\alpha[y]$$

FFBRST ....

Now if we calculate the Green's function of $\Theta[A, c, \bar{c}]$ between vacuum and a state with a ghost and a gauge field it has finite value [as opposed to infinitesimal].

And also Θ is nilpotent .

Properties of FFBRST

- **FFBRST is also Symmetry of the effective action, S_{eff}^{FP}.**
- **Nilpotent, $\delta^2 = 0$**
- **However the PI measure is NOT invariant FFBRST!!**

The measure,

$$\mathcal{D}A_\mu^\alpha \mathcal{D}c^\alpha \mathcal{D}\bar{c}^\alpha = \mathcal{D}A_\mu^{\alpha\prime} \mathcal{D}c^{\alpha\prime} \mathcal{D}\bar{c}^{\alpha\prime} J[A', c', \bar{c}']$$

Finite BRST

It has been shown that the Jacobian, $J[A, c, \bar{c}]$ **can always be replaced by** $e^{iS_1[A,c,\bar{c}]}$**. Where** $S_1[A, c, \bar{c}]$ **is local functions of fields and can be considered as part of the action.**

$$S^A_{eff} = S_1 + S_{eff}$$

An example:

$$\int \mathcal{D}A^\alpha_\mu \mathcal{D}c^\alpha \mathcal{D}\bar{c}^\alpha \; e^{iS^L_{eff}} \longrightarrow \int \mathcal{D}A^\alpha_\mu \mathcal{D}c^\alpha \mathcal{D}\bar{c}^\alpha \; e^{i(S^L_{eff}+S_1)}$$

FFBRST....

Where S_1 depends on the choice of the FFBRST parameter Θ. If we take a

$$\Theta[A, c, \bar{c}] = i\int d^4y\bar{c}^\alpha[\partial\cdot A^\alpha - \eta\cdot A^\alpha]$$

then S_1 is such that,

$$S^L_{eff} + S_1 = S^A_{eff}$$

We would like to show that this finite transformation could be an exact symmtry of some extended theory, like field/antifield formulations.

Field/Antifield formulation

Batalin & Vilkovisky addressed the question, given a set of classical fields $\phi^i(x)$ and a classical action $S[\phi^i]$, what is the full quantum action $W(\phi)$ such that the functional integral

$$Z[J] = \int \mathcal{D}\phi \, e^{iW(\phi)+J(\phi)}$$

determines all physical quantities.

To answer this question they extended the action, $W(\phi, \phi^*)$ by introducing antifields, ϕ^*, corresponding to each field ϕ with opposite statistic. Generically ϕ denotes all the fields involved in the theory. The sum of ghost number associated to a field and its antifield is equal to -1.

Field/Antifield formulation

This extended quantum action satisfies certain rich mathematical relation called quantum master equation

(4) $$\Delta e^{iW[\phi,\phi^*]} = 0 \ \textbf{with} \ \Delta \equiv \frac{\partial_r}{\partial\phi}\frac{\partial_r}{\partial\phi^*}(-1)^{\epsilon+1}.$$

Master equation reflects the gauge symmetry in the zeroth order of antifields and in the first order of antifields it reflects nilpotency of BRST transformation. This equation can also be written in terms of antibrackets as,

(5) $$(W, W) = 2i\Delta W,$$

Field/Antifield formulation

where the antibracket is defined as

$$(X,Y) \equiv \frac{\partial_r X}{\partial\phi}\frac{\partial_l Y}{\partial\phi^*} - \frac{\partial_r X}{\partial\phi^*}\frac{\partial_l Y}{\partial\phi}. \tag{6}$$

Different effective actions belonging to the same theory are solutions of master equations.

Field/Antifield formulation

The generating functional can be written as

$$Z[\Phi^\star] = \int D\Phi e^{iW_\Psi\left[\Phi, \Phi^* = \frac{\partial\Psi}{\partial\Phi}\right]}, \tag{7}$$

Ψ **is the gauge fixed fermion and has Grassman parity 1 and ghost number -1. The antifields are defined as**

$$\Phi^* = \frac{\delta\Psi}{\delta\Phi} \tag{8}$$

The generating functional, $Z[\Phi^*]$ **is independent of the choice of** Ψ

Field/Antifield formulation

We have constructed FFBRST transformation by chosing appropriate finite parameter in such a way that Jacobian contribution in the path integral only changes the gauge fixed fermions. We have shown this in 1-form as well as in 2-form gauge theories by considering several explicit examples.

Field/Antifield : In 1-form theory

The generating functional in 1-form gauge theory can be written compactly as,

(9) $$Z[\Phi^*] = \int D\Phi \exp\left[iW_\Psi(\Phi, \Phi^*)\right],$$

where

(10) $$W_\Psi = S_0(\Phi) + \delta_{brst}\Psi.$$

Ψ **is the gauge fixed fermion and can be written in this case as**

(11) $$\Psi = \int d^4x\ \bar{c}^\alpha \left[\frac{\lambda}{2} B^\alpha - \partial \cdot A^\alpha\right].$$

we apply FFBRST transformation given with the finite field dependent parameter $\Theta(A, c, \bar{c}, B)$ obtainable from

In 1- form theories

(12) $\Theta'(A, c, \bar{c}, B) = i \int d^4y \; \bar{c}^\alpha \left[\gamma_1 \lambda B^\alpha + (\partial \cdot A^\alpha - \eta \cdot A^\alpha)\right],$

The generating functional is changed to

(13) $$Z[\tilde{\Phi}^*] = \int D\Phi \exp\left[iW_{\Psi_1}(\Phi, \tilde{\Phi}^*)\right],$$

(14) $$W_{\Psi_1} = S_0(\Phi) + \delta_{brst}\Psi_1,$$

with $\tilde{\Phi}^* = \frac{\delta \Psi_1}{\delta \Phi}$**. The gauge fixed fermion is changed from** $\Psi \rightarrow \Psi_1$ **given as**

(15) $$\Psi_1 = \int d^4x \; \bar{c}^\alpha \left[\frac{\xi}{2} B^\alpha - \eta \cdot A^\alpha\right],$$

1-form theories ....

with $\xi = \lambda(1+2\gamma)$**. However** $Z[\tilde{\Phi}^*]$ **is independent of the choice of** Ψ

(16) $$Z[\Phi^*] \; - - - - - \overset{FFBRST}{- - - - - - - -} \longrightarrow Z[\tilde{\Phi}^*]$$

2-form gauge theory

The effective action for Abelian gauge theory for rank-2 antisymmetric tensor field $B_{\mu\nu}$ defined as

$$S = \int d^4x \left[\frac{1}{12} F_{\mu\nu\rho} F^{\mu\nu\rho} - i \partial_\mu \tilde{\rho}_\nu (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) + \partial_\mu \tilde{\sigma} \partial^\mu \sigma \right.$$
$$\left. + \beta_\nu (\partial_\mu B^{\mu\nu} + \lambda_1 \beta^\nu - \partial^\nu \varphi) - i \tilde{\chi} \partial_\mu \rho^\mu - i \chi (\partial_\mu \tilde{\rho}^\mu - \lambda_2 \tilde{\chi}) \right],$$

where $F_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$,
$B_{\mu\nu}$ is the antisymmetric tensor field of rank-2,
$(\rho_\mu, \tilde{\rho}_\mu)$ are anticommuting vector fields (ghost), $(\sigma_\mu, \tilde{\sigma}_\mu)$ are commuting scalar field, $(\chi, \tilde{\chi})$ are anticommuting scalar fields, and (β_μ, φ) are commuting vector and scalar field respectively.

Field/Antifield: 2-form gauge theory

The generating functional can be expressed compactly as

(17) $$Z[\Phi^*] = \int D\Phi \exp\left[iW_{\Psi_3}(\Phi, \tilde{\Phi}^\star)\right],$$

where

(18) $$W_{\Psi_3} = S_0(\Phi) + \delta_{brst}\Psi_3.$$

Ψ_3 is the gauge fixed fermion given as

$$\Psi_3 = -i\int d^4x\,[\tilde{\rho}_\nu\,(\partial_\mu B^{\mu\nu} + \lambda_1\beta^\nu) + \tilde{\sigma}\partial_\mu\rho^\mu + \varphi\,(\partial_\mu\tilde{\rho}^\mu - \lambda_2\tilde{\chi})]\,.$$

(19)

The antifields $\Phi^\star$ corresponding to generic field Φ for this particular theory can be obtained from the gauge fixed fermion using $\tilde{\Phi}^\star = \frac{\delta\Psi_3}{\delta\Phi}$.

2-Form gauge theory

Now we choose a FFBRST parameter corresponding to

$$\Theta' = \int d^4x \, [\gamma_1 \tilde{\rho}_\nu (\partial_\mu B^{\mu\nu} - \eta_\mu B^{\mu\nu} - \partial^\nu \varphi - \eta^\nu \varphi) + \gamma_2 \lambda_1 \tilde{\rho}_\nu \beta^\nu$$
$$+ \gamma_1 \tilde{\sigma} (\partial_\mu \rho^\mu - \eta_\mu \rho^\mu) + \gamma_2 \lambda_2 \tilde{\sigma} \chi] \quad (20)$$

and apply it to Z$[\Phi^\star]$ **to get** $Z[\tilde{\Phi}^\star]$ **where**

$$Z[\tilde{\Phi}^\star] = \int D\phi \exp[i W_{\Psi_4}(\Phi, \tilde{\Phi}^\star],$$
$$W_{\Psi_4} = S_0(\Phi) + \delta_{brst} \Psi_4,$$
$$\Psi_4 = -i \int d^4x \, [\tilde{\rho}_\nu \left(\eta_\mu B^{\mu\nu} + \lambda_1 \beta^\nu\right) + \tilde{\sigma} \eta_\mu \rho^\mu + \varphi \left(\eta_\mu \tilde{\rho}^\mu - \lambda_2 \tilde{\chi}\right)$$

FFAnti-BRST

- **Anti-BRST transformations are analogous to BRST transformations where the role of ghosts and anti-ghosts fields are interchanged**
- **Formal nilpotent symmetry transformations for the generating functionals in field/antifield formulation can also be constructed using FFanti-BRST transformations.**
- **FFanti-BRST transformations with finite field dependent parameter corresponding to**

$$\Theta' = -i\gamma \int d^4x\, c^\alpha(\partial \cdot A^\alpha - \partial_j A^{j\alpha}) \tag{21}$$

Finte anti-BRST

changes the gauge fixed fermion only and becomes the symmetry of the generating functional for 1-form theory in field/antifield formulation

$$Z[\Phi^*] \overset{FFBRST}{-------------\longrightarrow} Z[\tilde{\Phi}^*] \quad (22)$$

Conclusions

- FFBRST transformation which relates generating functionals in usual gauge theories are shown to be the symmetry of the generating functionals in the field/antifield formulation.
- The finite parameter is chosen in such a way that the Jacobian in the path integral measure is adjusted to change the gauge fixed fermions only.
- Results are shown to be valid in both 1-form and 2-form gauge theories.
- FFanti-BRST transformation is also constructed and it plays the same role as FFBRST