



# **Bulk Viscosity In Realistic Strange Stars**

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# Outline of the talk

*Strange  
Stars*

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graph TD; A([Strange Stars]) --> B(Bulk viscosity); A --> C(MIT Bag Model); C --> D(Realistic Model);
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*Spin Evolution*

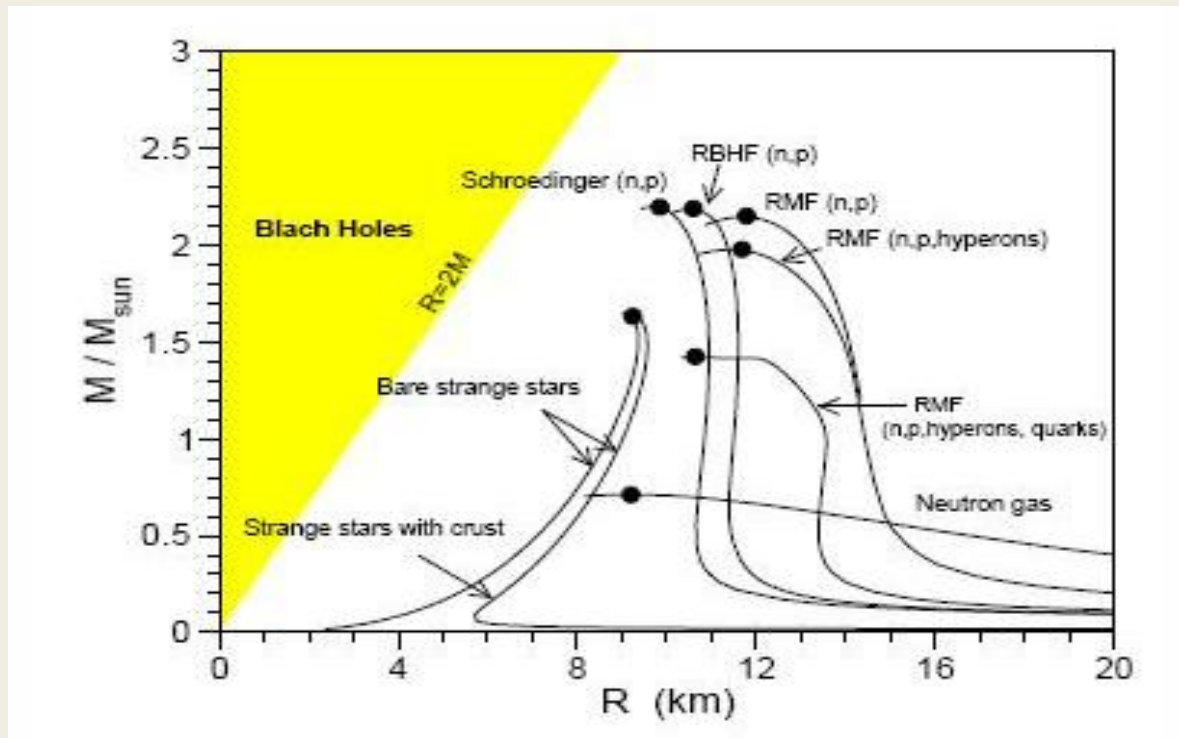
*Bulk viscosity*

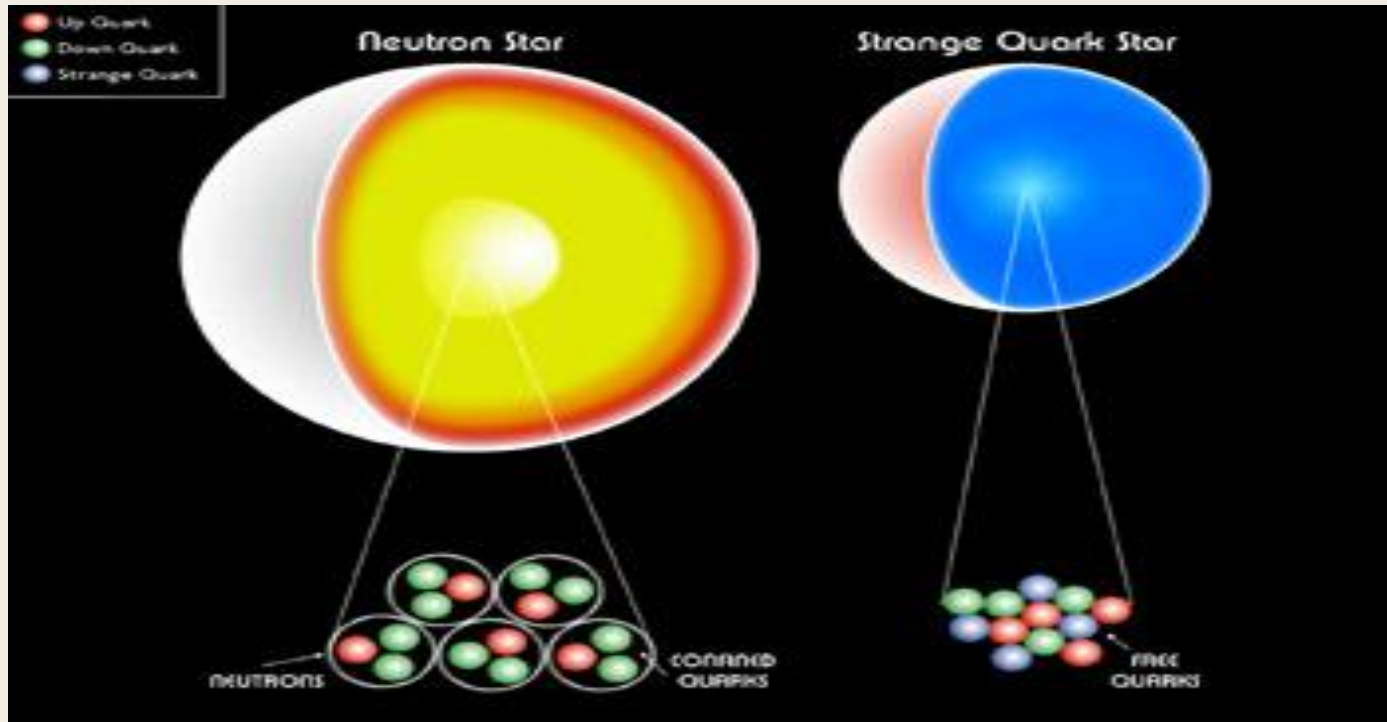
*MIT Bag Model*

*Realistic Model*

# Motivation

Observations to date cannot distinguish neutron stars(NS) from strange star(SS).





Bulk viscosity may play a decisive role

# Bulk Viscosity

change in volume/  
density of a fluid

destroys equilibrium  
configuration.

internal processes  
set up to restore  
the equilibrium

reactions are not  
reversible, thus  
increase entropy

dissipation of energy must be  
determined by bulk viscosity .

# The Bag Model

- **In this model the quarks are asymptotically free within a large bag.**
- **However, other crucial properties of QCD, e.g confinement, chiral symmetry breaking are not natural.**

# Realistic Model (ReSS)

Quarks are assumed to interact among themselves via a phenomenological potential

$$V_{ij} = \frac{12\pi}{27} \left[ \frac{1}{\ln \left( 1 + \frac{(k_i - k_j)^2}{\Lambda^2} \right)} - \frac{\Lambda^2}{(k_i - k_j)^2} + \frac{\Lambda'^2}{(k_i - k_j)^2} \right] \frac{1}{(k_i - k_j)^2}$$

In this potential confinement and asymptotic freedom are built in.

The screening effect in medium is incorporated by the screening length  $D$  where

$$(D^{-1})^2 = \frac{2\alpha_0}{\pi} \sum_{i=u,d,s} k_i^f \sqrt{(k_i^f)^2 + M_i^2} + 7.152\alpha_0 T$$

where  $k_i^f$  is the Fermi momentum of the  $i^{\text{th}}$  type of quarks and  $\alpha_0$  is the perturbative quark gluon coupling.

Quark masses depend on density

$$M_i = m_i + M_Q \operatorname{sech} \left( \nu \frac{n_B}{n_0} \right), \quad i = u, d, s$$

**Pressure P is then calculated as**

$$P(\varepsilon) = \sum (\mu_i n_i - \varepsilon_i) + (\mu_e n_e - \varepsilon_e)$$

The quarks satisfy  $\beta$ -equilibrium and charge-neutrality conditions.

The pressure  $P = 0$  at the minimum of  $E/A$ . This defines the surface of the star i.e at  $r = R$ ,  $P = 0$ .

Note: The energy density  $\varepsilon$  and chemical potentials  $\mu$  will have contributions from quarks (both kinetic & potential) as well as electrons (only kinetic).

## Bulk viscosity of SQM

Bulk viscosity results due to change in volume (i.e. in density) of a fluid.

Internal processes are set up in the element to restore the equilibrium.

As a result the reactions



**continue.**

The dissipation of energy is determined by the bulk viscosity  $\zeta$ .

# Coefficient of Bulk Viscosity $\zeta$

Lindblom L. and Owen B. J., 2002, PRD, 65, 063006

$$\zeta = - \frac{n \beta^2 \tau}{(1 + \omega \tau)^2} \left( \frac{\partial p}{\partial n_i} \right)_{n_B} \frac{\partial x_i}{\partial n_B}$$

$\omega$  the frequency of the perturbation,  $x_i$  the equilibrium value of number fraction.

$$\left( \frac{\partial p}{\partial n_i} \right)_{n_B} = n_j \alpha_{ij}$$

where

$$\alpha_{ij} = \left( \frac{\partial \mu_i}{\partial n_j} \right)_{n_{k \neq j}}$$

$\alpha_{ij}$ 's can be calculated numerically.

To get bulk viscosity, relaxation time  $\tau$  has to be calculated.  
 $\tau$  depends on the reaction by which equilibrium is achieved.

Nonleptonic reaction



When equilibrium is destroyed  $\delta\mu = \mu_d - \mu_s$

From number conservation  $\delta n_s = -\delta n_d$ ,  $\delta n_d$  independent variable.

Then the inverse of the relaxation time of the reaction is

$$\frac{1}{\tau} = \frac{\delta\Gamma}{\delta\mu} \frac{\delta\mu}{\delta n_d}$$

where  $\delta\Gamma$  is the difference between forward and reverse reaction rate

After tedious simplification, we get

$$\delta \Gamma = \frac{l}{192\pi^3} \langle |M|^2 \rangle k_{fs} (kT)^2 \delta \mu$$

where  $\langle |M|^2 \rangle$  is the angle averaged value of  $|M|^2$  and

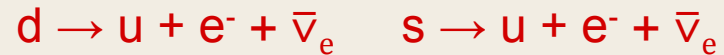
$$\sum |M_A|^2 = 32G_F^2 \sin^2 2\theta_c (k_d \cdot k_{u_i})(k_{u_f} \cdot k_s)$$

The change in the number density of any species could be expressed in terms of  $\delta n_d$ .

For the reaction under consideration,

$$\frac{\delta \mu}{\delta n_d} = \alpha_{dd} - \alpha_{ds} - \alpha_{sd} + \alpha_{ss} = \alpha \text{ (say)}$$

## Leptonic reactions



In  $d \rightarrow u + e^- + \bar{\nu}_e$ , we have  $\delta n_u = \delta n_e = -\delta n_d$ ,

$\delta n_d$  independent variable. Hence,

$$\delta\mu = \delta\mu_d - \delta\mu_u - \delta\mu_e$$

Thus

$$\frac{\delta\mu}{\delta n_d} = \alpha_{dd} - \alpha_{du} - \alpha_{ud} + \alpha_{uu} + \alpha_{ee}$$

Now, for the reaction  $s \rightarrow u + e^- + \bar{\nu}_e$

the result will be similar, only **d** quark replaced by **s** quark.

Thus, we finally get

$$\delta \Gamma = \frac{17}{480\pi^3} G_F^2 \cos^2 \theta_c \mu_u \mu_d \mu_e (kT)^2 \delta \mu \quad S$$

where  $S = \left[ 1 - \frac{X_d - X_u - X_e}{2 \mu_u \mu_e} \right]$

with  $X_{i=} (\mathbf{k}_{fi})^2$

## Results

Parameters used:

$$\Lambda = 100 \text{ fm}^{-1}, M_q = 310 \text{ MeV}, m_u = 4 \text{ MeV}, m_d = 7 \text{ MeV}, m_s = 150 \text{ MeV}.$$

With all these parameters EoS is derived.

$\tau$  is computed for both non-leptonic and leptonic reactions.

Next, we compute the value of  $\zeta$ .

Here we need  $\omega$  the frequency of the external perturbation.

Assume  $\omega \sim$  the frequency of  $l=2$  and  $m=2$  mode of r-mode oscillation,

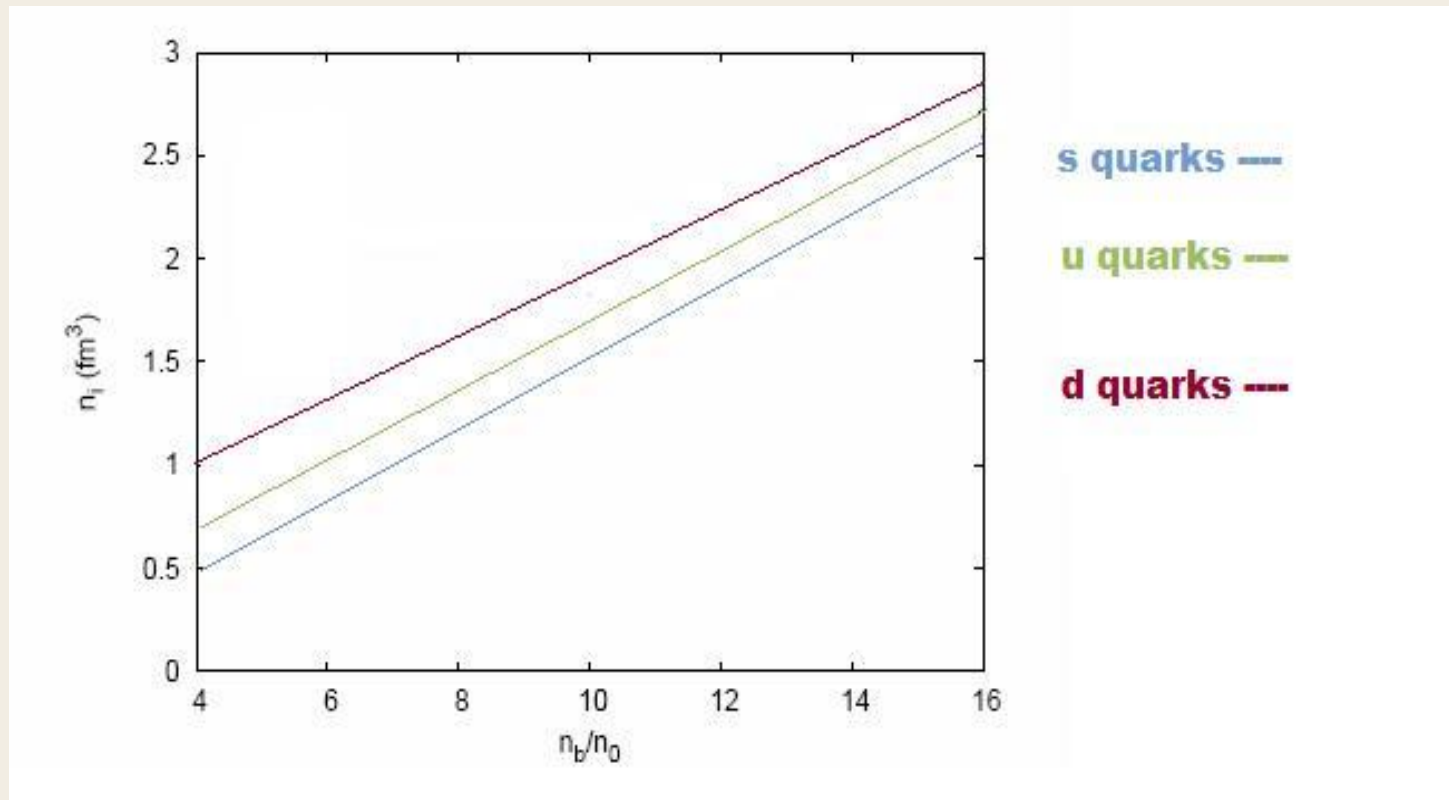
$\therefore \omega = 2/3 \Omega$ , where  $\Omega = 3000 \text{ s}^{-1}$  is the rotational frequency of the star.

# Different Parameters of EoS from the ReSS model.

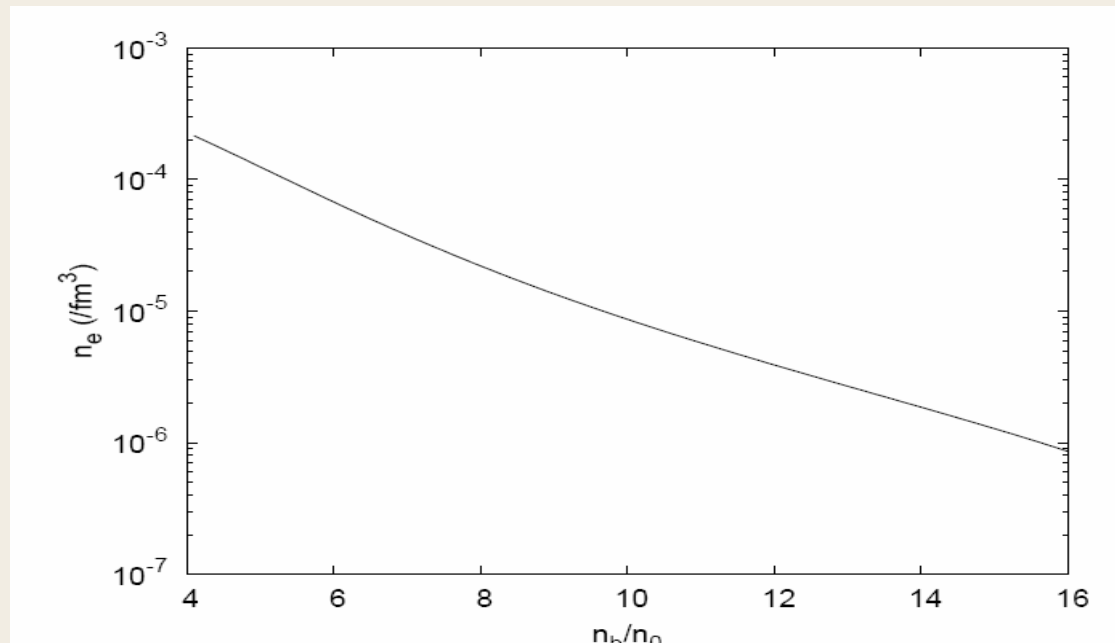
EoS	$\nu$	$M_Q$ (MeV)	$\square$ (MeV)	$\alpha_0$	$n_s/n_0$	$\epsilon_{\min}$ (MeV)
EoS1	0.333	310	100	0.20	4.586	888
EoS2	0.333	310	100	0.25	4.595	911
EoS3	0.286	310	100	0.20	5.048	926

$n_0 = 0.17 \text{ fm}^{-3}$  is the baryonic density of normal matter

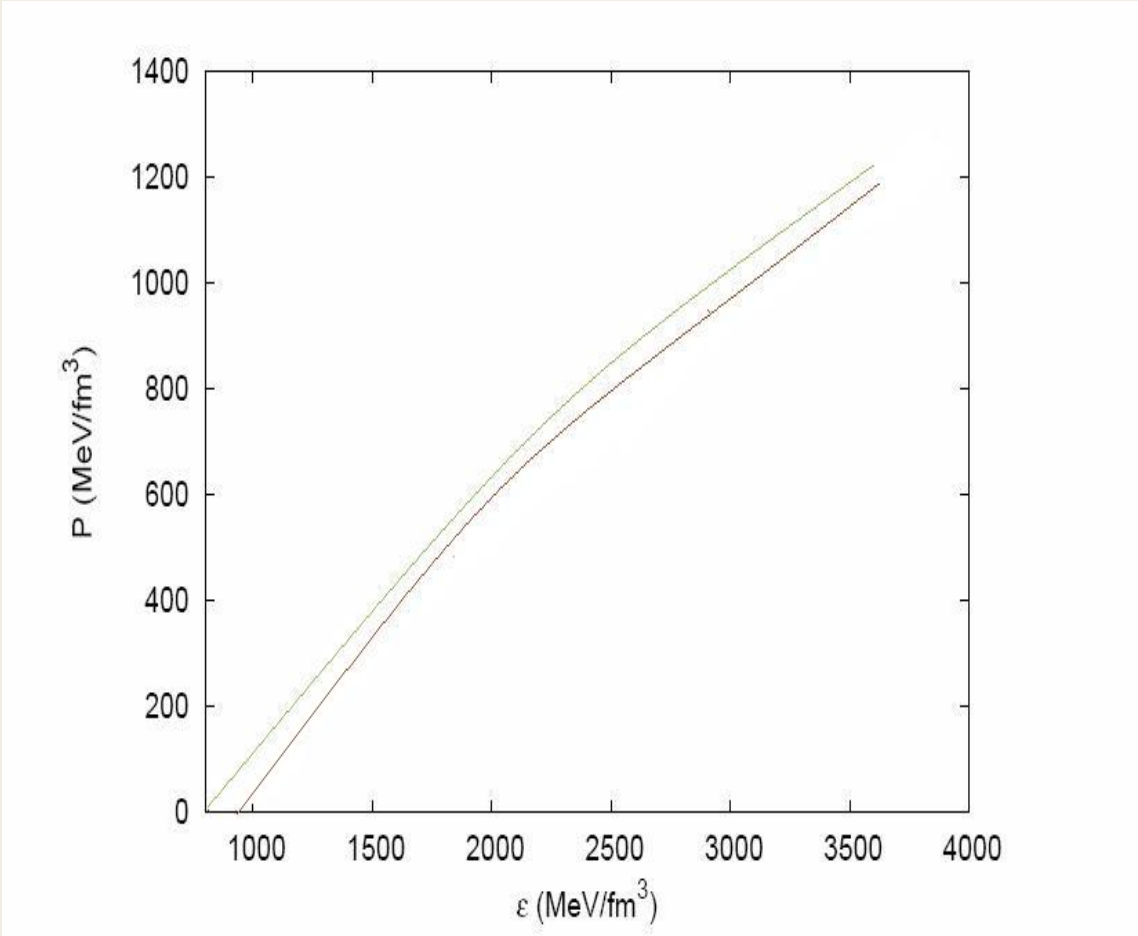
## Variation of number density of baryons inside ReSS



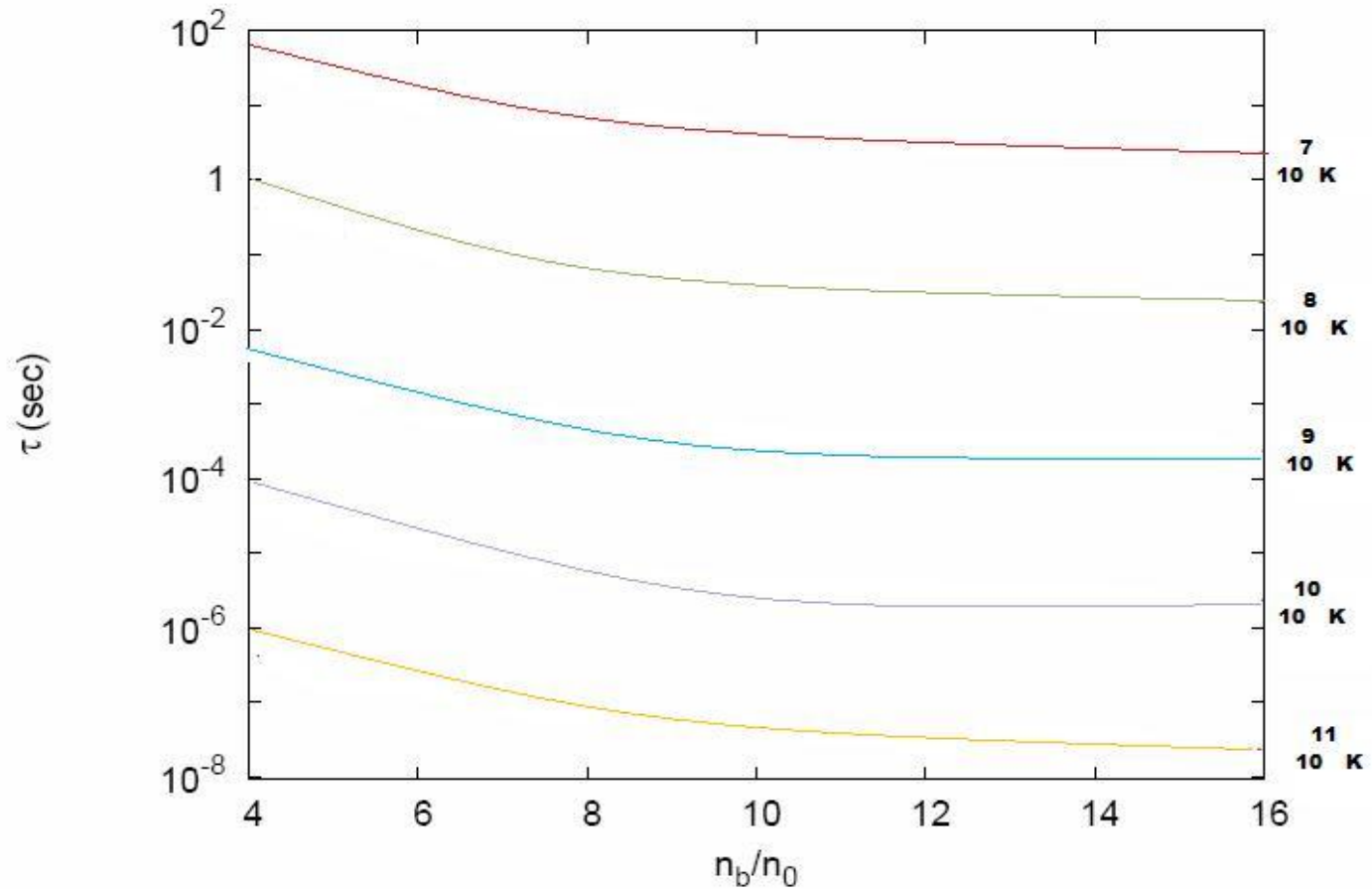
## Variation of number density of electrons inside ReSS



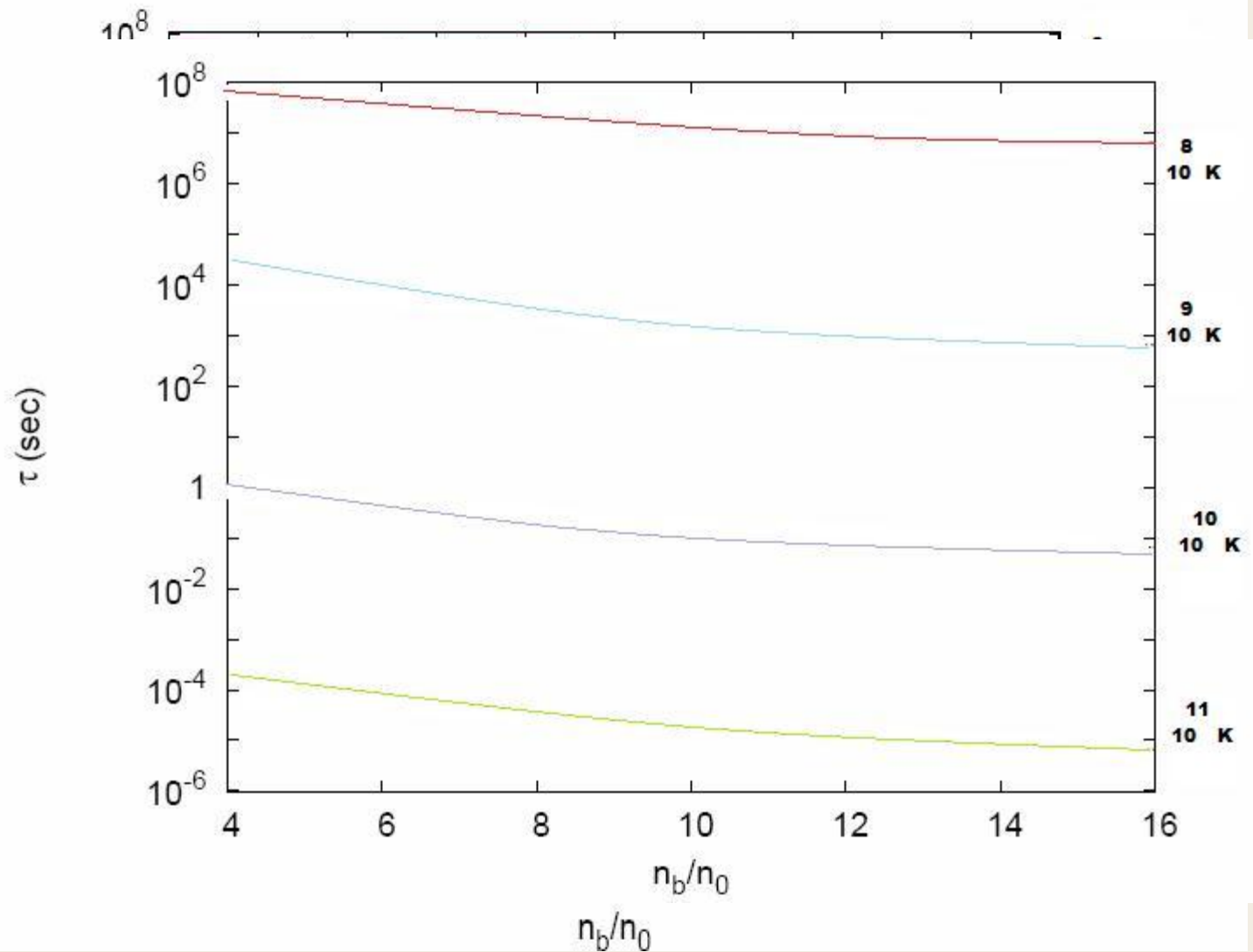
# Variation of pressure with energy density inside ReSS



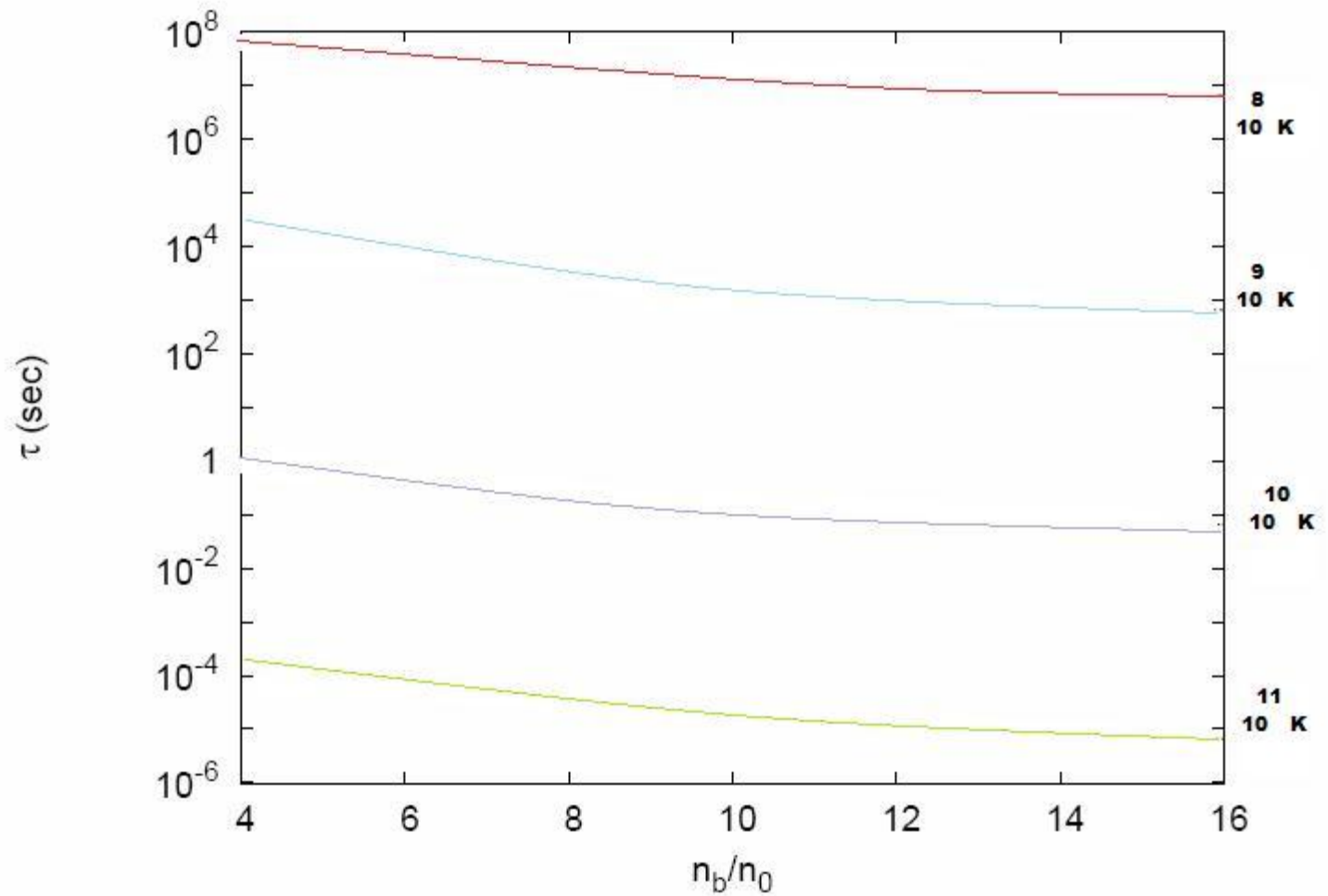
# Relaxation time for nonleptonic process



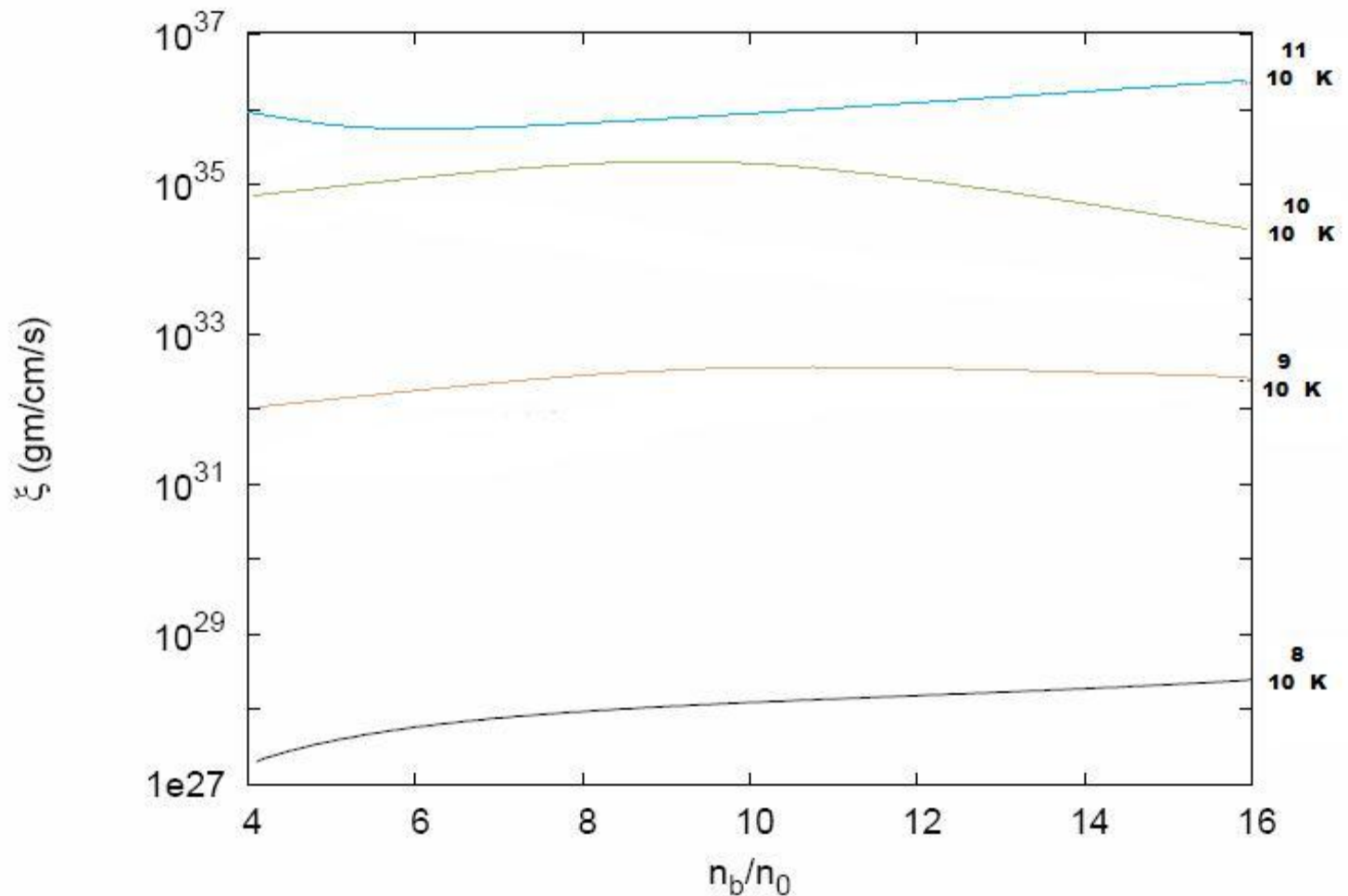
# Relaxation time for leptonic process ( with d quarks)



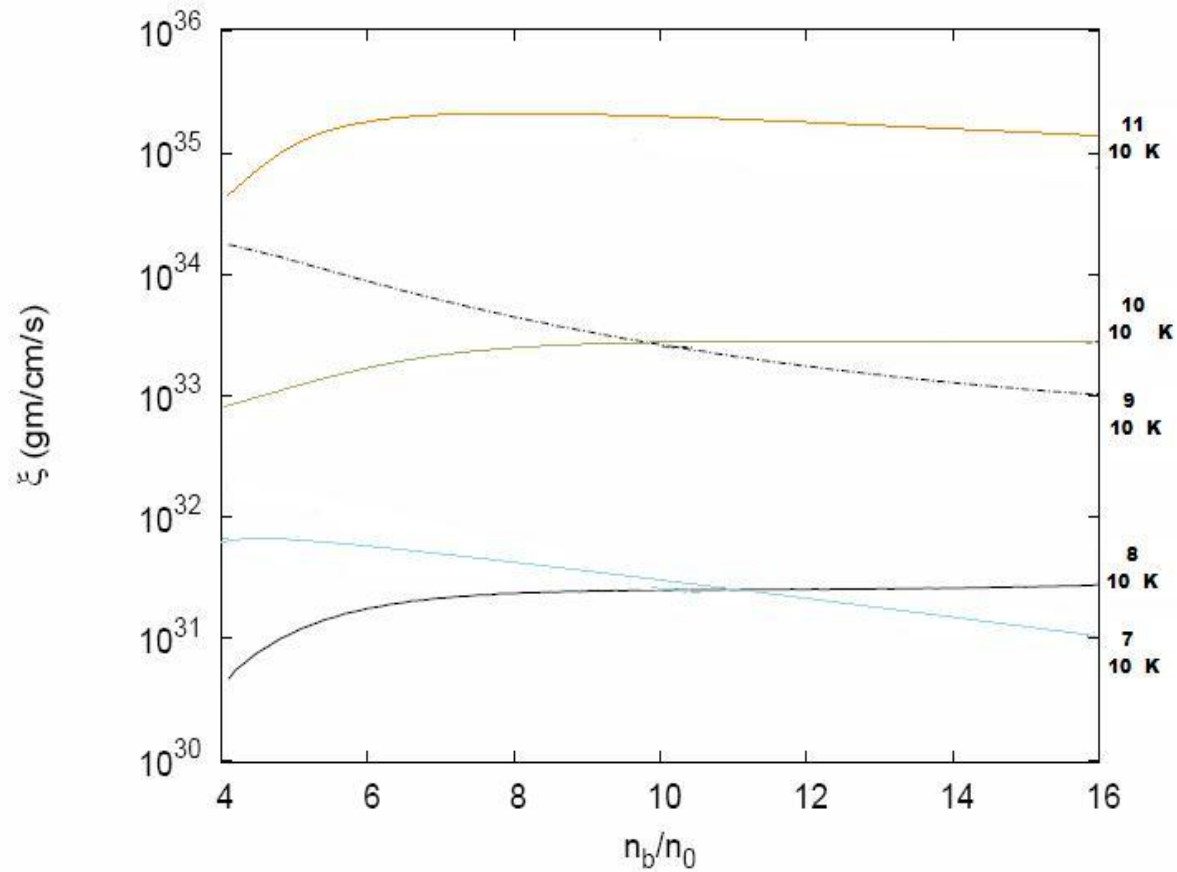
# Relaxation time for leptonic process ( with s quarks)



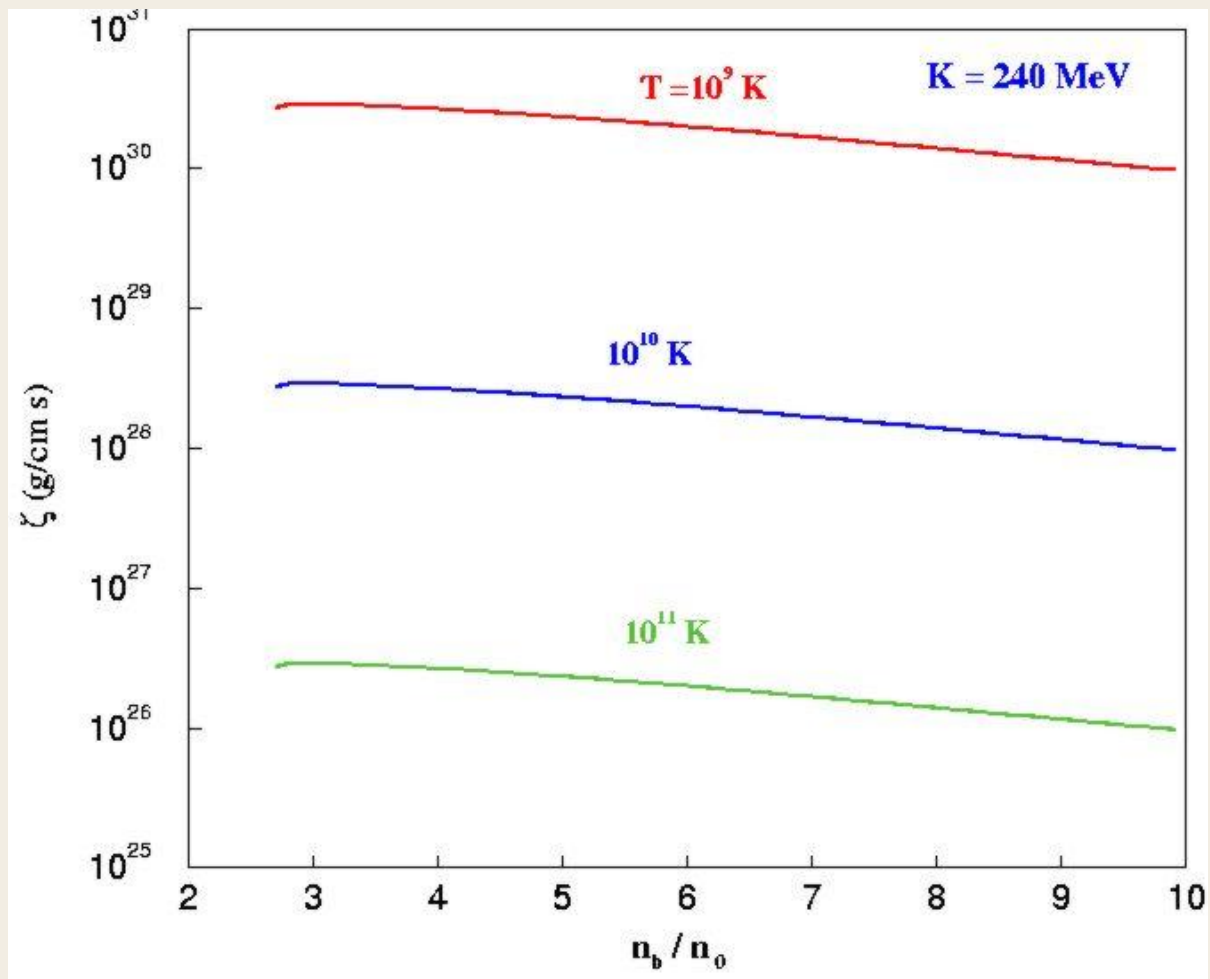
# Bulk viscosity for leptonic process



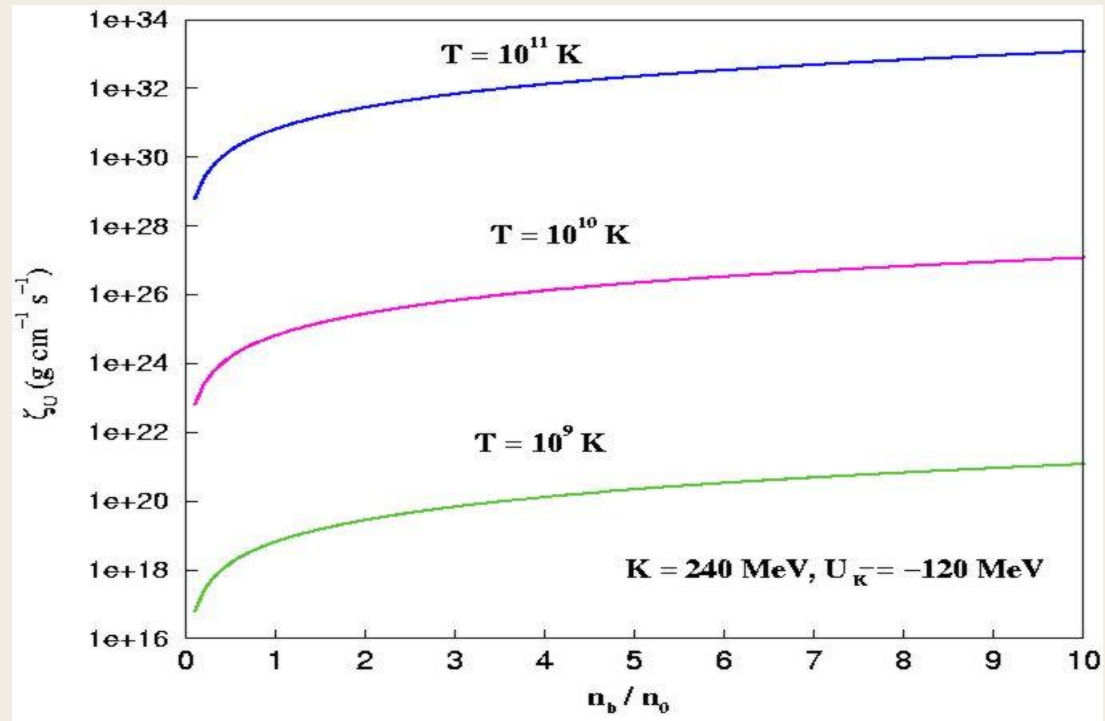
# Bulk viscosity for non-leptonic process



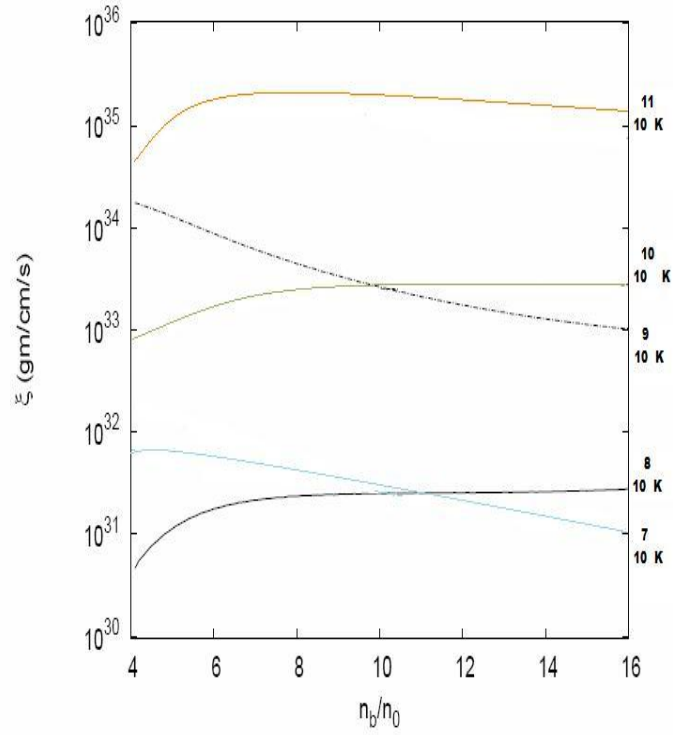
# Bulk viscosity coefficient for non leptonic process :Bag Model



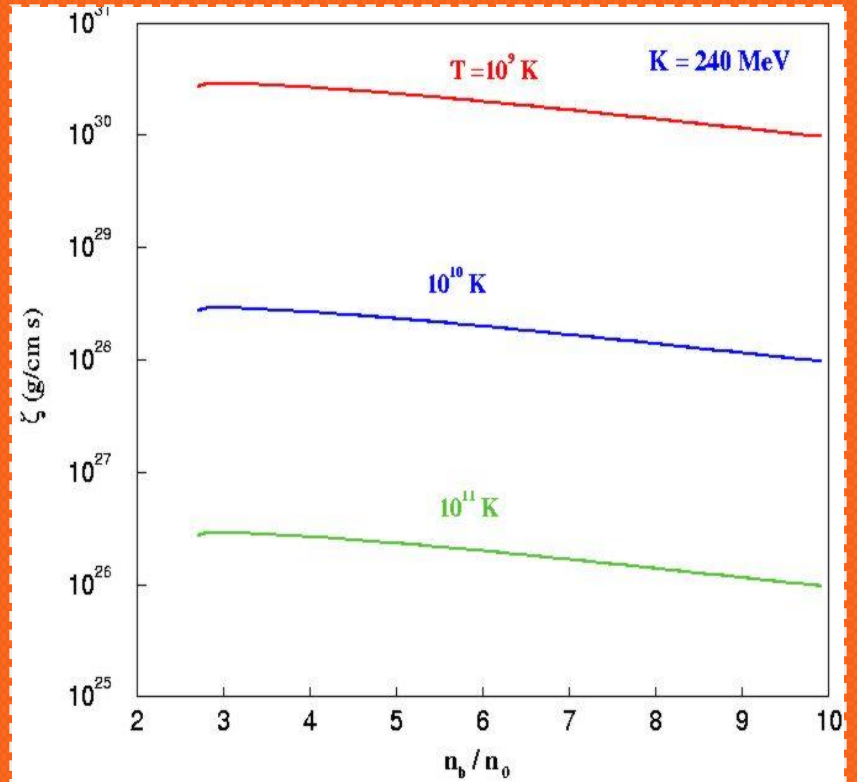
# Bulk viscosity for leptonic process: Bag Model



## ReSS Model

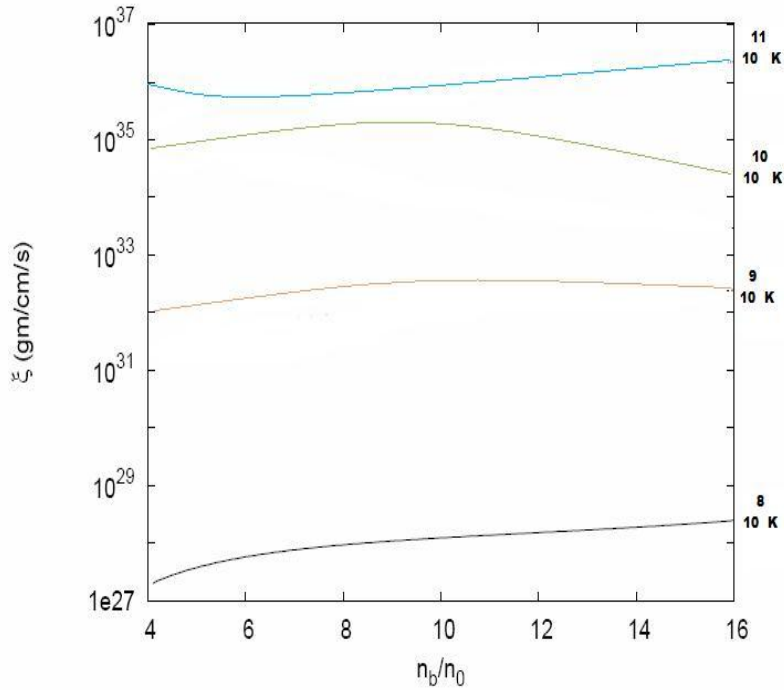


## Bag Model

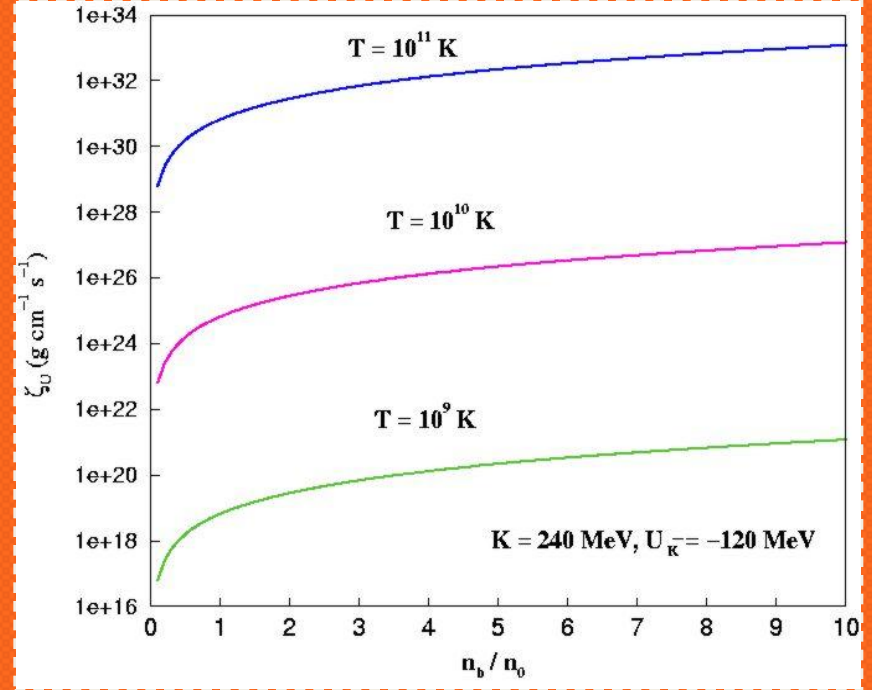


# Bulk Viscosity : Non leptonic

## ReSS Model



## Bag Model



# Bulk Viscosity :leptonic case

# Conclusions

- *The bulk viscosity coefficient calculated from ReSS model (due to the weak process as well as strong process) is at least two orders of magnitude larger than that obtained from MIT Bag Model.*

*Bulk viscosity may play an important role.*

# Work In Progress

