Scale Invariance and Cosmological Constant Problem

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History

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu}, \tag{1}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}, \tag{2}$$

- 1930 λ : Removed
- 1960 λ again was introduced to describe the dynamical nature of the universe.

The Problem

$$< T_{\mu\nu} > \quad = \quad - < \rho > g_{\mu\nu} \tag{3}$$

$$\lambda_{effective} \quad = \quad \lambda + 8\pi G < \rho > \tag{4}$$

where $\lambda_{effective}$ is effective cosmological constant.

$$H_0 = 2.133h \times 10^{-42} GeV^4, \frac{|k|}{a^2} \leq H_0^2, |\rho - < \rho > | \leq \frac{3H_0^2}{8\pi G}. \tag{5}$$

Using FRW metric the Einstein equation gives,

$$\lambda_{effective} \leq H_0^2, \tag{6}$$

$$\rho_v \leq 10^{-47} GeV^4, \tag{7}$$

We get very large value for $< \rho >$ from QCD and electroweak theory.

Why Scale Invariance?

- We impose scale invariance since it does not permit a cosmological constant term in action.
- Hence it might impose some constraints on the cosmological constant in the full quantum theory .
- We construct a theory where scale invariance is broken spontaneously.
- In this case scale transformations may not be anomalous.

References:

1. P. Jain et al.
2. F. Englert, C. Truffin and R. Gastmans
3. M. Shaposhnikov and D. Zenhausern.

Global Scale Invariance

$$\phi(x) \rightarrow \phi'(x') = \frac{1}{\Lambda}\phi(x), \quad (8)$$

$$x^\mu \rightarrow x'^\mu = \Lambda x^\mu, \quad (9)$$

$$A_\mu(x) \rightarrow A'_\mu(x') = \frac{1}{\Lambda}A_\mu(x), \quad (10)$$

$$L_0 = \frac{1}{2}g^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} \quad (11)$$

$$L_0 = \frac{1}{\Lambda^4}L_0 \quad (12)$$

$$S = \frac{1}{2}\int L_0|g|^{1/2}d^4x \rightarrow invariant \quad (13)$$

Global Scale Transformations in d dimension

Scale Transformation $\longrightarrow$ Pseudo Scale Transformation + General Coordinate Transformation

$$x^\mu \longrightarrow x^\mu, \tag{14}$$

$$\phi \longrightarrow \frac{\phi}{\Lambda}, \tag{15}$$

$$g^{\mu\nu} \longrightarrow \frac{g^{\mu\nu}}{\Lambda^{(b(d))}}, \tag{16}$$

$$\psi \longrightarrow \frac{\psi}{\Lambda^{c(d)}} \tag{17}$$

$$A_\mu \longrightarrow A_\mu \tag{18}$$

$$where, \tag{19}$$

$$b(d) = \frac{4}{d-2}, \tag{20}$$

$$c(d) = \frac{d-1}{d-2} \tag{21}$$

Theory in d dimension

$$\begin{aligned}\mathcal{S} &= \int d^dx\sqrt{-g}\Big[\frac{\beta}{8}\chi^2R + \frac{1}{2}g^{\mu\nu}(\partial_\mu\chi)(\partial_\nu\chi) - \frac{1}{4}\lambda\chi^4(\chi^2)^{-\delta} \\ &+ \mathcal{L}_{Matter}\Big]\end{aligned} \tag{22}$$

where $\delta = (d-4)/(d-2)$,

$$\begin{aligned}R^\mu_{\nu\alpha\beta} &= -\partial_\beta\Gamma^\mu_{\nu\alpha} + \partial_\alpha\Gamma^\mu_{\nu\beta} + \Gamma^\mu_{\gamma\alpha}\Gamma^\gamma_{\nu\beta} - \Gamma^\mu_{\gamma\beta}\Gamma^\gamma_{\nu\alpha}, \\ R_{\nu\beta} &= R^\mu_{\nu\beta\mu}\ , \qquad R = R_{\nu\beta}g^{\nu\beta}.\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{L}_{Matter} &= g^{\mu\nu}(D_\mu\mathcal{H})^\dagger(D_\nu\mathcal{H}) + \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{\lambda_1}{4}(\mathcal{H}^\dagger\mathcal{H} - \lambda_2\Phi^2)^2(\Phi^2)^{-\delta} \\ &- \frac{1}{4}g^{\mu\nu}g^{\alpha\beta}(\mathcal{A}^i_{\mu\alpha}\mathcal{A}^i_{\nu\beta} + \mathcal{B}_{\mu\alpha}\mathcal{B}_{\nu\beta} + \mathcal{G}^j_{\mu\alpha}\mathcal{G}^j_{\nu\beta})(\Phi^2)^\delta + \mathcal{L}_{\rm fs},\end{aligned} \tag{24}$$

$$\mathcal{L}_{\rm fermions} = \left(\overline{\Psi}_{\rm L}i\gamma^\mu\mathcal{D}_\mu\Psi_{\rm L} + \overline{\Psi}_{\rm R}i\gamma^\mu\mathcal{D}_\mu\Psi_{\rm R}\right) - \left[g_Y\overline{\Psi}_{\rm L}\mathcal{H}\Psi_{\rm R}(\Phi^2)^{-\delta/2} + h.c.\right], \tag{25}$$

The covariant derivative of fermion fields:

$$\mathcal{D}_\mu \Psi_{\rm L,R} = \left(\tilde{D}_\mu + \frac{1}{2}\omega_\mu^{ab}\sigma_{ab}\right)\Psi_{\rm L,R} \ , \tag{26}$$

and spin connection can be written as,

$$\omega_{\mu ab} \ = \ \frac{1}{2}(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu})e_a^{\ \nu} - \frac{1}{2}(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu})e_b^{\ \nu} - \frac{1}{2}e_a^{\ \rho}e_b^{\ \sigma}(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho})e^c_{\ \mu} \ . \tag{27}$$

Dimensionful Parameters

G	The Gravitational Constant,
M	Masses of particles,
ρ	Energy density corresponding to cosmological constant.
???	In Scale Invariant Theory.

Cosmological Symmetry Breaking

$$\Phi(x,t) = \eta(t) + \phi(x,t) \tag{28}$$

All dimension ful parameters are generated by cosmological symmetry breaking.

cosmological constant by CCB

$$\frac{\beta\chi^2 R}{8} + \frac{1}{2}g^{\mu\nu}(\partial_\mu\chi)(\partial_\nu\chi) - \frac{1}{4}\lambda\chi^4(\chi^2)^{-\delta}$$

we expand:

$$\chi = \chi_0 + \hat{\chi}, \tag{29}$$

- The value of λ can be set by comparing the value of observed value of cosmological constant.

Electroweak Scale

$$V_M = -\frac{\lambda_1}{4}(\mathcal{H}^\dagger\mathcal{H} - \lambda_2\Phi^2)^2(\Phi^2)^{-\delta} \tag{30}$$

$$\mathcal{H}^\dagger\mathcal{H} = \lambda_2\Phi^2 \tag{31}$$

We assume that classically,

$$\Phi = \Phi_0 \tag{32}$$

where Φ_0 is a dimensionful parameter. We denote the classical Higgs field by $\mathcal{H}_0$. We may express it as,

$$\mathcal{H}_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \tag{33}$$

where v is the electroweak symmetry breaking scale. We shall choose λ_2 and Φ_0 such that we generate the correct electroweak scale.

Method

$$\mathcal{S} = \int d^d x \sqrt{-g} \left(\frac{\lambda}{8\pi G} \right) \quad (34)$$

$$< 0 | T^{\mu}_{\mu} | 0 > \propto 4\lambda \quad (35)$$

- The value of trace of energy momentum tensor of fields in the vacuum gives the contribution to cosmological constant for corresponding fields.
- We do the theory in d dimension with scale invariance so we preserve the symmetry up to full quantum theory. So, any result valid for classical field is valid for quantum fields as well.

Equation of Motions

$$D^\mu D_\mu \mathcal{H} = -\frac{\lambda_1}{2}(\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)\mathcal{H}(\Phi^2)^{-\delta} - g_Y \bar{\Psi}_{\rm R} \Psi_{\rm L} (\Phi^2)^{-\delta/2} \tag{36}$$

The scalar field Φ equation of motion is given by,

$$\begin{aligned} \Phi \partial^\mu \partial_\mu \Phi &= \lambda_1 \lambda_2 (\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)\Phi^2 (\Phi^2)^{-\delta} + \frac{\lambda_1 \delta}{2}(\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \Phi^2)^2 (\Phi^2)^{-\delta} \\ &- \frac{\delta}{2} B^{\alpha\mu} B_{\alpha\mu} (\Phi^2)^{\delta} + \delta\, g_Y \left[\overline{\Psi}_{\rm L} \mathcal{H} \Psi_{\rm R} (\Phi^2)^{-\delta/2} + h.c. \right] \end{aligned} \tag{37}$$

Finally the equation of motion for the fermion fields may be written as,

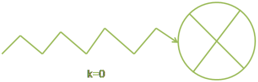
$$i\gamma^\mu D_\mu \Psi_{\rm R} - g_Y \mathcal{H}^\dagger \Psi_{\rm L} (\Phi^2)^{-\delta/2} = 0, \tag{38}$$

$$i\gamma^\mu D_\mu \Psi_{\rm L} - g_Y \mathcal{H} \Psi_{\rm R} (\Phi^2)^{-\delta/2} = 0. \tag{39}$$

Matter Energy Momentum Tensor

$$
\begin{aligned}
T_{\alpha\beta} \quad = \quad & -g_{\alpha\beta}\mathcal{L} + (D_\alpha\mathcal{H})^\dagger(D_\beta\mathcal{H}) + (D_\beta\mathcal{H})^\dagger(D_\alpha\mathcal{H}) \\
& +\partial_\alpha\Phi\partial_\beta\Phi - g^{\mu\nu}B_{\alpha\mu}B_{\beta\nu}(\Phi^2)^\delta \\
& +\frac{1}{2}\bar{\Psi}_{\rm L}i\gamma_\alpha D_\beta\Psi_{\rm L} + \frac{1}{2}\bar{\Psi}_{\rm R}i\gamma_\alpha D_\beta\Psi_{\rm R} + \frac{1}{2}\bar{\Psi}_{\rm L}i\gamma_\beta D_\alpha\Psi_{\rm L} + \frac{1}{2}\bar{\Psi}_{\rm R}i\gamma_\beta D_\alpha\Psi_{\rm R} \\
& -\frac{1}{4}(\overline{\psi_L}i\gamma_\beta\psi_L)_{;\alpha} - \frac{1}{4}(\overline{\psi_L}i\gamma_\alpha\psi_L)_{;\beta} - \frac{1}{4}(\overline{\psi_R}i\gamma_\beta\psi_R)_{;\alpha} - \frac{1}{4}(\overline{\psi_R}i\gamma_\alpha\psi_R)_{;\beta} \qquad (40)
\end{aligned}
$$

Result in the Vacuum



$$< 0|T^{\mu}_{\mu}|0 >^{Matter} = 0, \tag{41}$$

- Here we have used the equation of motion of fields.

Summary and Future Work

- There is no fine tuning problem as we get null contribution from the matter sector.The result is valid for full quantum theory.
- Electroweak masses are generated by spontaneous symmetry breaking.
- Planck constant and Cosmological constant is generated by cosmological symmetry breaking
- We are extending the theory for local scale invariance.