Gauge Invariances in Non-Commutative Theories

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Outline

Non-Commutativity

- Arises in perturbative string theory.
- Dynamics of D-branes in a constant background magnetic field can be described by a non-commutative gauge theory.

- First introduced by *H S Snyder* to regulate short-distance singularities in Quantum field theory
- This structure in spacetime introduces a lower bound in the continuity of space-time.

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In this talk

- We look at some classical aspects (origins) of Non-commutative theories
- Are there any classical systems, which when quantised can give rise to non-commutativity in co-ordinates?
- If such systems exist, what can you do with them?
- Existence of such systems are indicated by Lagrangians

Some Literature

1. Lagrangians can be written down.
2. They indicate the corresponding theories are constrained systems.
3. One earlier paper ⟶ `R Jackiw`

   "*Physical Instances of Non-Commuting Co-ordinates* "

   NP B (Proc. Suppl.) 108 (2002) 30-36
4. `A A Deriglazov`

   "*Non-commutative Version of an Arbitrary Nondegenerated Mechanics* "

   hep-th 0208072 (2002)
5. Very recent paper ⟶ `F S Benfica & H O Girotti`

   "*Non-commutative Quantum Mechanics as a Gauge Theory* "

   PR D 798 (2009) 125024

Our Work

1. We look at two Lagrangians (one already considered in above paper)
2. Non-Commutativity of co-ordinates will be reproduced in different ways.
3. Using Dirac brackets.
4. Equivalent gauge theories are generated.
5. The Moyal product also naturally arises in these gauge theories.

What is non-commutativity?

Suppose $x^i, \quad i = 1, 2, \ldots$ are the co-ordinates for a system.

Then non-commutativity $\implies$ the commutator

$$\left[\hat{x}^i, \hat{x}^j\right] = i\theta^{ij}$$

where θ^{ij} is an antisymmetric constant matrix.

Continuity of corresponding space breaks down. An uncertainty relation is induced,

$$\Delta x^i \Delta x^j \geq i\theta_{ij}$$

In the space defined by such non-commuting co-ordinates, functions do not obey the usual product rule

$$(fg)(x) \neq f(x)g(x)$$

The relevant product is the Moyal product

$$(f * g)\,(x) = e^{i\theta^{ij}\partial_i^1\partial_j^2} f(x_1)g(x_2)|_{x_1=x_2}$$

1st Lagrangian : Equations of Motion & Phase Space

Consider the Lagrangian in first order form,

$$\mathtt{L} = v_i \dot{q}^i - h_0(q^i, v_j) + \dot{v}_i \theta^{ij} v_j \quad i, j = 1, \ldots, \mathtt{N}$$

- First given by A A Deriglazov
    - $q^i, \; v_j$ are configuration space variables
    - Equations of Motion

$$\dot{q}^i = 2\theta^{ij} \dot{v}_j + \frac{\partial h_0}{\partial v_i} \quad \mathtt{and} \quad \dot{\mathrm{v}}_i = -\frac{\partial h_0}{\partial q^i}$$

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    - The canonical momenta

$$p_i = \frac{\partial \mathtt{L}}{\partial \dot{q}^i} = v_i \quad \mathtt{and} \quad \pi^i = \frac{\partial \mathtt{L}}{\partial \dot{v}_i} = \theta^{ij} v_j$$

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    - No velocities, so constraints arise

Constraints & the Hamiltonian

- $\phi_i = (p_i - v_i) \approx 0$ *and* $\psi^i = \pi^i - \theta^{ij} v_j \approx 0$
- These define a surface $\sum_{\mathrm{2N}}$ in phase space
- These are second class constraints $\Longrightarrow$

$$\{\phi_i, \phi_j\} = 0, \ \left\{\phi_i, \psi^j\right\} = -\delta_i^{\ j}, \ \left\{\psi^i, \psi^j\right\} = -2\theta^{ij}$$

- Canonical Hamiltonian

$$\mathrm{H}_0 = h_0 + (p_i - v_i)\dot{q}^i + (\pi^i - \theta^{ij} v_j)\dot{v}_i$$

- Total Hamiltonian

$$\mathrm{H}_{\mathrm{T}} = h_0 + \left(\frac{\partial h_0}{\partial v_i} - 2\theta^{ij}\frac{\partial h_0}{\partial q^j}\right)\phi_i - \frac{\partial h_0}{\partial q^i}\psi^i$$

- Hamiltonian equations of motion same as Lagrange equations.

Dirac Brackets

- Defined whenever 2$^{\rm nd}$ class constraints are present
- $$\{A, B\}_{DB} = \{A, B\} - \{A, \Phi_\alpha\}\, \mathtt{C}^{-1}_{\alpha\beta}\, \{\Phi_\beta, B\}$$

  where $\Phi_\alpha \equiv \left(\phi_i, \psi^i\right)$ and $\mathtt{C}_{\alpha\beta} = \{\Phi_\alpha, \Phi_\beta\}$

  Here $\mathtt{C} = \begin{pmatrix} 0 & -\mathtt{I} \\ \mathtt{I} & -2\theta \end{pmatrix}$ and $\mathtt{C}^{-1} = \begin{pmatrix} -2\theta & \mathtt{I} \\ -\mathtt{I} & 0 \end{pmatrix}$
- $$\{q^i, q^j\}_{DB} = -2\theta^{ij}$$

  $\{q^i, v_j\}_{DB} = \delta^i{}_j,\ \{q^i, p_j\}_{DB} = -2\theta^i{}_j,\ \{q^i, \pi^j\}_{DB} = -\theta^{ij}$
- Transition to Quantum Mechanics $\Longrightarrow$ $[\hat{q}^i, \hat{q}^j] = -2i\theta^{ij}$
- First given by A A Deriglazov
- Truly independent variables — (q^i, p_i)

Equivalent Gauge Theory

- Original System has no gauge invariance $\Longrightarrow$ no 1^{st} class constraints
- Can introduce gauge invariance here, and can reproduce non-commutativity in a different way
- Look at $\{\phi_i, \phi_j\} = 0, \{\phi_i, \psi^j\} = -\delta_i^{\,j},\ \{\psi^i, \psi^j\} = -2\theta^{ij}$
- The ϕ_i form a *first class* constraint subset.
- The ψ^i form a *gauge-fixing-like* constraint subset.
- Redefine ϕ_i as $\chi_i = \left(\{\phi, \psi\}^{-1}\right)_i^{\,j} \phi_j = -\phi_i,$
- **Discard** (no longer consider) the ψ^i as constraints.
- We have a new (smaller) constraint surface $\sum_{\mathrm{N}}$, defined by only $\chi_i \approx 0$.

Gauge Invariant Hamiltonian & Other Variables

- On this $\sum_{\mathrm{N}}$, $\mathtt{H}_{\mathtt{T}}$ no longer ensures time-independence of χ_i.
- $\mathtt{H}_{\mathtt{T}}$ must be *modified* to $\widetilde{\mathtt{H}}_{\mathtt{T}}$, such that $\{\chi_i, \widetilde{\mathtt{H}}_{\mathtt{T}}\} = 0$
- We get $\widetilde{\mathtt{H}}_{\mathtt{T}} = e^{-\psi^i \partial_{q^i}} h_0$. where $\partial_{q^i} = \dfrac{\partial}{\partial q^i}$
- Other gauge invariant variables are $\widetilde{\mathtt{A}} =: e^{-\psi^i \hat{\chi}_i} : \mathtt{A}$

$$\Rightarrow \widetilde{q}^i = q^i - \psi^i; \quad \widetilde{v}_i = v_i; \quad \widetilde{p}_i = p_i; \quad \widetilde{\pi}^i = \pi^i - \psi^i$$

- All these have zero Poisson brackets with the χ_i
- Equivalent gauge theory given by the $\chi_i \approx 0$ and $\widetilde{\mathtt{H}}_{\mathtt{T}}$
- Gauge transformations are generated by the χ_i

Non-commutative co-ordinates once again

- The Poisson brackets between the gauge invariant quantities

$$\left\{\tilde{q}^i, \tilde{q}^j\right\} = -2\theta^{ij}; \quad \left\{\tilde{q}^i, \tilde{p}_j\right\} = \delta^i{}_j; \quad \left\{\tilde{q}^i, \tilde{\pi}^j\right\} = -\theta^{ij}$$

- On quantising, the gauge-invariant $\tilde{q}^i$ go over to non-commuting operators!!
- The space of gauge invariant co-ordinates is the same as that of non-commuting co-ordinates.
- Similar results obtained by *Benfica and Girotti*, using extension of phase space.
- Here **no extension of phase space**.
  All results are in the original phase space

The Moyal Product

- The ordinary product of two functions in non-commuting space can be rewritten as a *Moyal Product.*

- For two functions $\mathtt{A}, \mathtt{B}$ the gauge invariant functions $\widetilde{\mathtt{A}}, \widetilde{\mathtt{B}}$ are given by

$$\widetilde{\mathtt{A}}(q^i, v_j) = e^{-\psi^i \partial_{q_i}} \mathtt{A} = \mathtt{A}\left(\widetilde{q}^i, \widetilde{v}_i\right); \quad \widetilde{\mathtt{B}}(q^i, v_j) = e^{-\psi^i \partial_{q_i}} \mathtt{B} = \mathtt{B}$$

- Then $\widetilde{\mathtt{A}}(q^i, v_j)\widetilde{\mathtt{B}}(q^i, v_j) = \text{Moyal product}$

  $= (\mathtt{A} * \mathtt{B})\left(\widetilde{q}^i, \widetilde{v}_j\right)$

Proof of Moyal Product

Proof.

The quantities $\widetilde{\mathtt{A}}(q^i, v_j)$ & $\widetilde{\mathtt{B}}(q^i, v_j)$ can be redefined upto terms in the first class constraints χ_i.

$$\widetilde{\mathtt{A}}(q, v) = e^{-\psi^i \partial_{q_i} - \chi_i \partial_{v_i}} \mathtt{A}; \qquad \widetilde{\mathtt{B}}(q, v) = e^{-\psi^i \partial_{q_i} - \chi_i \partial_{v_i}} \mathtt{B};$$

The Fourier transforms of $\widetilde{\mathtt{A}}(q, v)$ & $\widetilde{\mathtt{B}}(q, v)$ are

$$\begin{aligned} \widetilde{\mathtt{A}}(q^i, v_j) &= \frac{1}{(2\pi)^{\mathrm{N}}} \int d^{\mathrm{N}} m\, d^{\mathrm{N}} n\, e^{-(\psi^i \partial_{q^i} + \chi_i \partial_{v_i})} e^{i\left(m_j q^j + n^j v_j\right)} \mathtt{A}(m, n) \\ &= \frac{1}{(2\pi)^{\mathrm{N}}} \int d^{\mathrm{N}} m\, d^{\mathrm{N}} n\, e^{-(\psi^i m_i + \chi_i n^i)} e^{i\left(m_i q^i + n^i v_i\right)} \mathtt{A}(m, n) \end{aligned}$$

$$\widetilde{\mathtt{B}}(q^i, v_j) = \frac{1}{(2\pi)^{\mathrm{N}}} \int d^{\mathrm{N}} m\, d^{\mathrm{N}} n\, e^{-(\psi^i r_i + \chi_i s^i)} e^{i\left(r_i q^i + s^i v_i\right)} \mathtt{A}(m, n)$$

☐

proof continued

Taking the product $\widetilde{\mathrm{A}}(q^i, v_j)\widetilde{\mathrm{B}}(q^i, v_j)$, the product on its RHS can be modified using Baker-Campbell-Hausdorff formula

$$\begin{aligned} e^{\mathrm{X}}e^{\mathrm{Y}} &= e^{\mathrm{X}+\mathrm{Y}}e^{-\frac{1}{2}\{\mathrm{X},\mathrm{Y}\}}e^{\frac{1}{6}(2\{\mathrm{Y},\{\mathrm{X},\mathrm{Y}\}\}+\{\mathrm{X},\{\mathrm{X},\mathrm{Y}\}\})}\ldots \\ \mathrm{X} &= -i\left(\psi^i m_i + \chi_i n^i\right)\,;\ \mathrm{Y} = -i\left(\psi^i r_j + \chi_i s^i\right) \end{aligned}$$

Using $\{\mathrm{X},\mathrm{Y}\} = -\left[2\theta^{ij} m_i r_j - m_i s^i + n^i r_i\right]$, $\{\mathrm{Y},\{\mathrm{X},\mathrm{Y}\}\} = \{\mathrm{X},\{\mathrm{X},\mathrm{Y}\}\} = 0$, we get,

$$\begin{aligned} = \quad & \frac{1}{(2\pi)^{2\mathrm{N}}}\int d^{\mathrm{N}}m\, d^{\mathrm{N}}n\, d^{\mathrm{N}}r\, d^{\mathrm{N}}s\ e^{i\left[(m_i+r_i)\widetilde{q}^i+(n^i+s^i)\widetilde{v}_i\right]} \\ & e^{\frac{1}{2}\left[2\theta^{ij} m_i r_j - m_i s^i + n^i r_i\right]}\mathrm{A}(m,n)\mathrm{B}(r,s) \end{aligned}$$

proof continued

which is the same as the Fourier transform of the usual Moyal product:

$$(\mathtt{A} * \mathtt{B})(\widetilde{q}^i, \widetilde{v}_j) = e^{\frac{i}{2}(-2\theta^{ij})\partial_{q^i}\partial_{q'^j}} e^{\frac{i}{2}(0)\partial_{v_i}\partial_{v'_j}} e^{\frac{i}{2}(\delta^i{}_j)\partial_{q^i}\partial_{v'_j}} e^{\frac{i}{2}(-\delta^i{}_j)\partial_{v_i}\partial_{q'^j}}$$

in which the $\widetilde{q}^i$ are usual non-commuting co-ordinates.

Thus the ordinary product of two gauge invariant functions can be rewritten as the Moyal products of the corresponding two non-invariant functions of non-commuting co-ordinates.

2^{nd} Lagrangian : Equations of Motion & Phase Space

- Using q^i alone as the $\mathtt{N}$ co-ordinates, consider

$$\mathtt{L} = \dot{q}^i(\theta^{-1})_{ij}q^j - h_0(q^i), \quad i,j = 1,2,\ldots,\mathtt{N}$$

where (θ^{-1}) is the inverse of the original θ

- Lagrange equations of motion $\longrightarrow$ $\dot{q}^i = -\frac{1}{2}\,\theta^{ij}\,\dfrac{\partial h_0}{\partial q^j}$
- The canonical momenta are $p_i = \dfrac{\partial \mathtt{L}}{\partial \dot{q}^i} = (\theta^{-1})_{ij}q^j$

span a $2\mathtt{N}$-dimensional phase space

- Fundamental PBs — $\{q^i, q^j\} = 0$; $\{q^i, p_j\} = \delta^i{}_j$
- No velocities again; so constraints appear

Constraints & the Hamiltonian

- There are N constraints – $\phi'_i = p_i - (\theta^{-1})_{ij} q^j \approx 0$ defining an N-dimensional surface $\sum'_{\mathrm{N}}$.
- Second class constraints – $\mathrm{C}_{ij} = \{\phi'_i, \phi'_j\} = -2(\theta^{-1})_{ij}$
- Canonical Hamiltonian is
$$\mathrm{H}_0(q,p) = h_0 + \sum_{i,j} \left[p_i - (\theta^{-1})_{ij} q^j\right] \dot{q}^i$$
- Total Hamiltonian is $\mathrm{H}_{\mathrm{T}} = h_0 - \frac{1}{2} \sum_{i,j} \phi'_i \, \theta^{ij} \, \frac{\partial h_0}{\partial q^j}$
- Hamilton's equations of motion
$$\dot{q}^i = \{q^i, \mathrm{H}_{\mathrm{T}}\} = -\frac{1}{2} \theta^{ij} \, \frac{\partial h_0}{\partial q^j}; \qquad \dot{p}_i = -\frac{1}{2} \, \frac{\partial h_0}{\partial q^j}$$
on the surface $\sum_{\mathrm{N}}$.

Dirac Brackets

- Here $\{A, B\}_{DB} = \{A, B\} - \{A, \phi'_i\}(\mathbb{C}^{-1})^{ij}\{\phi'_j, B\}$

  where $\mathbb{C}^{-1}$ is the matrix $-\frac{1}{2}\theta$

- Applying this we get $\{q^i, q^j\}_{DB} = -\frac{1}{2}\theta^{ij}$ and

  $$\{q^i, p_j\}_{DB} = \frac{1}{2}\delta^i{}_j, \quad \{p_i, p_j\}_{DB} = \frac{1}{2}(\theta^{-1})_{ij}$$

- Again upon quantisation, these Dirac brackets go over to commutators, and we get non-commutative co-ordinates.

  Note: There is no gauge invariance here

Equivalent Gauge Theory for the Second System

- Once again, we can introduce gauge invariance here.
- Note – The number of constraints here is less than in the earlier case.
- Gauge invariance can be induced by either enlarging the phase space, or by remaining with this phase space ($2\mathtt{N}$-dimensional).
- We work in the same phase space.
- However, must now split the $\mathtt{N}$ constraints ϕ_i' into two subsets, at least one of which obeys the *first class* property. The other subset must behave a gauge-fixing subset.
- As it is, a general (constant) matrix θ doesn't permit this splitting.

The classification of constraints

- Classification in the above sense is thus **non-trivial.**
- No problem at all for just **two** constraints. Either one is taken as first class, the other gauge-fixing like.
- A <u>single</u> classification throughout is needed.
- May be possible here since θ is a constant matrix.
- Consider a matrix $\mathtt{S}$ such that $\mathtt{S}^{\mathtt{T}}\theta^{-1}\mathtt{S} = (\theta')^{-1}$, where $(\theta')^{-1}$ is of the form $(\theta')^{-1} = \begin{pmatrix} 0 & \mathtt{E} \\ -\mathtt{E} & 0 \end{pmatrix}$
- The submatrix $\mathtt{E}$ is of dimension $\frac{\mathtt{N}}{2} \times \frac{\mathtt{N}}{2}$.
- The same matrix $\mathtt{S}$ can be used to transform the constraint set ϕ' as $\phi' \longrightarrow \phi'' = \mathtt{S}^{\mathtt{T}}\phi'$

Classification of constraints contd.

- For the constraints ϕ'_i, since formally $\{\phi', \phi'\}, = -2\theta^{-1}$ the above two transformations can be used to get

$$\begin{aligned}\{\phi'', \phi''\} = \{\mathrm{S}^{\mathrm{T}}\phi', \phi'\mathrm{S}\} &= \mathrm{S}^{\mathrm{T}}\{\phi', \phi'\}\mathrm{S} \\ &= -2\left(\mathrm{S}^{\mathrm{T}}\theta^{-1}\mathrm{S}\right) = -2(\theta')^{-1}\end{aligned}$$

- The form of the matrix $(\theta')^{-1}$ implies that the modified constraints ϕ'' can now be divided into two subsets,

$$\phi''_\alpha = \chi'_\alpha; \qquad \phi''_{\alpha+\frac{\mathrm{N}}{2}} = \psi'_\alpha \qquad (\alpha = 1, 2, \ldots, \frac{\mathrm{N}}{2})$$

- Classification is obvious if we look at the PBs,

$$\{\chi'_\alpha, \chi'_\beta\} = 0; \qquad \{\chi'_\alpha, \psi'_\beta\} = -2\mathrm{E}_{\alpha\beta}; \qquad \{\psi'_\alpha, \psi'_\beta\} = 0$$

Choice of the first class subset

- Can choose either the χ' or the ψ' as the first class subset, and the remaining as gauge-fixing-like subset.
- Redefine the χ' as $\chi''_\alpha = -\frac{1}{2}\,(\mathtt{E})^{-1}_{\alpha\beta}\,\chi'_\beta$ & $\psi''_\alpha = \psi'_\alpha$.
- Retain χ''_α as first class constraint subset and discard the ψ''_α.
- The constraint surface, defined by only $\chi''_\alpha \approx 0$, is $\sum_{\frac{\mathtt{N}}{2}}$
- However, on this $\sum_{\frac{\mathtt{N}}{2}}$, $\mathtt{H}_\mathtt{T}$ is not invariant under transformations generated by the χ''_α!

$$\{\chi''_\alpha, \mathtt{H}_\mathtt{T}\} \neq 0.$$

Gauge Invariant Hamiltonian

- In terms of the $\chi'_\alpha, \ \psi'_\alpha$ rewrite the $\mathtt{H_T}$ as

$$\begin{aligned}\mathtt{H_T} &= h_0 + \sum_{ij} u^i \phi'_i = h_0 + \sum_{ij} u'^i \phi''_i \\ &= \frac{1}{2}\sum_{i,\alpha}\left[\chi'_\alpha (\mathtt{E}^{-1})^{\alpha\beta}(\mathtt{S}^{\mathtt{T}})_{\beta+\frac{\mathtt{N}}{2}}^{\ i} - \psi'_\alpha (\mathtt{E}^{-1})^{\alpha\beta}(\mathtt{S}^{\mathtt{T}})_{\beta}^{\ i}\right]\frac{\partial h_0}{\partial q^i}\end{aligned}$$

  where the various transformations (above) are used.

- Formally the gauge invariant Hamiltonian is

$$\widetilde{\mathtt{H}}_{\mathtt{T}} =: e^{-\psi''\hat{\chi}''} : \mathtt{H_T}, \qquad [: \hat{\chi}'' :\equiv \{\chi'', ()\}] \qquad \chi''_\alpha = (\,(\mathtt{E})\,)^{-1}_{\alpha\beta}\chi'_\beta$$

- The gauge invariant Hamiltonian is

$$\widetilde{\mathtt{H}}_{\mathtt{T}} = e^{-\frac{1}{2}\left(\mathtt{E}^{-1}\mathtt{S}^{\mathtt{T}}\right)^{\alpha i}\psi''_\alpha \frac{\partial}{\partial q^i}} \ h_o$$

Gauge Invariant variables & Non-Commutativity

- For a general function $\mathtt{A}$, the gauge invariant counterpart is $\widetilde{\mathtt{A}} = e^{-\frac{1}{2}\left(\mathtt{E}^{-1}\mathtt{S}^{\mathtt{T}}\right)^{\alpha i}\psi''_\alpha \frac{\partial}{\partial q^i}}\ \mathtt{A}$
- Then $\widetilde{q}^i = q^i - \frac{1}{2}\left(\mathtt{E}^{-1}\mathtt{S}^{\mathtt{T}}\right)^{\alpha i}\psi''_\alpha$, and

  $$\widetilde{p}_i = p_i + \frac{1}{2}\left(\mathtt{E}^{-1}\mathtt{S}^{\mathtt{T}}\right)^{\alpha j}\psi''_\alpha(\theta^{-1})_{ji}$$
- The $\widetilde{q}^i$ have non-zero Posson brackets $\Longrightarrow$ upon quantisation, they are non-commuting!

  $$\{\widetilde{q}^i, \widetilde{q}^i\} = -\frac{1}{2}\theta^{ij}; \quad \{\widetilde{q}^i, \widetilde{p}_j\} = \frac{1}{2}\delta^i{}_j; \quad \{\widetilde{p}_i, \widetilde{p}_j\} = \frac{1}{2}(\theta^{-1})_{ij}$$
- Thus, the space of gauge-invariant co-ordinates is **non-commuting.**

Gauge Invariance and The Moyal Product

- Here also, product of two gauge invariant functions can be rewritten as the Moyal product.
- Use the 1^{st} class χ''_i to write, for any $\mathtt{A}(q^i)$,

$$\widetilde{\mathtt{A}} = e^{-\frac{1}{2}\left(\mathtt{E}^{-1}\mathtt{S}^{\mathtt{T}}\right)^{\alpha i}\psi''_\alpha \frac{\partial}{\partial q^i} \; -\frac{1}{2}(\mathtt{S})^{i\;\alpha+\frac{\mathtt{N}}{2}}\chi''_\alpha \frac{\partial}{\partial q^i}} \; \mathtt{A},$$

with a similar expression for any other $\mathtt{B}(q^i)$.

- Take Fourier transforms for $\widetilde{\mathtt{A}}, \widetilde{\mathtt{B}}$, and take their product.
- Use Baker-Campbell-Hausdorff appropriately.

$$\begin{aligned}
\widetilde{\mathtt{A}}\widetilde{\mathtt{B}} &= \frac{1}{(2\pi)^{\mathtt{N}}}\int d^{\mathtt{N}}m d^{\mathtt{N}}n e^{i(m_i+n_i)\widetilde{q}^i} e^{-\frac{1}{2}\theta^{ij}m_i n_j}\mathtt{A}(m_i)\mathtt{B}(n_i) \\
&= \frac{1}{(2\pi)^{\mathtt{N}}}\int d^{\mathtt{N}}m d^{\mathtt{N}}n \; e^{-\frac{1}{2}\theta^{ij}\frac{\partial}{\partial\widetilde{q}^i}\frac{\partial}{\partial\widetilde{q}'^j}} e^{im_i\widetilde{q}^i} e^{in_i\widetilde{q}'^i}\mathtt{A}(m_i)\mathtt{B}(n_i)| \\
&= (\mathtt{A} * \mathtt{B})(\widetilde{q})
\end{aligned}$$

Summary

- Classical aspects of Non-Commutative Theories.
- What are the classical theories that can generate non-commutativity of co-ordinates upon quantisation?
- Two theories, given by two Lagrangians.
- Both are theories involving second class constraints.
- In both cases, Dirac brackets were used and non-commutative co-ordinates were generated.
- Equivalent gauge theories were constructed.
- Gauge invariant co-ordinates are non-commuting upon quantisation.
- In the space spanned by such gauge invariant co-ordinates, ordinary product of functions are the same as the Moyal product.

References

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