

Fate of the false monopoles: induced vacuum decay

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Outline

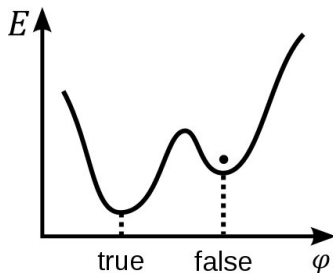
- 1 Vacuum stability in the presence of topological solitons
 - False vacuum decay via quantum tunneling
 - Quantum tunneling of solitons
- 2 Monopole decay
 - Classical instabilities of monopoles
 - Thin walled monopoles
 - Collective coordinates, Instantons, and Bounce action
- 3 Summary and open problems

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Fate of the false vacuum (Coleman 1977)

- Long-lived metastable vacua are crucial in models of SUSY breaking and string theory.



- $\frac{\Gamma}{V} = A \exp\left(\frac{-S_E[\bar{\phi}]}{\hbar}\right) \times (1 + O(\hbar))$.
- Large bounce action \iff sufficiently stable false vacuum.

Fate of the false vacuum

- $\frac{\Gamma}{V} = \left(\frac{S_E[\bar{\phi}]}{2\pi\hbar} \right)^2 \exp \left(-\frac{S_E[\bar{\phi}]}{\hbar} \right) \left| \frac{\det'[-\partial^2 + U''(\bar{\phi})]}{\det[-\partial^2 + U''(\phi_f)]} \right|^{-1/2} \times (1 + O(\hbar))$.
- S_E = Euclidean action of instanton configuration associated with tunneling.
- Instantons boundary conditions:
 - $\phi = | \text{False Vacuum} \rangle$ for $\tau = \pm\infty$
 - $\frac{\partial\phi}{\partial\tau} = 0$ for $\tau = 0$
- This analysis is for homogenous, translation invariant case!
- With topological solitons present, large S_E does not necessarily ensure stability of false vacuum.

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Stability for the translation non-invariant case

- Topological solitons in false vacua can *themselves* become dynamically unstable.
- Decay of the topological object implies decay of the false vacuum.
 - Classically stable solitons can tunnel into classically unstable solitons.
 - True vacuum (within core) then expands and fills the false vacuum outside the core.
 - The solitons are topologically stable but dynamically unstable.
- This type of tunneling and the associated instantons will be discussed here for monopoles in a false vacuum.
- Reported in Kumar, Paranjape, and Yajnik, PRD 82, 2010.

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Our main result

- Let B_0 denote bounce action for tunneling in translation invariant case (described by Coleman).
- Let B denote bounce action for instantons associated with monopole decay.
- $B \sim B_0 \left(1 - \frac{R_1}{R_2}\right)^{5/2}$.
- Bounce action can become vanishingly small!

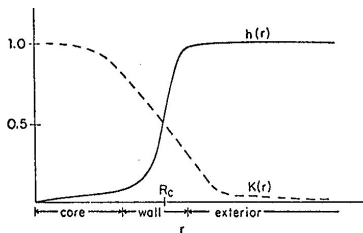
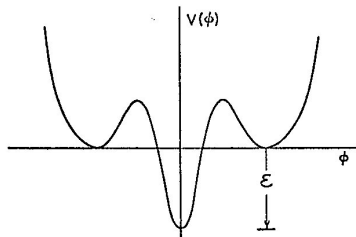
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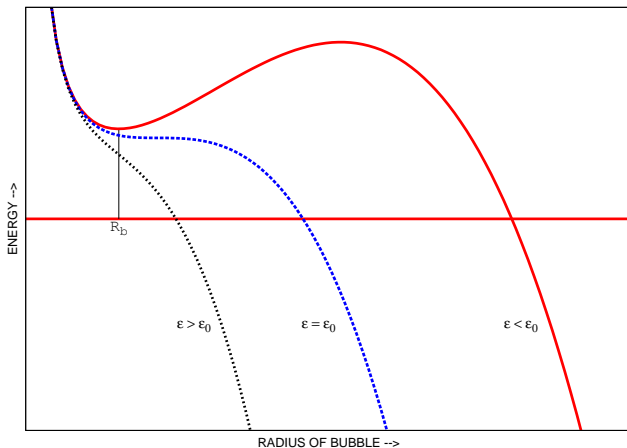
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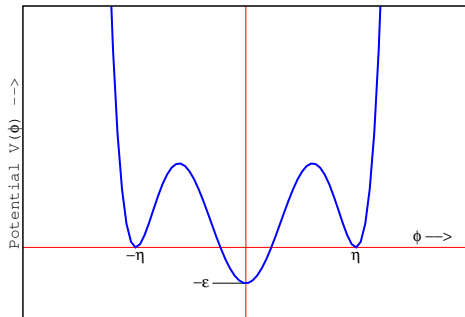


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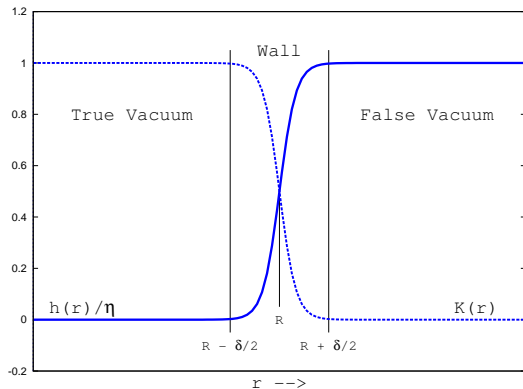
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Monopoles in a false vacuum



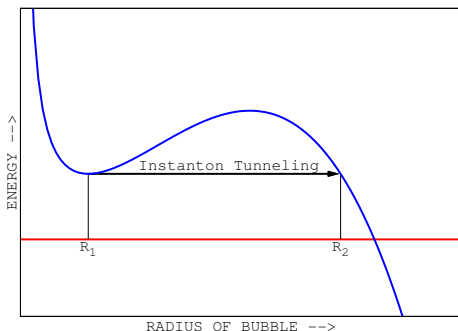
- We consider a $SU(2)$ gauge theory with a triplet scalar ϕ
- $V(\phi) = \lambda\phi^2(\phi^2 - a^2)^2 + \gamma^2\phi^2 - \epsilon$

Monopole profile and ansatz



- $\phi_a = \hat{r}_a h(r)$
 $A_\mu^a = \epsilon_{\mu ab} \hat{r}_b \frac{1-K(r)}{er}$
 $A_0 = 0$

Monopole mass



- $E(K, h)/4\pi = \int_0^\infty dr \left[\frac{(K')^2}{e^2} + \frac{(1-K^2)^2}{2e^2 r^2} + \frac{1}{2} r^2 (h')^2 + K^2 h^2 + r^2 V(h) \right]$
- $E(R) = -\alpha R^3 + 4\pi\sigma R^2 + \frac{C}{R}$

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- Use of thin wall approximation enables the use of one collective coordinate R describing the full dynamics of the monopole
- Field theory in $3 + 1$ dimensions reduces to one dimensional problem involving $R(t)$
- Instanton $R(\tau)$ describing monopole decay has $R = R_1$ for $\tau = \pm\infty$ and $R = R_2$ with $\frac{dR}{d\tau} = 0$ for $\tau = 0$.
- Full form of $R(\tau)$ is not necessary. Bounce action can be computed without solving for $R(\tau)$

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Bounce Action

- Monopole mass is derived for static case. Including time dependence,

- $\dot{\phi}^a = \hat{r}_a \frac{dh}{dR} \dot{R}$

$$\frac{1}{2} \dot{\phi}^a \dot{\phi}^a = \frac{1}{2} \left(\frac{dh}{dR} \right)^2 \dot{R}^2 = \frac{1}{2} \left(\frac{dh}{dr} \right)^2 \dot{R}^2$$

- $\dot{A}_\mu^a = \epsilon_{\mu ab} \hat{r}_b \left(\frac{-1}{er} \right) \frac{dK}{dR} \dot{R}$

$$\frac{1}{4} \dot{A}_\mu^a \dot{A}_\mu^a = \frac{1}{2e^2 r^2} \left(\frac{dK}{dr} \right)^2 \dot{R}^2$$

- The total Lagrangian is

$$L = 2\pi \int_0^\infty \left(r^2 \left(\frac{dh}{dr} \right)^2 \dot{R}^2 + \frac{1}{e^2} \left(\frac{dK}{dr} \right)^2 \dot{R}^2 \right) dr - E(R)$$

$$L = L(R, \dot{R})$$

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Bounce Action (2)

- $S = \int_{-\infty}^{\infty} dt L(R, \dot{R})$
 $S_E = \int_{-\infty}^{\infty} d\tau L(R, \dot{R})$ with $E(R) \rightarrow -E(R)$
- We pull out a factor of \dot{R} from $L(R, \dot{R})$ so that

$$S_E = \int_{-\infty}^{\infty} d\tau \frac{dR}{d\tau} f(R, \dot{R})$$

$$= 2 \int_{R_1}^{R_2} f(R, \dot{R}) dR$$
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- Using straightforward algebra, the integral over R can be done to yield the bounce action

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Bounce action (3)

- $B = \frac{9\sqrt{2}\pi}{140} \tilde{\lambda}^2 \tilde{a}^{16} \frac{\mu^{12}}{\epsilon^3} \left(1 - \frac{R_1}{R_2}\right)^{5/2} I\left(\frac{R_1}{R_2}, \frac{R_3}{R_2}\right)$

- $B = \frac{32\sqrt{2}}{105\pi} B_0 \left(1 - \frac{R_1}{R_2}\right)^{5/2} I\left(\frac{R_1}{R_2}, \frac{R_3}{R_2}\right)$

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Regime of classical instability

- As R_1 approaches R_2 , regime of classical instability is approached
- Solving $E(R) = 0$ after shifting by a constant E_0 enables the calculation of R_1 and R_2
- For our potential, we have

$$\frac{R_1}{R_2} = \frac{1}{(\tilde{\lambda}e)^{2/3}} \left(\frac{16}{27}\right)^{1/3} \frac{\epsilon}{\tilde{a}^{16/3}\mu^4}$$
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Concluding Remarks

- Metastable vacua which permit the existence of solitons can undergo vacuum decay catalyzed by these topological defects.
- The corresponding bounce action can be vanishingly small even when the bounce action for the homogeneous vacuum is large.
- Analysis here is under the thin-wall approximation. Applies equally well to vortices.
- Decay of cosmic strings in a similar manner is currently under study.

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