Probing CPT Violation in B Systems[1]

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[1] A. Kundu, S. Nandi and S. K. P; PR **D 81**, 076010 (2010)

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Contents:

Introduction

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- Then why study CPTV?

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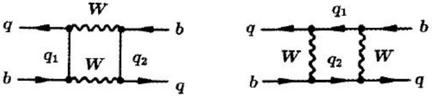

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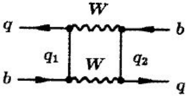
- 

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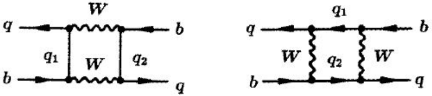
- 

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Definition of δ:

CPT violation$\rightarrow$ significant $\Delta\Gamma/\Gamma$ in generic M^0-$\overline{M}^0$ system, $M^0 = K^0, B^0$, or B_s.[Datta *et al.*[5]] Following them,

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}} \, ,$$

$$\mathcal{M} = \begin{pmatrix} M_0 - \mathrm{Re}(\delta') & M_{12} \\ M_{12}^* & M_0 + \mathrm{Re}(\delta') \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_0 + 2\mathrm{Im}(\delta') & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 - 2\mathrm{Im}(\delta') \end{pmatrix} ,$$

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  $$\text{or,} \quad \lambda = \left[H_{11} + H_{12}\alpha\left(y + \frac{\delta}{2}\right)\right], \quad \left[H_{22} - H_{12}\alpha\left(y + \frac{\delta}{2}\right)\right],$$
  where $y = \sqrt{1 + \frac{\delta^2}{4}}$ and $\alpha = \sqrt{H_{21}/H_{12}}$.
- $$|B_H\rangle = p_1|B^0\rangle + q_1|\overline{B}^0\rangle\,,$$
  $$|B_L\rangle = p_2|B^0\rangle - q_2|\overline{B}^0\rangle\,.$$

Formalism of CPTV

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$$\Gamma(B_q(t) \to f_{CP}) = [|f_+(t)|^2 + |\xi_{f_1}|^2 |f_-(t)|^2 + 2\mathrm{Re}\left(\xi_{f_1} f_-(t) f_+^*(t)\right)] |A_f|^2,$$

$$\Gamma(\overline{B_q}(t) \to f_{CP}) = [|f_-(t)|^2 + |\xi_{f_2}|^2 |\bar{f}_+(t)|^2 + 2\mathrm{Re}\left(\xi_{f_2} \bar{f}_+(t) f_-^*(t)\right)] \left|\frac{A_f}{\eta_2}\right|^2 ,$$

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$$\Gamma_T[f,t] = \Gamma(B_q(t) \to f) - \Gamma(\bar{B}_q(t) \to f)$$

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- Using these:

$$Br[f] = \frac{|A_f|^2}{2}\left[\frac{1}{\Gamma_s}\{2 - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\}\right.$$
$$\left. + \frac{\Gamma_s}{(\Delta m)^2 + (\Gamma_s)^2}\mathrm{Im}(\delta)\mathrm{Im}(\xi_f) + \frac{\Delta\Gamma_s}{(\Gamma_s)^2}\mathrm{Re}(\xi_f)\right],$$

$$\Gamma_U[f,t] = \left[(2 - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f))\cosh\left(\frac{\Delta\Gamma_q t}{2}\right)\right.$$
$$+ \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\cos(\Delta m_q t)$$
$$\left. + 2\mathrm{Re}(\xi_f)\sinh\left(\frac{\Delta\Gamma_q t}{2}\right)\right]|A_f|^2 e^{-\Gamma_q t}.$$

- Contd..

$$\Gamma_T[f,t] = \Bigg[-\mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\cosh\left(\frac{\Delta\Gamma_q t}{2}\right) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\cos(\Delta m_q t) - \mathrm{Re}(\delta)\sinh\left(\frac{\Delta\Gamma_q t}{2}\right) + \{2\mathrm{Im}(\xi_f) - \mathrm{Im}(\delta)\}\sin(\Delta m_q t)\Bigg]|A_f|^2 e^{-\Gamma_q t}.$$

- One can extract $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ by comparing the untagged and tagged analyses when $|\mathrm{Re}(\xi_f)|^2 + |\mathrm{Im}(\xi_f)|^2 = 1$.

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CP Asymmetry

The time-dependent CP asymmetry:

$$A_{CP}(f,t) = \frac{\Gamma_T[f,t]}{\Gamma_U[f,t]} = \frac{\Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)}{\Gamma(B_q(t)\to f) + \Gamma(\bar{B}_q(t)\to f)},$$

and the time-independent CP asymmetry:

$$A_{CP}(f) = \frac{\int_0^\infty dt\ \Gamma_T[f,t]}{\int_0^\infty dt\ \Gamma_U[f,t]} = \frac{\int_0^\infty dt\ [\Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)]}{\int_0^\infty dt\ [\Gamma(B_q(t)\to f) + \Gamma(\bar{B}_q(t)\to f)]}.$$

Deviation from SM $\rightarrow$ NP.

Analysis: B_s System

- For the B_s system, we take

$$\Delta m_s = 17.77 \pm 0.12 \mathrm{ps}^{-1}, \Delta\Gamma_s = 0.096 \pm 0.039 \mathrm{ps}^{-1}\,,$$
$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.147 \pm 0.060\,, \frac{1}{\Gamma_s} = 1.530 \pm 0.009 \mathrm{ps}\,,$$
$$\mathrm{Re}(\xi_f) = 0.99\,, \mathrm{Im}(\xi_f) = -0.04\,.$$

- We take $|\mathrm{Re}(\delta)|, |\mathrm{Im}(\delta)| < 0.1$. we fix Δm_s and $\Delta\Gamma_s$ to their respective central SM values.

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A_{CPT} vs. $\mathrm{Re}(\delta)$

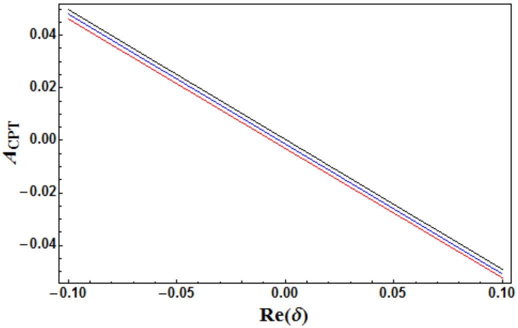

Figure: Variation of A_{CPT} with $\mathrm{Re}(\delta)$ for the B_s system. The three lines, from top to bottom, are for $\mathrm{Im}(\delta) = -0.1, 0$ and 0.1 respectively.

$\sin(2\beta_s)$ vs. $\mathrm{Im}(\delta)$

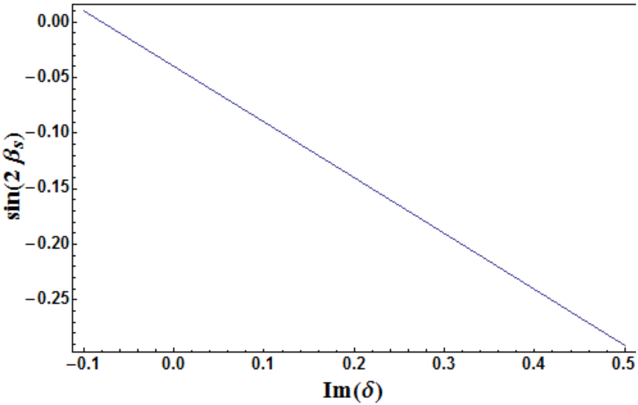

Figure: Variation of $\sin(2\beta_s)$ with $\mathrm{Im}(\delta)$.

Analysis: B_d System

- For the B_d system, we take

$$\Delta m_d = 0.507 \mathrm{ps}^{-1}\,,\ \Delta\Gamma_d = 0\,,\ \mathrm{Re}(\xi_f) = 0.72\,,\ \mathrm{Im}(\xi_f) = 0.695\,.$$

- $\sin(2\beta_d) = 0.695 \pm 0.020$ (CKM expectation).
  $\sin(2\beta_d) = 0.668 \pm 0.028$ (Measured: $b \to c\bar{c}s$ channel).

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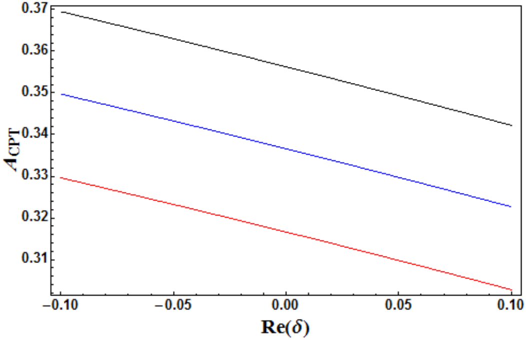
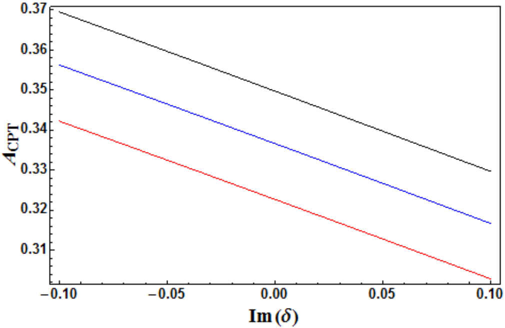


Figure: Variation of A_{CPT} with $\mathrm{Re}(\delta)$ for the B_d system. The three lines, from top to bottom, are for $\mathrm{Im}(\delta) = -0.1, 0$ and 0.1 respectively.



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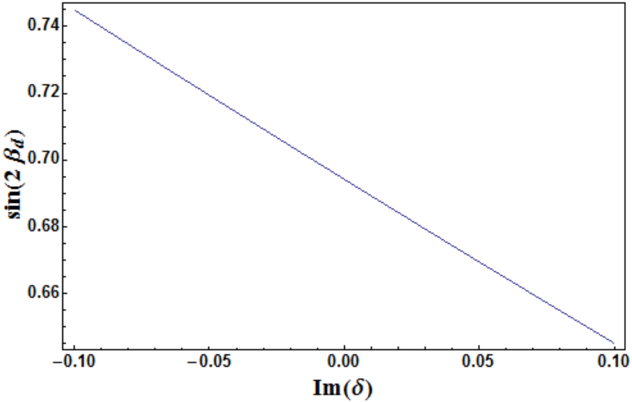

Figure: Variation of $\sin(2\beta_d)$ with $\mathrm{Im}(\delta)$.

Summary and Conclusion

- CPT, even if it exists, should be quite small.
- Still possible to measure a small CPTV from $\Gamma_T[f,t]$ and $\Gamma_U[f,t]$. In particular, from the coefficients of the trigonometric terms $\sin(\Delta m t)$ and $\cos(\Delta m t)$.
- Best to focus upon the single-amplitude observables: $B_d \to J/\psi K_S$ and $B_s \to J/\psi \phi$ or $B_s \to D_s^+ D_s^-$ [6]

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Thank you

Backup

- Assuming $\delta \ll 1$ and expanding any function $f(\delta)$ using Taylor series expansion and dropping all the terms $\mathcal{O}(\delta^n)$ for $n > 2$, we get:

$$
\begin{aligned}
&\Gamma_U[f,t] \\
&= \Bigg[\left\{ (1+|\xi_f|^2)\Big(1+\frac{(\mathrm{Im}(\delta))^2}{4}\Big) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f) \right\} \cosh\left(\frac{\Delta\Gamma_q t}{2}\right) \\
&- \left\{ (1+|\xi_f|^2)\frac{(\mathrm{Im}(\delta))^2}{4} - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f) \right\} \cos(\Delta m_q t) \\
&+ \left\{ 2\mathrm{Re}(\xi_f) - \frac{1}{2}(1-|\xi_f|^2)\mathrm{Re}(\delta) - \frac{1}{4}\mathrm{Re}(\xi_f)\big((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2\big) \right\} \\
&\sinh\left(\frac{\Delta\Gamma_q t}{2}\right) - \frac{1}{2}\mathrm{Im}(\delta)\left\{(1-|\xi_f|^2) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\sin(\Delta m_q t) \Bigg] \\
&|A_f|^2 e^{-\Gamma_q t}.
\end{aligned}
$$

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Thus, for the B_s system, where the hyperbolic functions are not negligible, we get (keeping up to first order of terms in $\Delta\Gamma_s$):

$$
\begin{aligned}
Br[f] &= \frac{1}{2}\int_0^\infty dt\;\Gamma[f,t] \\
&= \left[\frac{1}{\Gamma_s}\left\{(1+|\xi_f|^2)(1+\frac{(\mathrm{Im}(\delta))^2}{4}) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\}\right. \\
&- \frac{\Gamma_s}{(\Delta m)^2+(\Gamma_s)^2}\left\{(1+|\xi_f|^2)\frac{(\mathrm{Im}(\delta))^2}{4} - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\} \\
+\frac{\Delta\Gamma_s}{2(\Gamma_s)^2}&\left\{2\mathrm{Re}(\xi_f) - \frac{1}{2}(1-|\xi_f|^2)\mathrm{Re}(\delta) - \frac{1}{4}\mathrm{Re}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2\right) \\
&\left. - \frac{1}{2}\mathrm{Im}(\delta)\left\{(1-|\xi_f|^2) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\frac{\Delta m}{(\Delta m)^2+(\Gamma_s)^2}\right]\frac{|A_f|^2}{2}
\end{aligned}
$$

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One can also tag the B mesons and define a tagged decay rate asymmetry

$$
\begin{aligned}
\Gamma_T[f,t] &= \Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)\\
&= \Bigg[\left\{(1-|\xi_f|^2)\frac{(\mathrm{Re}(\delta))^2}{4} - \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\cosh\left(\frac{\Delta\Gamma_q t}{2}\right)\\
&+ \left\{(1-|\xi_f|^2)(1-\frac{(\mathrm{Re}(\delta))^2}{4}) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\cos(\Delta m_q t)\\
&- \frac{1}{2}\mathrm{Re}(\delta)\left\{(1+|\xi_f|^2) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\}\sinh\left(\frac{\Delta\Gamma_q t}{2}\right)\\
&+ \Bigg\{2\mathrm{Im}(\xi_f)\\
&- \frac{1}{2}\mathrm{Im}(\delta)(1+|\xi_f|^2) - \frac{1}{4}\mathrm{Im}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2)\Bigg\}\\
&\sin(\Delta m_q t)\Bigg]|A_f|^2 e^{-\Gamma_q t}.
\end{aligned}
$$

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One could even relax the assumption of $H_{21} = H_{12}^*$. However, there are two points that one must note.

- First, the effect of expressing $H_{12} = h_{12} + \bar{\delta}$, $H_{21} = h_{12}^* - \bar{\delta}$ appears as $\bar{\delta}^2$ in $\sqrt{H_{12}H_{21}}$, the relevant expression, and can be neglected if we assume $\bar{\delta}$ to be small.
- Second point(more important): CPT constrains only the diagonal elements and puts no constraint whatsoever on the off-diagonal elements. It has been shown[7] that $H_{12} \neq H_{21}^*$ leads to T violation, and only $H_{11} \neq H_{22}$ leads to unambiguous CPT violation.

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- It has been shown by **Greenberg** [8] that CPT violation necessarily means Lorentz violation, and bounds on Lorentz-violating parameters using the data on B mesons can be found in the paper by **Kostelecky and Kooten** [9] and references therein. Here we assume a simplistic parametrization, which was earlier followed by **Datta et al.**[10].
- The convention of **The BABAR Collaboration** [11] leads to $z_0 = \delta/2$, where z_0 is a measure of CPT violation as used there. The limits imply that $|z_0| \ll 1$. Even if the origin of CPT violation is something different, it is not unrealistic to assume $|\delta| \ll 1$.

[8] Phys. Rev. Lett. **89**, 231602 (2002),
[9] arXiv:1007.5312 [hep-ph].
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