Detecting Physics Beyond the Standard Model through b $\to$ s μ^+ μ^- transition

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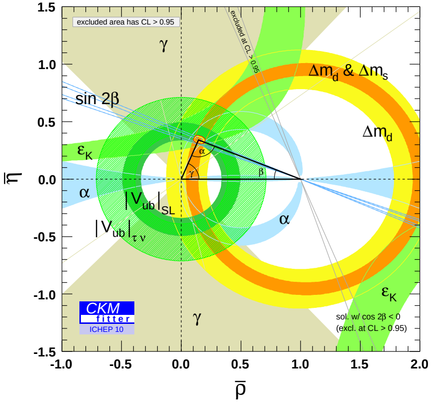
Based on :

1) New-Physics Contributions to the Forward-Backward Asymmetry in $B \to K^* \mu^+ \mu^-$ [JHEP 02 (2010) 053]

2) New Physics in $b \to s\, \mu^+ \mu^-$: CP-Conserving Observables [arXiv:1008.2367 [hep-ph]]

Collaborators :

A. K. Alok, A. Datta, A. Dighe, M. Duraisamy, D. London, J. Matias, M. Nagashima, A. Szynkman, S. Uma Sankar



INPUTS

$|V_{ud}|$, $|V_{us}|$
$|V_{cb}|$, $|V_{ub}|$
$B \to \tau\nu$
Δm_d, Δm_s
ϵ_K
$\sin 2\beta$
α
γ

Wolfenstein parameters:			
$A = 0.812^{+0.013}_{-0.027}$	$\lambda = 0.22543 \pm 0.00077$	$\bar{\rho} = 0.144 \pm 0.025$	$\bar{\eta} = 0.342^{+0.016}_{-0.015}$

Sides and angles:				
$R_u = 0.371^{+0.015}_{-0.013}$	$R_t = 0.922^{+0.025}_{-0.026}$	$\alpha = (91.0 \pm 3.9)^\circ$	$\beta = (21.76^{+0.92}_{-0.82})^\circ$	$\gamma = (67.2 \pm 3.9)^\circ$

B_s system	
$\beta_s = (1.041^{+0.050}_{-0.048})^\circ$	$BF(B_s \to \mu\mu)[10^{-9}] = 3.073^{+0.070}_{-0.190}$

All measurements consistent with their predictions within $\pm 1\sigma$

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$B \to \tau\nu$

2.8 σ

$\Longrightarrow$ Severe constraints on MFV models with a charged Higgs.

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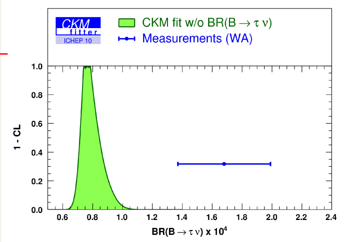

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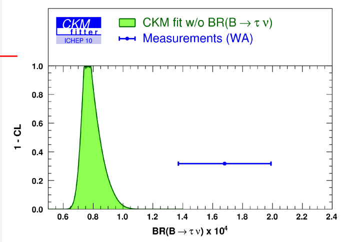
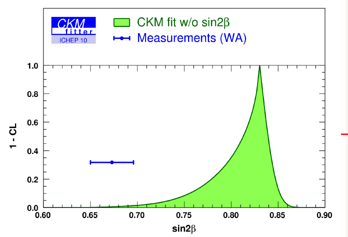
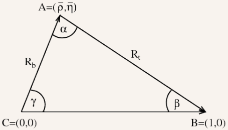


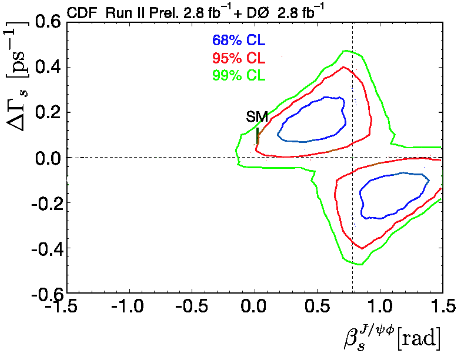
sin(2β) ⇉





2.6 σ

$\beta_s(B_s \to J/\psi\phi)$



The combined experimental constraints by CDF and DØ through $B_s \to J/\psi\phi$

$$\Delta\Gamma_s^{\rm SM} = (0.096 \pm 0.039)\ \mathrm{ps}^{-1} \qquad \beta_s^{J/\psi\phi(\rm SM)} = \mathrm{Arg}\left(-\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right) \approx 0.019 \pm 0.001$$

$$\begin{aligned} \Delta\Gamma_s &\in \pm(0.154^{+0.054}_{-0.070})\ \mathrm{ps}^{-1} \\ \beta_s^{J/\psi\phi} &\in (0.39^{+0.18}_{-0.14}) \cup (1.18^{+0.14}_{-0.18}) \end{aligned}$$

Anomalous like-sign dimuon charge asymmetry A^b_{sl}

$$A^b_{sl} \equiv \frac{\Gamma(b\bar{b} \to \mu^+\mu^+X) - \Gamma(b\bar{b} \to \mu^-\mu^-X)}{\Gamma(b\bar{b} \to \mu^+\mu^+X) + \Gamma(b\bar{b} \to \mu^-\mu^-X)}$$

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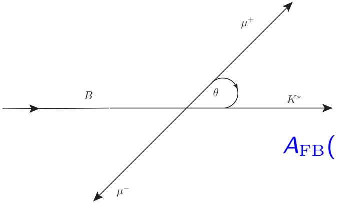
$A_{sl}^b = 0.00957 \pm 0.00251(\text{stat}) \pm 0.00146(\text{syst})$ DØ arXiv:1005.2757[hep-ex]

$A_{sl}^b(\text{SM}) = -2.3^{0.5}_{-0.6} \times 10^{-4}$ Lenz, Nierste, JHEP 0706:072, 2007

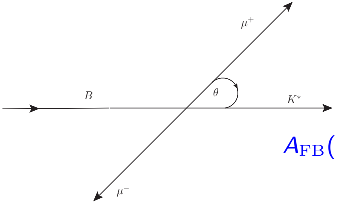
3.2 σ deviation from SM central value.



$$A_{\mathrm{FB}}(q^2) = \frac{\int_0^1 d\cos\theta \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d^2\Gamma}{dq^2 d\cos\theta}}{\int_0^1 d\cos\theta \frac{d^2\Gamma}{dq^2 d\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d^2\Gamma}{dq^2 d\cos\theta}}$$

$$(q = p_+ + p_-)$$

Forward-Backward Asymmetry: $B \to K^*\mu^+\mu^-$



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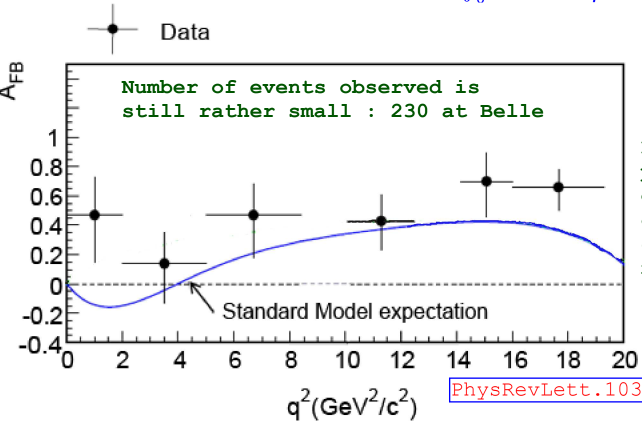
$(\mathrm{q} = \mathrm{p}_+ + \mathrm{p}_-)$



LHCb collaboration predicts 7200 signal events with a data set of 2 fb^{-1}, which corresponds to one nominal year of running.

PhysRevLett.103.171801

$B \to \pi K$ puzzle

- The extracted value of the weak phase $\gamma \sim (35.3 \pm 7.1)^\circ$ from the $B \to \pi K$ data disagrees with that of other independent measurements $\gamma \sim (66.8^{+5.4}_{-3.8})^\circ$

Baek, Chiang, London 2009

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Polarization fraction in $B \to \phi K^*$

- Ratio of transverse and longitudinal polarizations of final states is expected to be $f_T/f_L \ll 1$.
It is observed that $f_T/f_L \approx 1$

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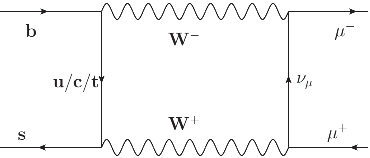
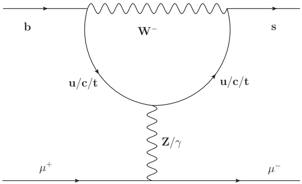
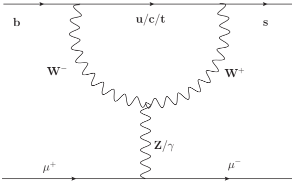
$\Longrightarrow$ **All discrepancies are in the $b \to s$ sector.**

b $\to$ s $\mu^+ \mu^-$ in the Standard Model : Free Quark picture







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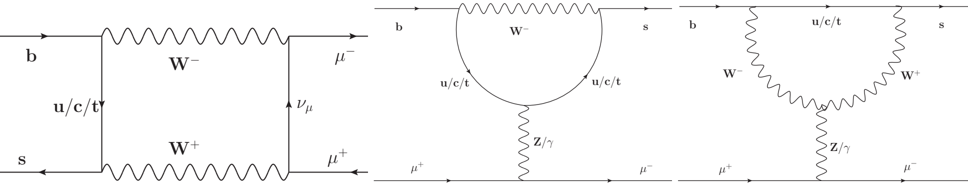

$$-\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left\{ C_7^{eff} \frac{e}{16\pi^2} [\bar{s}\sigma_{\mu\nu}(m_s P_L + m_b P_R) b] F^{\mu\nu} \right.$$

$$+ C_9^{eff} \frac{\alpha_{em}}{4\pi} [\bar{s}\gamma_\mu P_L b] [\bar{\mu}\gamma_\mu \mu]$$

$$\left. + C_{10}^{eff} \frac{\alpha_{em}}{4\pi} [\bar{s}\gamma_\mu P_L b] [\bar{\mu}\gamma_\mu \gamma_5 \mu] \right\}$$

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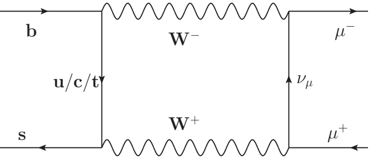

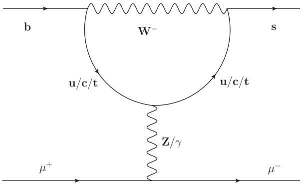

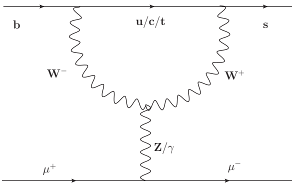

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- SM Wilson coefficients at $\mu = 4.8$ GeV in next-to-next-to-leading order(NNL0) accuracy :

$$C_7^{\text{eff}} = -0.304 \quad , \quad C_9^{\text{eff}} = 4.211 \quad , \quad C_{10}^{\text{eff}} = -4.103$$

Bobeth,Misiak,Urban 00
Misiak,Steinhauser 04
Gorbahn,Haisch 05
Gorbahn,Haisch,Misiak 05
Czakon,Haisch,Misiak 06

New Physics Operators

$$
\begin{aligned}
\mathcal{H}_{\mathrm{eff}}^{VA} \quad = \quad & -\frac{4G_F}{\sqrt{2}} \frac{\alpha_{em}}{4\pi} V_{ts}^* V_{tb} \times \\
& \Big\{ R_V \, [\bar{s}\gamma_\mu P_L b] \, [\bar{\mu}\gamma_\mu \mu] + R_A [\, \bar{s}\gamma_\mu P_L b] \, [\bar{\mu}\gamma_\mu \gamma_5 \mu] \\
& + R_V' \, [\bar{s}\gamma_\mu P_R b] \, [\bar{\mu}\gamma_\mu \mu] + R_A' \, [\bar{s}\gamma_\mu P_R b] \, [\bar{\mu}\gamma_\mu \gamma_5 \mu] \Big\}
\end{aligned}
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&+ R'_V\,[\bar{s}\gamma_\mu P_R b]\,[\bar{\mu}\gamma_\mu \mu] + R'_A\,[\bar{s}\gamma_\mu P_R b]\,[\bar{\mu}\gamma_\mu\gamma_5\mu]\Big\} \\
\mathcal{H}_{\mathrm{eff}}^{SP} \quad &= \quad -\frac{4G_F}{\sqrt{2}}\,\frac{\alpha_{em}}{4\pi}\,V_{ts}^{*}V_{tb}\,\times \\
&\Big\{ R_S\;[\bar{s}P_R b]\;[\bar{\mu}\mu] + R_P\;[\bar{s}P_R b]\;[\bar{\mu}\gamma_5\mu] \\
&+ R'_S\;[\bar{s}P_L b\;[\bar{\mu}\mu] + R'_P\;[\bar{s}P_L b]\;[\bar{\mu}\gamma_5\mu]\Big\}
\end{aligned}
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$$\mathcal{H}_{\rm eff}^{T} = -\frac{4G_F}{\sqrt{2}}\,\frac{\alpha_{em}}{4\pi}\,V_{ts}^* V_{tb}\,\times$$
$$\Big\{C_T\,[\bar{s}\sigma_{\mu\nu} b]\,[\bar{\mu}\sigma^{\mu\nu}\mu] + \quad C_{TE}\,i\epsilon^{\mu\nu\alpha\beta}\,[\bar{s}\sigma_{\mu\nu} b]\,[\bar{\mu}\sigma_{\alpha\beta}\mu]\Big\}$$

New Physics Operators

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- We take all new Wilson Coefficients real and hence do not discuss CP violation.

New Physics in $b \to s\ \mu^+\mu^-$: CP-Violating Observables, in preparation
A. K. Alok, A. Datta, A. Dighe, M. Duraisamy, DG, D. London, S. UmaSankar

From Free Quarks to Hadrons

Deacy Modes	Branching Ratios
$B_s^0(b\,\bar{s}) \to \mu^+\mu^-$	$\lesssim 3.6 \times 10^{-8}$(90% CL)

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- Mass scales involved :

$M_{B_d} = 5.28$ GeV, $M_{B_s} = 5.37$ GeV , $M_K = 498$ MeV, $M_{K^*} = 896$ MeV

$m_b(\mu = m_b) = 4.2$ GeV($\overline{MS}$) , $m_s(\mu = 2$ GeV$) = 101$ MeV ($\overline{MS}$)

$m_\mu = 105$ MeV

- $$\alpha_s(\mu = m_b) = \mathbf{0.22}$$

- Large background due to
  $B \to \{K, K^*, X_s\} J/\psi \to \{K, K^*, X_s\}\ l^+\ l^-$
  $B \to \{K, K^*, X_s\} \psi' \quad \to \{K, K^*, X_s\}\ l^+\ l^-$
  is vetoed explicitly by cuts on the dilepton invariant mass distribution $\Longrightarrow$ 6 GeV$^2 < (q^2)_{cut} <$ 14.4 GeV^2

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- Low q^2 region : 1 GeV^2 - 6 GeV^2

  High q^2 region : $\geq$ 14.4 GeV^2

Constraints on New Physics

- Branching Ratio of $B_s^0(b\ \bar{s}) \to \mu^+\mu^-$:

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$$\begin{cases} (1.60 \pm 0.50) \times 10^{-6} & (\text{low } q^2) \\ (0.44 \pm 0.12) \times 10^{-6} & (\text{high } q^2) \end{cases}$$

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- $B(\bar{B}_d^0 \to \bar{K} \mu^+\mu^-)_{\text{Measured}} =$

  $$\begin{cases} (1.36 \pm 0.24) \times 10^{-7} & (\text{low } q^2) \\ (1.36 \pm 0.28) \times 10^{-7} & (\text{high } q^2) \end{cases}$$

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  $$\Longrightarrow |R_S - R_S'|^2 + |R_P - R_P'|^2 \lesssim 0.44$$

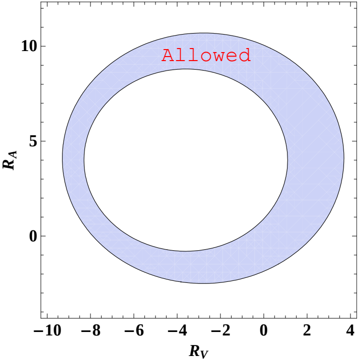
- $B(\bar{B}_d^0 \to X_s \mu^+\mu^-)_{\text{Measured}} =$

  $$\begin{cases} (1.60 \pm 0.50) \times 10^{-6} & (\text{low } q^2) \\ (0.44 \pm 0.12) \times 10^{-6} & (\text{high } q^2) \end{cases}$$

- $B(\bar{B}_d^0 \to \bar{K} \mu^+\mu^-)_{\text{Measured}} =$

  $$\begin{cases} (1.36 \pm 0.24) \times 10^{-7} & (\text{low } q^2) \\ (1.36 \pm 0.28) \times 10^{-7} & (\text{high } q^2) \end{cases}$$

$$\Longrightarrow \quad |C_T|^2 + 4|C_{TE}|^2 \lesssim 1.0$$



Decay Amplitude for $\mathrm{B(p_1)} \to \mathrm{K^*(p_2, \epsilon)}\ \mu^+(\mathrm{p_+})\ \mu^-(\mathrm{p_-})$:

$$\frac{\alpha G_F}{2\sqrt{2}\pi} V_{ts}^* V_{tb} \times \Big[\langle K^*(p_2, \epsilon)\, | \bar{s}\gamma_\mu (1-\gamma_5) b |\, B(p_1) \rangle$$

$$\Big\{ C_9^{\mathrm{eff}}\, \bar{u}(p_-)\gamma_\mu v(p_+) + C_{10}\, \bar{u}(p_-)\gamma_\mu\gamma_5 v(p_+) \Big\}$$

$$-\, 2\, \frac{C_7^{\mathrm{eff}}}{q^2}\, m_b \langle K^*(p_2, \epsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu (1+\gamma_5) b | B(p_1) \rangle\, \bar{u}(p_-)\gamma_\mu v(p_+) \Big]$$

Decay Amplitude for $B(p_1) \to K^*(p_2,\epsilon)\ \mu^+(p_+)\ \mu^-(p_-)$:

$$\frac{\alpha G_F}{2\sqrt{2}\pi} V_{ts}^* V_{tb} \times \Big[\langle K^*(p_2,\epsilon)\,|\bar{s}\gamma_\mu(1-\gamma_5)b|\,B(p_1)\rangle$$

$$\Big\{ C_9^{\rm eff}\,\bar{u}(p_-)\gamma_\mu v(p_+) + C_{10}\,\bar{u}(p_-)\gamma_\mu\gamma_5 v(p_+) \Big\}$$

$$-\,2\,\frac{C_7^{\rm eff}}{q^2}\,m_b \langle K^*(p_2,\epsilon)|\bar{s}\,i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_1)\rangle \bar{u}(p_-)\gamma_\mu v(p_+)\Big]$$

Form Factors

Form Factors

Form Factors ⇒

Decay Amplitude for $\mathrm{B(p_1) \to K^*(p_2, \epsilon)\ \mu^+(p_+)\ \mu^-(p_-)}$:

$$\frac{\alpha G_F}{2\sqrt{2}\pi} V_{ts}^* V_{tb} \times \Big[\langle K^*(p_2,\epsilon)\,|\bar{s}\gamma_\mu(1-\gamma_5)b|\,B(p_1)\rangle$$
$$\Big\{ C_9^{\text{eff}}\,\bar{u}(p_-)\gamma_\mu v(p_+) + C_{10}\,\bar{u}(p_-)\gamma_\mu\gamma_5 v(p_+)\Big\}$$
$$- 2\,\frac{C_7^{\text{eff}}}{q^2}\, m_b \langle K^*(p_2,\epsilon)|\bar{s}\, i\sigma_{\mu\nu} q^\nu(1+\gamma_5)b|B(p_1)\rangle\,\bar{u}(p_-)\gamma_\mu v(p_+)\Big]$$

- Low $\mathrm{q^2}$: QCD (Improved) Factorization(QCDF) Form Factors
- High $\mathrm{q^2}$: Light Cone Sum Rule(LCSR) Form Factors

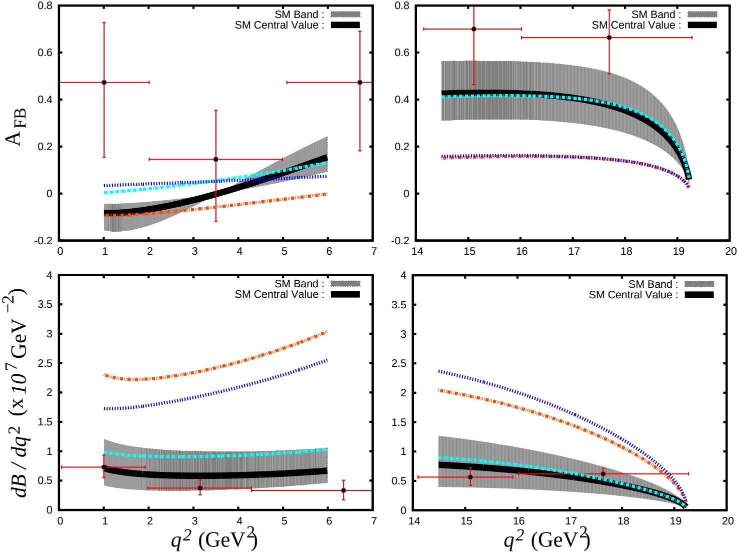
$B_d^0 \to K^* \mu^+ \mu^-$

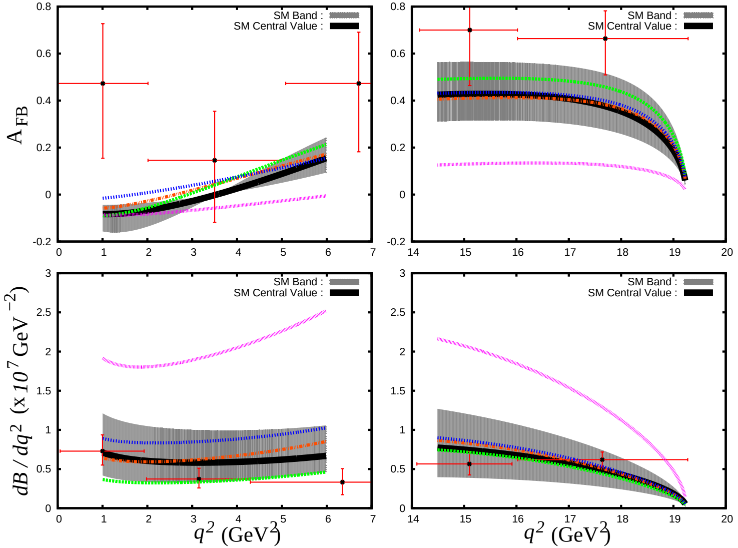
$B_d^0 \to K^* \mu^+ \mu^-$

- Scalar, Pseudo scalar : No deviation from SM $\Longrightarrow$ robust prediction

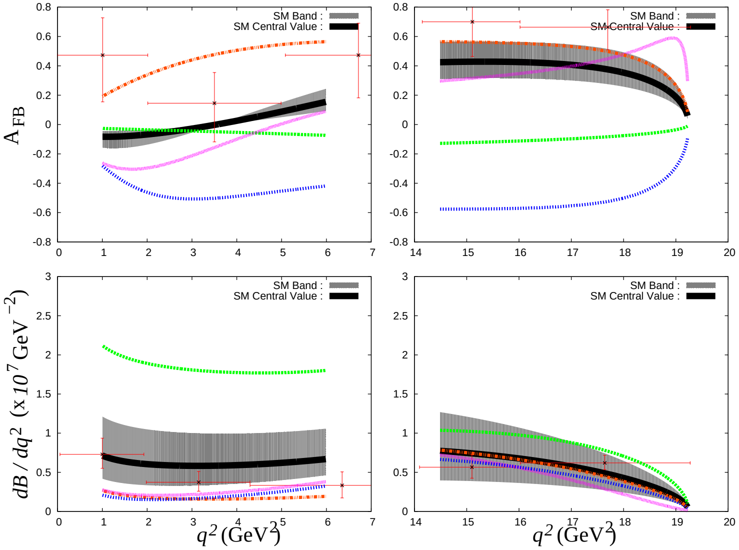
$B_d^0 \to K^* \mu^+ \mu^-$

- Scalar, Pseudo scalar : No deviation from SM $\Longrightarrow$ robust prediction
- Tensor :




SM Band :
SM Central Value :
A_{FB}
dB/dq^2 (x 10^7 GeV $^{-2}$)
q^2 (GeV^2)
q^2 (GeV^2)
0.8
0.6
0.4
0.2
0
-0.2
3
2.5
2
1.5
1
0.5
0
0 1 2 3 4 5 6 7
14 15 16 17 18 19 20

Successful scenario with new Vector-Axial Vector couplings



$B_d^0(b\,\bar{d}) \to X_s \mu^+ \mu^-$

- Forward Backward Asymmetry has a zero crossing in the SM: $(q^2)_0 = 3.5 \pm 0.12$ GeV^2 (NNLO+QED) Huber, Hurth, Lunghi 08
- A_{FB} is not measured yet but a precision of 5% is expected in the Super B factory.
- Effect of New Physics : Qualitatively same as in $B_d^0 \to K^* \mu^+ \mu^-$

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$B_s^0(b\,\bar{s}) \to \mu^+\mu^-\gamma$

- A_{FB} has a zero crossing in the low q^2 region : $q_0^2 \sim 4.3$ GeV^2
- A_{FB} is independent of form factors in the LEET limit $\Longrightarrow$ Deviation from SM will be difficult to accommodate by Form Factor uncertainties.
- SP operators do not contribute to this decay. Tensor operators can only suppress the A_{FB} from its SM value.
- Large deviation from SM is only possible by new VA operators.

$B_d^0(b\ \bar{d}) \to K(s\ \bar{d})\mu^+\mu^-$

Axial Vector Current

- $\left\langle \bar{K}(p_2) \left| \bar{s}\gamma_\mu\gamma_5 b \right| \bar{B}_d^0(p_1) \right\rangle = 0 \Longrightarrow A_{FB} = 0$ in SM.

$$\begin{aligned} \text{Integrated Asymmetry } \langle A_{FB} \rangle &= (0.15^{+0.21}_{-0.23} \pm 0.08)(\mathrm{BaBar}) \\ &\quad (0.10 \pm 0.14 \pm 0.01)(\mathrm{Belle}) \end{aligned}$$

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- New Physics VA operators do not contribute to A_{FB}.

$B_d^0(b\ \bar{d}) \to K(s\ \bar{d})\mu^+\mu^-$

- $\left\langle \bar{K}(p_2)\left|\bar{s}\gamma_\mu\gamma_5 b\right|\bar{B}_d^0(p_1)\right\rangle = 0 \Longrightarrow A_{FB} = 0$ in SM.

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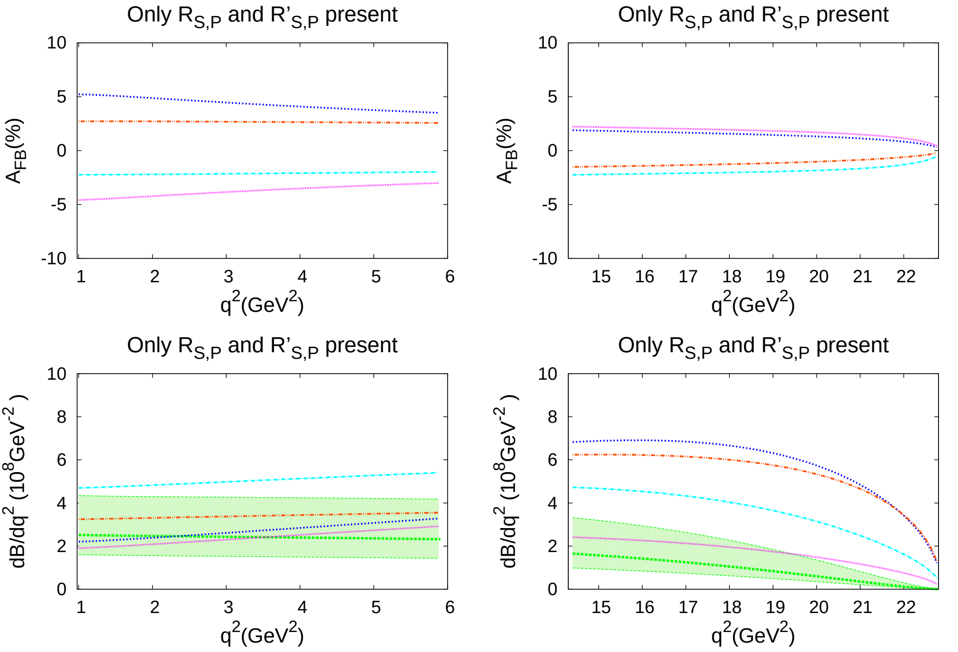
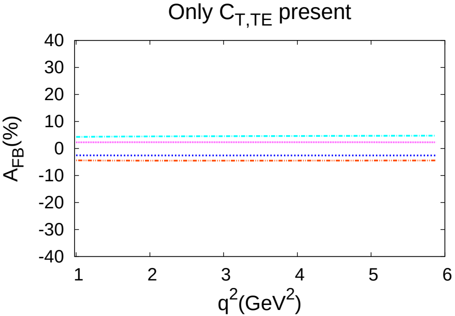
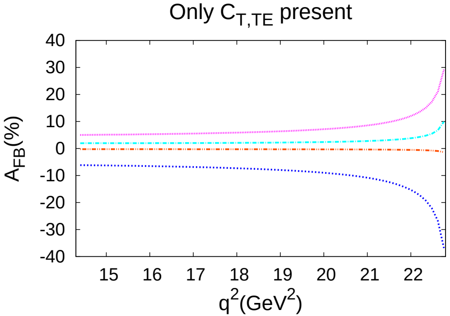
- New Physics VA operators do not contribute to A_{FB}.

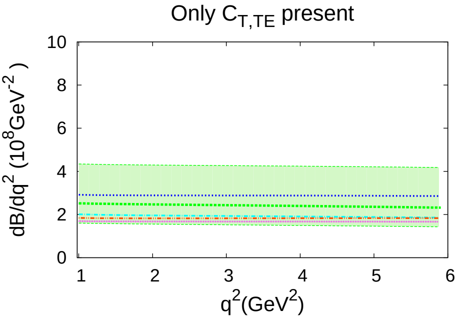
- Deviation from SM is possible only due to SP and/or T operators.

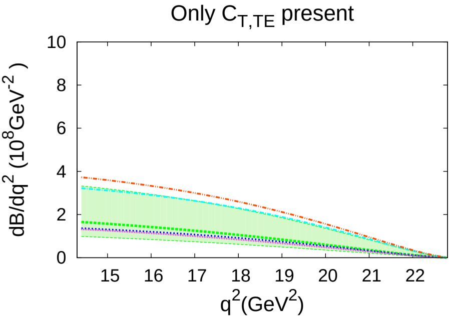

Only R_S,P and R'_S,P present
A_FB(%)
q^2(GeV^2)
Only R_S,P and R'_S,P present
A_FB(%)
q^2(GeV^2)
Only R_S,P and R'_S,P present
dB/dq^2 (10^8 GeV^-2)
q^2(GeV^2)
Only R_S,P and R'_S,P present
dB/dq^2 (10^8 GeV^-2)
q^2(GeV^2)

$B_d^0(b\ \bar{d}) \to K(s\ \bar{d})\mu^+\mu^-$

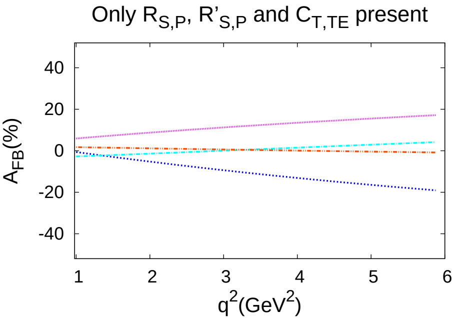


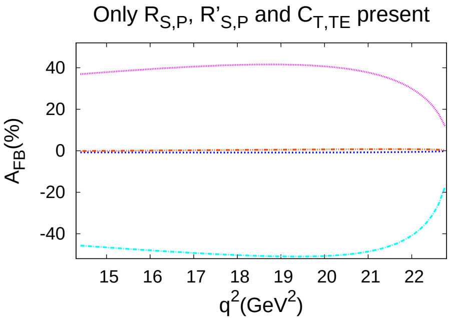


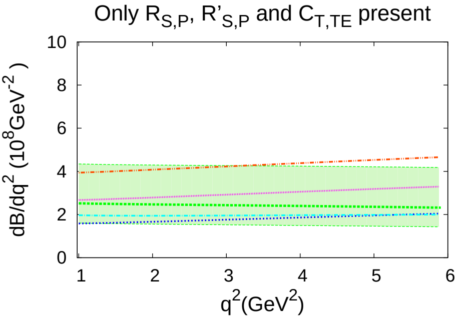


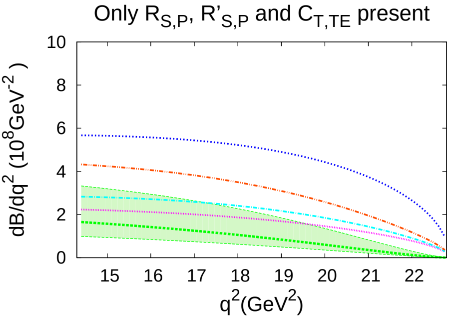


$B_d^0(b\ \bar{d}) \to K(s\ \bar{d})\mu^+\mu^-$

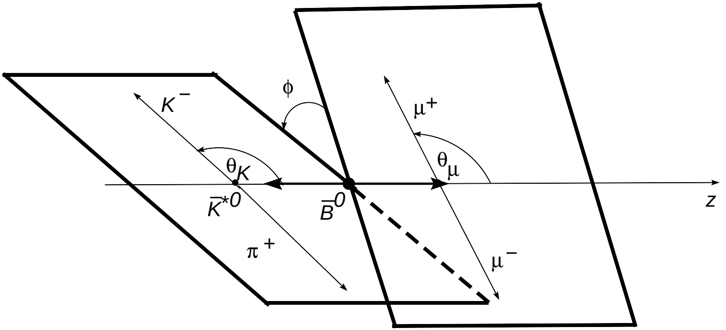








$B_d^0 \to K^*(\to K\ \pi)\mu^+\mu^-$: Fun with angles



Longitudinal and Transverse Polarisation Fractions f_L & f_T

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_\mu d\cos\theta_K d\phi} = N_F \Bigg\{ 2\left(I^0_{VA} + I^0_{VS} + I^0_{VT} + I^0_{AP} + I^0_{AT} + I^0_{SP} + I^0_{ST} + I^0_{PT} + I^0_{TT} \right) \cos^2\theta_K$$

$$+ \left(I^{++}_{VA} + I^{++}_{VT} + I^{++}_{AT} + I^{++}_{TT} \right) \sin^2\theta_K + \left(I^{+-}_{VA} + I^{+-}_{TT} \right) \sin^2\theta_K + \left(I^{0\pm}_{VA} + I^{0\pm}_{VS} + I^{0\pm}_{ST} + I^{0\pm}_{PT} + I^{0\pm}_{TT} \right) \sin 2\theta_K \Bigg\}$$

Longitudinal and Transverse Polarisation Fractions f_L & f_T

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_\mu d\cos\theta_K d\phi} = N_F\left\{2\left(I^0_{VA}+I^0_{VS}+I^0_{VT}+I^0_{AP}+I^0_{AT}+I^0_{SP}+I^0_{ST}+I^0_{PT}+I^0_{TT}\right)\cos^2\theta_K\right.$$

$$\left.+\left(I^{++}_{VA}+I^{++}_{VT}+I^{++}_{AT}+I^{++}_{TT}\right)\sin^2\theta_K+\left(I^{+-}_{VA}+I^{+-}_{TT}\right)\sin^2\theta_K+\left(I^{0\pm}_{VA}+I^{0\pm}_{VS}+I^{0\pm}_{ST}+I^{0\pm}_{PT}+I^{0\pm}_{TT}\right)\sin 2\theta_K\right\}$$

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\mu} = \frac{8\pi N_F}{3}\left\{\left(I^0_{VA}+I^0_{VS}+I^0_{VT}+I^0_{AP}+I^0_{AT}+I^0_{SP}+I^0_{ST}+I^0_{PT}+I^0_{TT}\right)+\left(I^{++}_{VA}+I^{++}_{VT}+I^{++}_{AT}+I^{++}_{TT}\right)\right\}$$

Longitudinal and Transverse Polarisation Fractions f_L & f_T

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\mu\, d\cos\theta_K\, d\phi} = N_F \Big\{ 2\Big(I^0_{VA} + I^0_{VS} + I^0_{VT} + I^0_{AP} + I^0_{AT} + I^0_{SP} + I^0_{ST} + I^0_{PT} + I^0_{TT} \Big) \cos^2\theta_K$$

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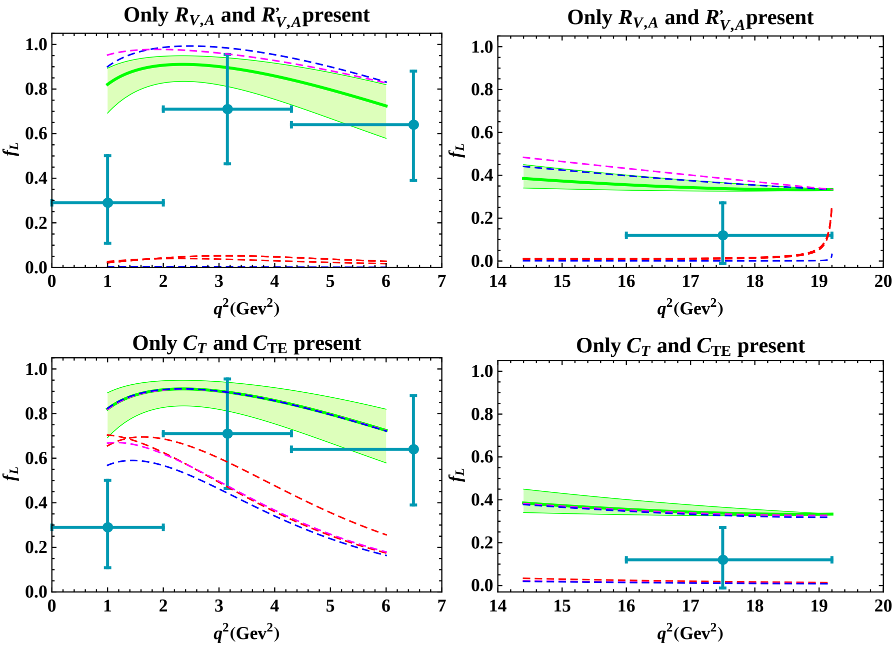
$$\frac{d^2\Gamma}{dq^2\, d\cos\theta_\mu} = \frac{8\pi N_F}{3} \Big\{ \Big(I^0_{VA} + I^0_{VS} + I^0_{VT} + I^0_{AP} + I^0_{AT} + I^0_{SP} + I^0_{ST} + I^0_{PT} + I^0_{TT} \Big) + \Big(I^{++}_{VA} + I^{++}_{VT} + I^{++}_{AT} + I^{++}_{TT} \Big) \Big\}$$

$$\frac{d\Gamma}{dq^2} = \frac{8\pi N_F}{3} (A_L + A_T)$$

$$\begin{aligned} A_L &= \int_{-1}^{1} d\cos\theta_\mu \Big(I^0_{VA} + I^0_{VS} + I^0_{VT} + I^0_{AP} + I^0_{AT} + I^0_{SP} + I^0_{ST} + I^0_{PT} + I^0_{TT} \Big), \\ A_T &= \int_{-1}^{1} d\cos\theta_\mu \Big(I^{++}_{VA} + I^{++}_{VT} + I^{++}_{AT} + I^{++}_{TT} \Big) \end{aligned}$$

$$f_L = \frac{A_L}{A_L + A_T} \quad , \qquad f_T = \frac{A_T}{A_L + A_T}$$

Longitudinal Polarisation Fraction f_L



Angular Asymmetries $A_T^{(2)}$ and A_{LT}

$A_T^{(2)}$

$$\frac{d^2\Gamma}{dq^2 d\phi} = \frac{1}{2\pi}\frac{d\Gamma}{dq^2}\left[1 + f_T\left(A_T^{(2)}\cos 2\phi + A_T^{(im)}\sin 2\phi\right)\right]$$

- In the Large Energy(of the final state meson) and Heavy Quark limit $A_T^{(2)}$ is ≈ 0 in the low-q^2 region in SM. Kruger, Matias 2005

A_{LT}

$$\frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K}$$

Angular Asymmetries $A_T^{(2)}$ and A_{LT}

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A_{LT}

$$\int_{-\pi/2}^{\pi/2} d\phi \frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K}$$

Angular Asymmetries $A_T^{(2)}$ and A_{LT}

$A_T^{(2)}$

$$\frac{d^2\Gamma}{dq^2 d\phi} = \frac{1}{2\pi}\frac{d\Gamma}{dq^2}\left[1 + f_T\left(A_T^{(2)}\cos 2\phi + A_T^{(im)}\sin 2\phi\right)\right]$$

- In the Large Energy(of the final state meson) and Heavy Quark limit $A_T^{(2)}$ is ≈ 0 in the low-q^2 region in SM.

A_{LT}

$$\frac{\{\int_0^1 d\cos\theta_K - \int_{-1}^0 d\cos\theta_K\}(\int_{-\pi/2}^{\pi/2} d\phi \frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K})}{\{\int_0^1 d\cos\theta_K + \int_{-1}^0 d\cos\theta_K\}(\int_{-\pi/2}^{\pi/2} d\phi \frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K})}$$

Angular Asymmetries $A_T^{(2)}$ and A_{LT}

$A_T^{(2)}$

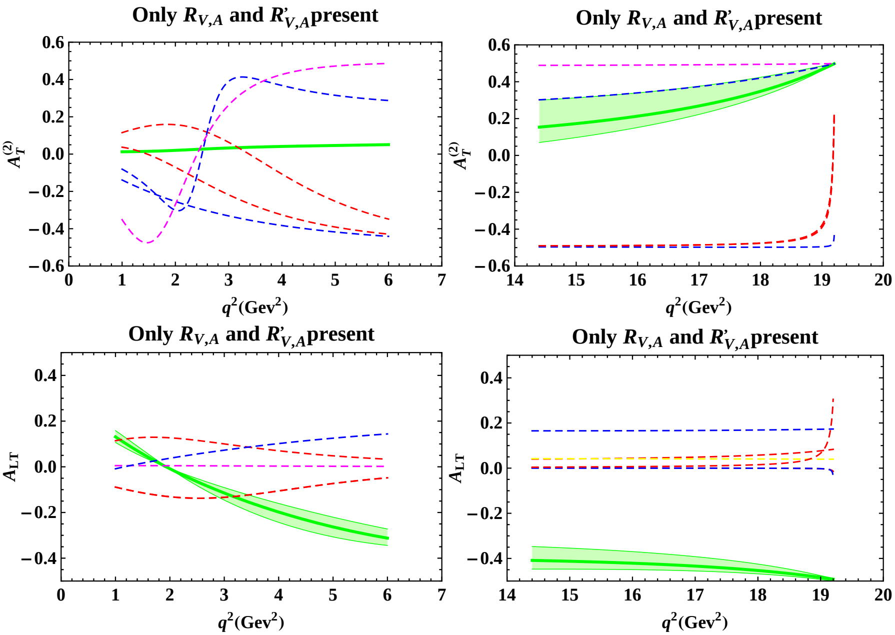
$$\frac{d^2\Gamma}{dq^2 d\phi} = \frac{1}{2\pi}\frac{d\Gamma}{dq^2}\left[1 + f_T\left(A_T^{(2)}\cos 2\phi + A_T^{(im)}\sin 2\phi\right)\right]$$

- In the Large Energy(of the final state meson) and Heavy Quark limit $A_T^{(2)}$ is ≈ 0 in the low-q^2 region in SM.

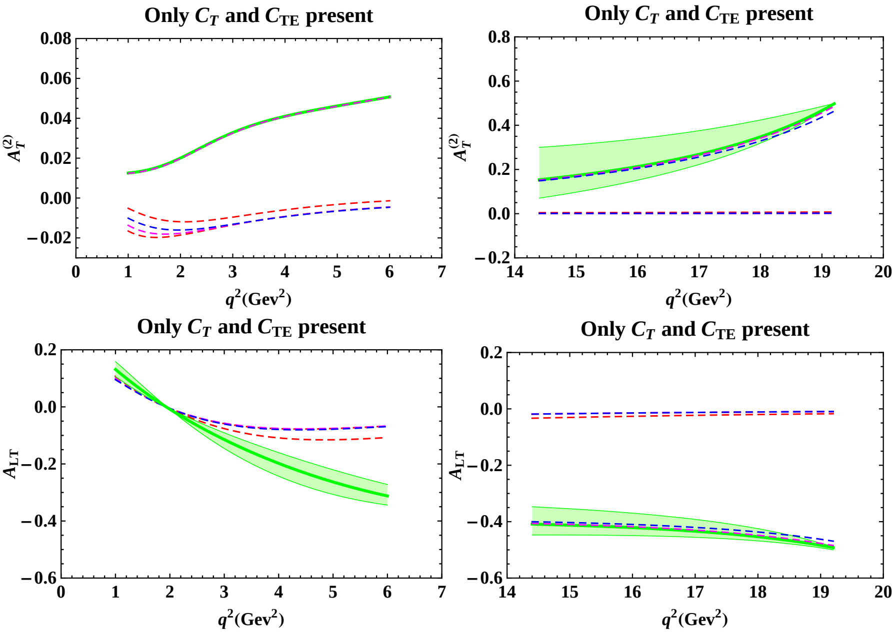
A_{LT}

$$\frac{\{\int_0^1 d\cos\theta_K - \int_{-1}^0 d\cos\theta_K\}(\int_{-\pi/2}^{\pi/2} d\phi \frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K})}{\{\int_0^1 d\cos\theta_K + \int_{-1}^0 d\cos\theta_K\}(\int_{-\pi/2}^{\pi/2} d\phi \frac{d^3\Gamma}{dq^2 d\phi d\cos\theta_K})}$$

- A_{LT} has a zero crossing in the SM. $(q^2)_0 \approx 1.96$GeV.
- Zero crossing point is independent of Form Factors in the Large Energy and Heavy Quark limit $\Longrightarrow$ Excellent observable to look for New Physics.


Only $R_{V,A}$ and $R'_{V,A}$present
$A_T^{(2)}$
q^2(Gev^2)
Only $R_{V,A}$ and $R'_{V,A}$present
$A_T^{(2)}$
q^2(Gev^2)
Only $R_{V,A}$ and $R'_{V,A}$present
A_{LT}
q^2(Gev^2)
Only $R_{V,A}$ and $R'_{V,A}$present
A_{LT}
q^2(Gev^2)

Angular Asymmetries $A_T^{(2)}$ and A_{LT} : Tensor



Summary

- B decays in particular, the $b \to s$ sector is an excellent place to look for New Physics.
- We have done a comprehensive analysis of various decay modes involving $b \to s$ transition with all possible New Physics lorentz structures in a very model independent way.
- We have considered observables and also constructed an observable with small hadronic uncertainties so that unambiguous predictions of the exixtance of new physics can be made.
- Even if the current deviations from Standard Model do not persist, or show some other features, our analysis enables us to recognize which kind of new physics structure could account for the observed deviation.

Stay tuned....

Questions/Comments ....


Thanks

Appendix-1: Effective Hamiltonian

$$Q_1 = (\bar{s}_i u_j)_{\mathrm{V-A}} (\bar{u}_j d_i)_{\mathrm{V-A}}\,, \quad Q_2 = (\bar{s}u)_{\mathrm{V-A}} (\bar{u}d)_{\mathrm{V-A}}$$

$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V-A}}$$

$$Q_4 = (\bar{s}_i d_j)_{\mathrm{V-A}} \sum_q (\bar{q}_j q_i)_{\mathrm{V-A}}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V+A}}$$

$$Q_6 = (\bar{s}_i d_j)_{\mathrm{V-A}} \sum_q (\bar{q}_j q_i)_{\mathrm{V+A}}$$

$$Q_7 = \frac{3}{2} (\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V+A}}$$

$$Q_8 = \frac{3}{2} (\bar{s}_i d_j)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_j q_i)_{\mathrm{V+A}}$$

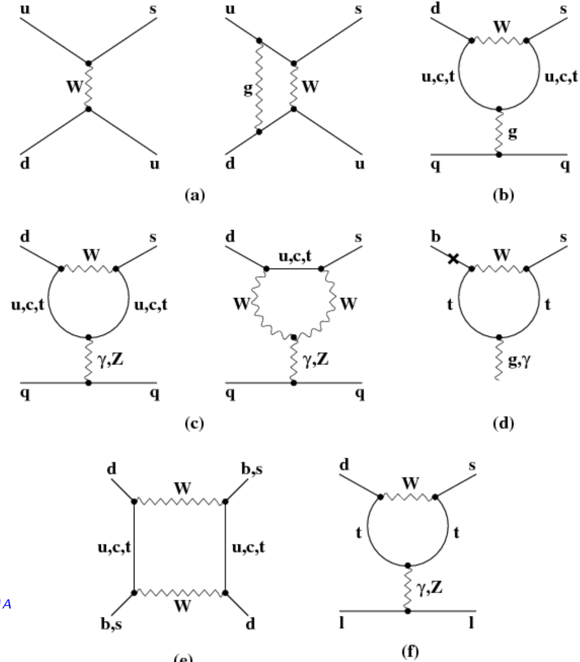
$$Q_9 = \frac{3}{2} (\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V-A}}$$

$$Q_{10} = \frac{3}{2} (\bar{s}_i d_j)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_j q_i)_{\mathrm{V-A}}$$

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) b_i F_{\mu\nu}$$

$$Q_{8G} = \frac{g}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) T^a_{ij} b_j G^a_{\mu\nu}$$

$$Q_{9V} = (\bar{b}s)_{V-A} (\bar{l}l)_V\,, \quad Q_{10A} = (\bar{b}s)_{V-A} (\bar{l}l)_A$$



$$\begin{aligned}
\left\langle \bar{K}(p_2)\left|\bar{s}\gamma_\mu b\right|\bar{B}^0_d(p_1)\right\rangle &= (2p_1-q)_\mu f_+(z) + (\frac{1-k^2}{z})\, q_\mu [f_0(z)-f_+(z)] \,, \\
\left\langle \bar{K}(p_2)\left|\bar{s} i\sigma_{\mu\nu} q^\nu b\right|\bar{B}^0_d(p_1)\right\rangle &= -\Big[(2p_1-q)_\mu q^2 - (m_B^2-m_K^2) q_\mu\Big] \frac{f_T(z)}{m_B+m_K} \,, \\
\left\langle \bar{K}(p_2)\left|\bar{s} b\right|\bar{B}^0_d(p_1)\right\rangle &= \frac{m_B(1-k^2)}{\hat{m}_b} f_0(z) \,, \\
\left\langle \bar{K}(p_2)\left|\bar{s}\sigma_{\mu\nu} b\right|\bar{B}^0_d(p_1)\right\rangle &= i\Big[(2p_1-q)_\mu q_\nu - (2p_1-q)_\nu q_\mu\Big] \frac{f_T(z)}{m_B+m_K}
\end{aligned}$$

Heavy quark limit : $\not{v} b = b$, $v^\mu = p_B^\mu / m_B$.
Multiply v^μ in both sides of the form factor equations.
After some algebra .. $(k = m_k/m_B, z = q^2/m_B^2)$

$$\begin{aligned}
f_T &= f_+(1+k) f_+ \\
f_0 &= \frac{1-k^2-z}{1-k^2} f_+
\end{aligned}$$

Basics ▸ ⇉

$$\mathcal{L}_{\mathrm{int}}^{\mathrm{CC}} = -\frac{g_2}{\sqrt{2}} \begin{pmatrix} \bar{u}_L, & \bar{c}_L, & \bar{t}_L \end{pmatrix} \gamma^\mu \, V_{\mathrm{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^\dagger + \mathrm{h.c.}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

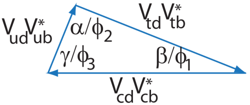
$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$s_{12} \equiv \lambda = 0.22, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$$

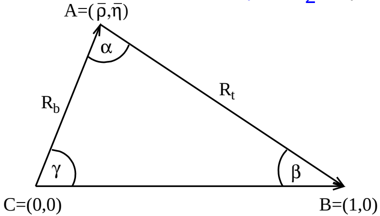
$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

(Wolfenstein Parametrization)

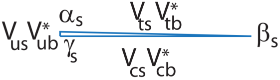
$$0 = V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = O(\lambda^3) + O(\lambda^3) + O(\lambda^3)$$



$$\bar{\rho} = \rho(1 - \frac{1}{2}\lambda^2)$$
$$\bar{\eta} = \eta(1 - \frac{1}{2}\lambda^2)$$



$$0 = V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = O(\lambda^4) + O(\lambda^2) + O(\lambda^2)$$



$$\alpha \equiv \arg(\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}), \beta \equiv \arg(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}), \gamma \equiv \arg(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}) \quad \beta_s \equiv \arg(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*})$$

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = (1 - \frac{\lambda^2}{2})\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \qquad R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$

$$V_{td} = |V_{td}|e^{-i\beta} \qquad V_{ub} = |V_{ub}|e^{-i\gamma}$$

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1$$