The meson-nucleon sigma terms in the chiral constituent quark model

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Outline

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Naive Quark Model

- **Internal Structure:** Knowledge has been rather limited because of confinement and it is still a big challenge to perform the calculations from the first principles of QCD.
- **Naive Quark Model** is able to provide a intuitive picture and successfully accounts for many of the low-energy properties of the hadrons in terms of the valence quarks.

The driving question "Proton Spin Problem"

- 1988 European Muon Collaboration (Valence quarks carry 30% of proton spin)
- Naive Quark Model contradicts this results (Based on Pure valence description: proton = 2u + d)
  "Proton spin crisis"
- Confirmed by the measurements of polarized structure functions of proton in the deep inelastic scattering (DIS) experiments by SMC, E142-3 and HERMES experiments.
- Provides evidence that the valence quarks of proton carry only a small fraction of its spin suggesting that they should be surrounded by an indistinct sea of quark-antiquark pairs.

Quark Sea

- Several interesting facts revealed regarding the flavor distribution functions.
- 1991 NMC result: Asymmetric nucleon sea ($\bar{d} > \bar{u}$)
  Recently confirmed by E866 and HERMES.
- Flavor structure of the nucleon is not limited to u and d quarks only.
- The measured quark sea asymmetry established that the study of the structure of the nucleon is intrinsically a nonperturbative phenomena.
- Non-perturbative effects explained only through the generation of “quark sea”

Strangeness at high energies

- Recent indications of Strangeness.
- Electromagnetic form factors SAMPLE at MIT-Bates , G0 at JLab, A4 at MAMI and HAPPEX at JLab. These experiments have provided considerable insight on the role played by strange quarks when the nucleon interacts at high energies.
- The strange spin polarization of the nucleon is well established through the measurements of polarized structure functions in DIS experiments.

Strangeness at low energies

- Strangeness content in the nucleon y_N has been indicated in the context of low-energy experiments.
- The CCFR Collaboration and more recently the NuTeV collaboration have given fairly good deal of information regarding the integrals of strangeness dependent quark ratios in the nucleon given as $\frac{2\bar{s}}{u+d} = 0.099^{+0.009}_{-0.006}$ and $\frac{2\bar{s}}{\bar{u}+\bar{d}} = 0.477^{+0.063}_{-0.053}$.
- A large value of pion-nucleon sigma term indicating a non zero strangness fraction f_s is also indicative of the presence of strange quarks in the nucleon.
- Even a considerable progress in the past few years to estimate the strangeness matrix elements does not lead to a consensus regarding the various mechanisms which can contribute to y_N or f_s.

Outline

1. Introduction
   - Internal structure of the baryons
   - Meson-nucleon sigma terms
2. Chiral Constituent Quark Model
   - The flavor structure
3. The Pion-Nucleon Sigma Term
   - OZI sum rule
4. Meson-Baryon Sigma Terms
   - Kaon-nucleon sigma term
   - Eta-nucleon sigma term
5. Results
   - New Observations
   - Application Potential
6. Summary and Conclusions

- Currently, there is enormous interest in determining the meson-nucleon sigma terms.
- These are the fundamental parameters to test the chiral symmetry breaking effects and thereby determine the scalar quark content of the baryons.
- These cannot be measured directly from experiments and are known to have intimate connection with the dynamics of the non-valence quarks.
- They are theoretically interesting because there is a discrepancy in the value derived from the meson-nucleon scattering experiments and from the hadron spectroscopy.
- The meson-nucleon sigma terms also provide restriction on the contribution of strangeness to the parameters measured in low-energy.

Purpose

- To understand the implications of chiral symmetry breaking for the scalar matrix elements of the nucleon within the chiral constituent quark model.
- To phenomenologically estimate the quantities affected by the hidden strangeness component in the nucleon.
- Strangeness content in the nucleon y_N, strangeness fraction f_s, the SU(3) antisymmetric and symmetric parameters for the flavor structure (F_s and D_s), the total flavor singlet and non-singlet contents (G_s^0, G_s^3, G_s^8, etc.).
- To study the meson-nucleon sigma terms (σ_{KN}, $\sigma_{\eta N}$) as well as the meson-baryon sigma terms for Σ and Ξ baryons which have not been observed experimentally and are expected to have large contributions from the quark sea.

Chiral Constituent Quark Model

- χCQM initiated by Weinberg and developed by Manohar and Georgi to explain the successes of NRQM.
- "Quark sea" generation $q_{\pm} \rightarrow GB^0 + q'_{\mp} \rightarrow (q\bar{q}') + q'_{\mp}$
- Incorporates *confinement* and *chiral symmetry breaking.*
- "Justifies" the idea of constituent quarks and scope of the model extended in the context of **"proton spin crisis"**

Methodology

- "Quark sea" generation $q_{\pm} \rightarrow GB^{0} + q^{'}_{\mp} \rightarrow (q\bar{q}^{'}) + q^{'}_{\mp}$ where GB represents the Goldstone boson and $q\bar{q}^{'} + q^{'}$ constitute the "quark sea".
- The effective Lagrangian describing the interaction between quarks and a nonet of GBs, can be expressed as $\mathcal{L} = g_8 \bar{q} \Phi q + g_1 \bar{q} \frac{\eta^{'}}{\sqrt{3}} q = g_8 \bar{q} \left(\Phi + \zeta \frac{\eta^{'}}{\sqrt{3}} I \right) q$
- $\Phi' = \begin{pmatrix} \phi_{uu} u\bar{u} + \phi_{ud} d\bar{d} + \phi_{us} s\bar{s} & \varphi_{ud} u\bar{d} & \varphi_{us} u\bar{s} \\ \varphi_{du} d\bar{u} & \phi_{du} u\bar{u} + \phi_{dd} d\bar{d} + \phi_{ds} s\bar{s} & \varphi_{ds} d\bar{s} \\ \varphi_{su} s\bar{u} & \phi_{sd} s\bar{d} & \phi_{su} u\bar{u} + \phi_{sd} d\bar{d} + \phi_{ss} s\bar{s} \end{pmatrix}$

- $$\begin{aligned} \phi_{uu} &= \phi_{dd} = \frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}\,, \quad \phi_{ss} = \frac{2\beta}{3} + \frac{\zeta}{3}\,, \quad \phi_{us} = \phi_{ds} = \phi_{su} = \phi_{sd} = -\frac{\beta}{3} + \frac{\zeta}{3}\,, \\ \phi_{du} &= \phi_{ud} = -\frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}\,, \quad \varphi_{ud} = \varphi_{du} = 1\,, \quad \varphi_{us} = \varphi_{ds} = \varphi_{su} = \varphi_{sd} = \alpha\,. \end{aligned}$$
- The parameter $a(=|g_8|^2)$ denotes the transition probability of chiral fluctuation of the splittings $u(d) \rightarrow d(u) + \pi^{+(-)}$, whereas $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$ respectively denote the probabilities of transitions of $u(d) \rightarrow s + K^{-(o)}$, $u(d,s) \rightarrow u(d,s) + \eta$, and $u(d,s) \rightarrow u(d,s) + \eta^{'}$.

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The scalar matrix elements of the nucleon

- The flavor structure of the nucleon is defined as
  $\hat{N} \equiv \langle N|q\bar{q}|N\rangle$
  $|N\rangle$ is the nucleon wavefunction and $q\bar{q}$ is the number operator for the scalar quark content.
- The sum of the quark and antiquark numbers
  $q\bar{q} = \sum_{q=u,d,s}(n_q q + n_{\bar{q}}\bar{q}) = n_u u + n_{\bar{u}}\bar{u} + n_d d + n_{\bar{d}}\bar{d} + n_s s + n_{\bar{s}}\bar{s}$
  $n_{q(\bar{q})}$ being the number of $q(\bar{q})$ quarks.

- The pion-nucleon sigma term ($\sigma_{\pi N}$) affected by the contributions of the quark sea

$$\sigma_{\pi N} = \hat{m}\langle N|\bar{u}u + \bar{d}d|N\rangle = \hat{m}\left(3 + 6a\left(\phi_{uu}^2 + \phi_{ud}^2 + \varphi_{ud}^2\right)\right)$$

- $\hat{m} = \frac{(m_u+m_d)}{2}$ is the average value of current u and d quark masses evaluated at fixed gauge coupling and $q\bar{q}$ is the scalar quark content.
- $\sigma_{\pi N}$ provides restriction on the contribution of strange quarks in the nucleon. In terms of the strangeness content in nucleon y_N

$$\sigma_{\pi N} = \hat{m}\frac{\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle}{1 - 2y_N} = \frac{\hat{\sigma}}{1 - 2y_N}$$

$$\hat{\sigma} = \hat{m}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle = \hat{m}(3 + 6a(\phi_{uu}^2 + \phi_{ud}^2 + \varphi_{ud}^2 - 2\phi_{us}^2 - 2\varphi_{us}^2))$$

$$y_N = \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \frac{2a(\phi_{us}^2 + \varphi_{us}^2)}{1 + 2a(\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2 + \varphi_{ud}^2 + \varphi_{us}^2)}$$

Quark mass ratio

- $\sigma_{\pi N}$ is related to the hadron mass spectrum as well as the quark mass ratio

$$\hat{\sigma} = -\frac{3(M_{\Xi} - M_{\Lambda})}{(1 - \frac{m_s}{\hat{m}})}$$

  where M_{Ξ} and M_{Λ} are the baryon masses.

- The latest accepted quark mass ratio $\frac{m_s}{\hat{m}}$ range is 22-30

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- The strangeness fraction of the nucleon is related to the strangeness content in nucleon y_N as

$$f_s = \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d + \bar{s}s|N\rangle} = \frac{y_N}{1 - y_N} = \frac{\sigma_{\pi N} - \hat{\sigma}}{3\sigma_{\pi N} - \hat{\sigma}}$$

- The strangeness sigma term is

$$\sigma_s = m_s\langle N|\bar{s}s|N\rangle = \frac{1}{2} y_N \frac{m_s}{\hat{m}} \sigma_{\pi N}$$

- The validity of OZI rule in the case of NQM would imply $y_N = f_s = 0$ or $\hat{\sigma} = \sigma_{\pi N}$ and for $\frac{m_s}{\hat{m}} = 22$, $\sigma_{\pi N}$ comes out to be close to 28 MeV.
- The most recent analysis of experimental data gives higher values of $\sigma_{\pi N}$ which points towards a significant strangeness content in the nucleon.

Axial-vector matrix elements

- The SU(3) symmetric and antisymmetric scalar matrix elements characterize the weak matrix elements for the flavor structure and have implications for the strangeness contribution to the nucleon

$$F_S = \frac{1}{2}\langle N|\bar{u}u - \bar{s}s|N\rangle = 1 + a(2\phi_{uu}^2 + \phi_{du}^2 + \varphi_{du}^2 - 3\phi_{us}^2 - 3\varphi_{us}^2)$$

$$D_S = \frac{1}{2}\langle N|\bar{u}u - 2\bar{d}d + \bar{s}s|N\rangle = 3a\left(-\phi_{ud}^2 - \varphi_{du}^2 + \phi_{us}^2 + \varphi_{us}^2\right)$$

Singlet and non-singlet combinations of the flavor structure

- The singlet and non-singlet combinations of the flavor structure related to the weak couplings are expressed as

$$g_A^0 = \langle N|\bar{u}u + \bar{d}d + \bar{s}s|N\rangle = 3 + 6a(\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2 + \varphi_{ud}^2 + \varphi_{us}^2)$$

$$g_A^3 = \langle N|\bar{u}u - \bar{d}d|N\rangle = 1 + 2a(\phi_{uu}^2 - \phi_{ud}^2 - \varphi_{ud}^2)$$

$$g_A^8 = \langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle = 3 + 6a(\phi_{uu}^2 + \phi_{ud}^2 + \varphi_{ud}^2 - 2\phi_{us}^2 - 2\varphi_{us}^2)$$

Outline

- The kaon-nucleon sigma term in terms of the scalar quark content and $\sigma_{\pi N}$ is

$$\sigma_{KN} = \frac{\sigma_{KN}^{u} + \sigma_{KN}^{d}}{2} = \frac{\hat{m} + m_s}{2} \langle N|\bar{u}u + \bar{d}d + 2\bar{s}s|N\rangle = \frac{\hat{m} + m_s}{4\hat{m}} (2\sigma_{\pi N} - \hat{\sigma})$$

where

$$\sigma_{KN}^{u} = \frac{\hat{m} + m_s}{2} \langle N|\bar{u}u + \bar{s}s|N\rangle$$

$$\sigma_{KN}^{d} = \frac{\hat{m} + m_s}{2} \langle N|\bar{d}d + \bar{s}s|N\rangle .$$

Outline

- The η-nucleon sigma term can be expressed as

$$\sigma_{\eta N} = \frac{1}{3}\langle N|\hat{m}(\bar{u}u + \bar{d}d) + 2m_s\bar{s}s|N\rangle = \frac{1}{3}\hat{\sigma} + \frac{2(m_s + \hat{m})}{3\hat{m}} y_N \sigma_{\pi N}.$$

- The calculations can be extended to the calculations of meson-baryon sigma terms of Σ and Ξ baryons using the respective baryon wavefunctions and antiquark distribution functions.

Table: The χCQM results for the scalar matrix elements of the nucleon.

Quantity	Phenomenology	NQM	χCQM with SU(3) symmetry	χCQM with SU(3) symmetry breaking
$\langle N\|\bar{u}u\|N\rangle$	...	≤ 2	2.41	2.44
$\langle N\|\bar{d}d\|N\rangle$	...	≤ 1	1.75	1.68
$\langle N\|\bar{s}s\|N\rangle$	...	0.0	1.08	0.18
y_N	0.11 ± 0.07	0.0	0.26	0.044
f_s	0.10 ± 0.06	0.0	0.21	0.042
F_s	1.52 , 1.81	≤ 1	0.67	1.13
D_s	-0.52 , -0.57	0.0	0.0	-0.37
R_s	...	≤ 3	6.22	5.39
G_S^0	...	≤ 3	5.24	4.30
G_S^3	...	≤ 1	0.67	0.76
G_S^8	...	≤ 3	2.01	3.76

Table: The χCQM results for the meson-nucleon sigma terms at different values of the quark mass ratio $\frac{m_s}{\hat{m}}$ in the range $22 - 30$.

Quantity	NQM	χCQM with SU(3)			
		with SU(3) symmetry	with SU(3) symmetry breaking		
		$\frac{m_s}{\hat{m}} = 22$	$\frac{m_s}{\hat{m}} = 22$	$\frac{m_s}{\hat{m}} = 26$	$\frac{m_s}{\hat{m}} = 30$
$\hat{\sigma}$	28.57	28.57	28.57	24	20.69
σ_s	0	168.71	15.12	15.01	14.93
$\sigma_{\pi N}$	28.57	59.25	31.32	26.31	22.68
σ_{KN}	164.29	517.04	195.90	193.17	191.20
$\sigma_{\eta N}$	9.52	244.70	30.60	28.78	27.47

Table: The χCQM results for the meson-baryon sigma terms for the quark mass ratio $\frac{m_s}{\hat{m}} = 22$.

Baryon (B)	$\sigma_{\pi B}$	σ_{KB}	$\sigma_{\eta B}$
N	31.32	195.90	30.60
Σ	137.76	1419.97	846.65
Ξ	-17.96	-370.76	-347.17

Outline

- The non-zero values for y_N and f_s indicate that the chiral symmetry breaking is essential to understand the significant role played by the quark sea.
- SU(3) symmetry breaking results better as compared to SU(3) symmetry results.
- The quantities involving the strange quark content are very sensitive to SU(3) symmetry breaking. For example, y_N, f_s, F_s, D_s, G_s^0 and G_s^8.
- Results consistent with the experimental data as well as with other phenomenological models.
- Most widely accepted range $22 - 30$ has been used for $\frac{m_s}{\hat{m}}$ to calculate $\hat{\sigma}$, σ_s, $\sigma_{\pi N}$, σ_{KN}, and $\sigma_{\eta N}$. The possibility of readjusting the quark mass ratio to get higher value of σ term is ruled out.

Outline

Long term

Understanding the flavor structure of the proton will help to resolve the most challenging problems facing subatomic physics which include

- How can we distinguish between the *current quarks* and the *constituent quarks*?
- What is the role played by non-valence flavors in understanding the hadron internal structure?

Future DAΦNE experiments and the hyperon-antihyperon production in heavy ion collisions will allow a determination of KN sigma terms and hence could restrict the model parameters and provide better knowledge of strangeness content of the nucleon.

Since the strange quarks constitute purely sea degrees of freedom, the low-energy determination would undoubtedly provide vital clues to the nonperturbative aspects of QCD.

Summary and Conclusions

- χCQM phenomenologically estimates the scalar matrix elements having implications for chiral symmetry breaking, SU(3) symmetry breaking and hidden strangeness component in the nucleon.
- At leading order, the model envisages constituent quarks, the Goldstone bosons (π, K, η mesons) as appropriate degrees of freedom.
- A refinement in the analysis of $\pi - N$ scattering giving higher values of $\sigma_{\pi N}$ would strengthen the mechanism of chiral symmetry breaking generating the appropriate amount of strangeness in the nucleon. The σ_{KN} and $\sigma_{\eta N}$ terms are found to be quite sensitive to y_N.