

Chiral symmetry breaking in strong magnetic background

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- 1 Introduction
 - Chiral symmetry breaking in vacuum.
 - Chiral symmetry breaking in magnetic background.
- 2 Solution of Dirac equation in magnetic field.
- 3 Ansatz for the ground state and evaluation of the thermodynamic potential.
- 4 Mass gap equation.
- 5 Results and discussions.

- 1 Hadrons get mass in vacuum through $q\bar{q}$ pairing.
- 2 Chiral symmetry is restored at high temperature.

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 - Strong magnetic field exists in compact stars.
 - Relevant for heavy ion collision experiments (Mag. field $\sim 10^{18}$ Gauss).
- 4 NJL model for studying χ SB.

Dirac equation in magnetic field

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- 5 For $E < 0$, the energy levels are given by

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$$|vac\rangle = \mathcal{U} |0\rangle = e^{B^\dagger - B} |0\rangle$$

Where

$$B^\dagger = \sum_n \int d\mathbf{p}_x q_r^\dagger(n, \mathbf{p}_x) a_{r,s}(n, p_z) f(n, \mathbf{p}_x) \tilde{q}_s(n, -\mathbf{p}_x)$$

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- 3 $f(n, \mathbf{p}_x)$ is the condensate function. and

$$a_{r,s} = \frac{1}{\sqrt{p_z^2 + 2n|q_i|B}} \left[-\sqrt{2n|q_i|B} \delta_{r,s} - ip_z \delta_{r,-s} \right]$$

- 1 The chiral condensate term is given by

$$\begin{aligned}\langle \bar{\psi}\psi \rangle_i &= -\sum_{n=0}^{\infty} \frac{N_c |q_i| B \alpha_n}{(2\pi)^2} \int dp_z \frac{\cos 2\theta_{\pm}^i}{\epsilon_{ni}} [m_i \cos 2f_i + |p_i| \sin 2f_i] \\ &= -I_i\end{aligned}$$

where

$$\epsilon_{ni} = \sqrt{m_i^2 + p_z^2 + 2n|q_i|B}$$

and

$$\cos 2\theta_{\pm}^i = 1 + \sin^2 \theta_-^i + \sin^2 \theta_+^i$$

3 – *flavor* NJL Lagrangian

- ① The 3 – *flavor* NJL Lagrangian in presence of magnetic field with the Kobayashi-Maskawa-t'Hooft [KMT] vertex is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi} (i\partial - q_i B x \alpha_2 - m) \psi + G \sum_{A=0}^8 \left[(\bar{\psi} \lambda^A \psi)^2 + (\bar{\psi} \gamma^5 \lambda^A \psi)^2 \right] \\ & - K \left[\det\{\bar{\psi}(1 + \gamma^5)\psi\} + \det\{\bar{\psi}(1 - \gamma^5)\psi\} \right] \end{aligned}$$

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- 2 λ 's are the Gell Mann matrices satisfying

$$\sum_{A=0}^8 \lambda_{ij}^A \lambda_{kl}^A = 2\delta_{il}\delta_{jk}$$

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$$\Omega = T + V - \mu \langle \psi^\dagger \psi \rangle - \frac{S}{\beta}$$

Thermodynamic potential

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$$\Omega = T + V - \mu \langle \psi^\dagger \psi \rangle - \frac{S}{\beta}$$

- ② S is the entropy which is given by

$$S = - \sum_i \sum_n \frac{N_c \alpha_n |q_i| B}{(2\pi)^2} \int dp_z \{ \sin^2 \theta_-^i \ln \sin^2 \theta_-^i + - \leftrightarrow + \}$$
$$+ \int dp_z \{ \cos^2 \theta_-^i \ln \cos^2 \theta_-^i + - \leftrightarrow + \}$$

- 1 The thermodynamic potential is given by

$$\Omega = - \sum_{n=0}^{\infty} \sum_i \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z \sqrt{M_i^2 + |p_i|^2} + 2G \sum_i l_i^2 + 4K l_1 l_2 l_3$$

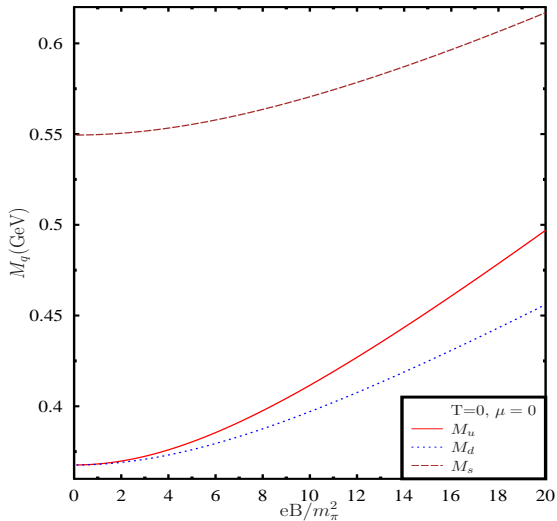
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- 2 The gap equation at $T = 0, \mu = 0$ is given by

$$M_i = m_i + 4G l_i + 2K |\epsilon_{ijk}| l_j l_k$$

$T = 0, \mu = 0$ case



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$$\begin{aligned}\Omega &= -\frac{2N_c}{(2\pi)^3} \sum_i \int d^3p \sqrt{\mathbf{p}_i^2 + M_i^2} + 2G \sum_i l_i^2 + 4Kl_1l_2l_3 \\ &- \frac{N_c}{2\pi^2} \sum_i |q_i B|^2 \left[\zeta'(-1, x_i) - \frac{1}{2}(x_i^2 - x_i) \ln x_i + \frac{x_i^2}{4} \right] \\ &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln \{1 + e^{-\beta(\omega_i - \mu_i)}\} \\ &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln \{1 + e^{-\beta(\omega_i + \mu_i)}\}\end{aligned}$$

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 &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln \{1 + e^{-\beta(\omega_i - \mu_i)}\} \\
 &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln \{1 + e^{-\beta(\omega_i + \mu_i)}\}
 \end{aligned}$$

Where $x_i = \frac{M_i^2}{2|q_i B|}$. and

$$\omega_{ni} = \sqrt{M_i^2 + |\mathbf{p}_i|^2}$$

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$T \neq 0, \mu \neq 0$ case

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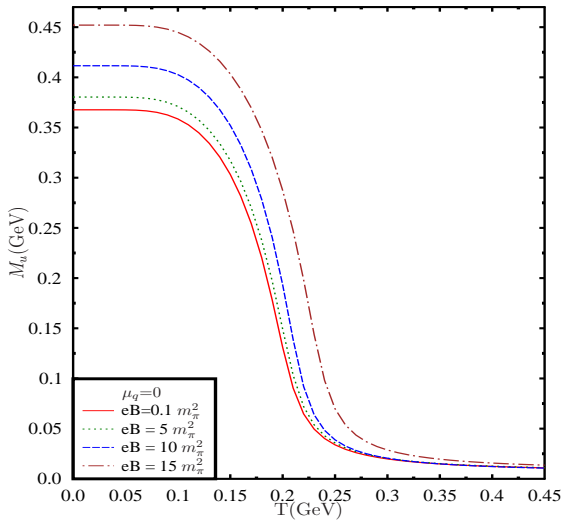
$$M_i = m_i + 4G l_i + 2K |\epsilon_{ijk}| l_j l_k$$

- ② The chiral condensate term is given by

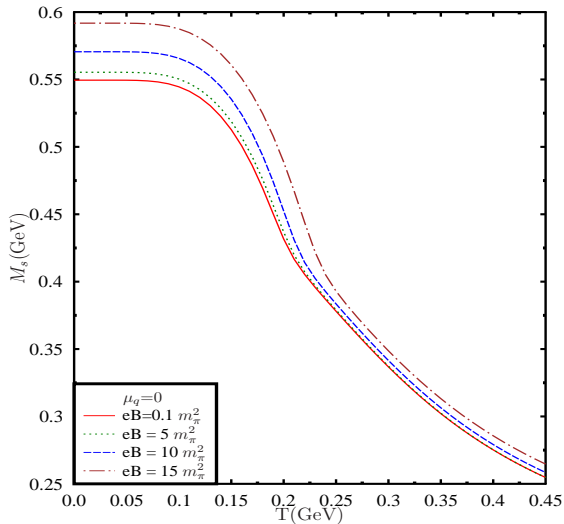
$$\begin{aligned} l_i &= l_{vac}^i + l_{field}^i + l_{med}^i \\ &= \frac{N_c M_i}{2\pi^2} \left[\Lambda \sqrt{\Lambda^2 + M_i^2} - m_i^2 \ln \left(\frac{\Lambda + \sqrt{\Lambda^2 + M_i^2}}{M_i} \right) \right] \\ &+ \frac{N_c M_i |q_i B|}{(2\pi)^2} \left[x_i (1 - \ln x_i) + \ln \Gamma(x_i) + \frac{1}{2} \ln \frac{x_i}{2\pi} \right] \\ &- \sum_{n=0}^{\infty} \frac{N_c |q_i| B \alpha_n}{(2\pi)^2} \int dp_z \frac{M_i}{\sqrt{M_i^2 + |p_i|^2}} (\sin^2 \theta_-^i + \sin^2 \theta_+^i) \end{aligned}$$

where Λ is the cut off in NJL model.

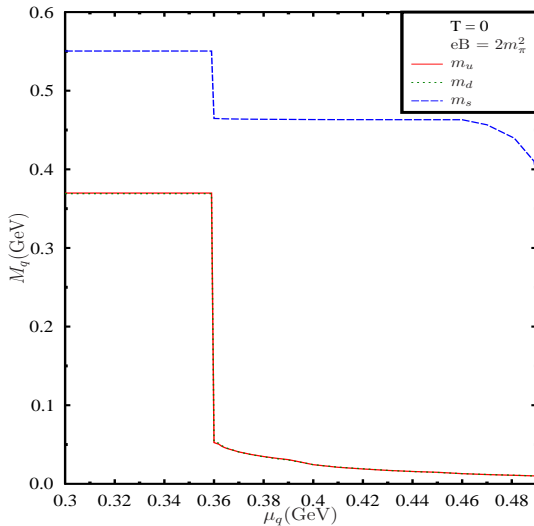
$T \neq 0, \mu = 0$ case



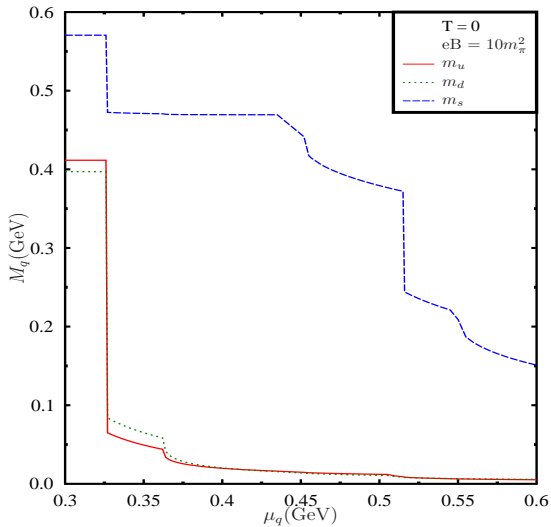
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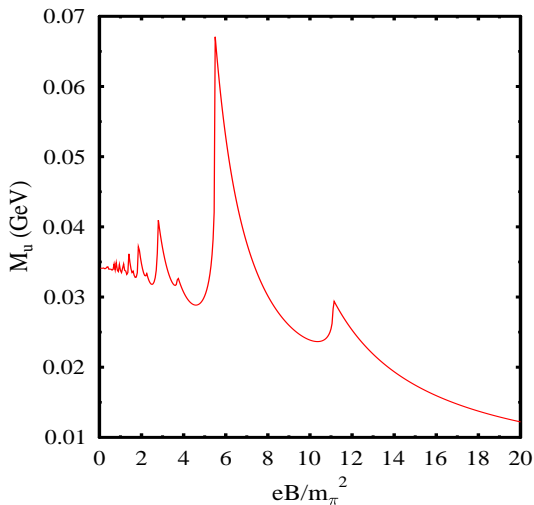


de Haas-van Alphen effect

- 1 At $T = 0$ gaps depend on the density of states of quarks at the Fermi surface.
- 2 Density of state has an oscillatory structure because of different quantization levels with varying magnetic field.
- 3 Gaps display magnetic oscillation.
- 4 Similar to de Haas-van Alphen effect in metals.

de Haas-van Alphen effect

- 1 de Haas-van Alphen effect. We have taken $\mu = 380$ MeV.

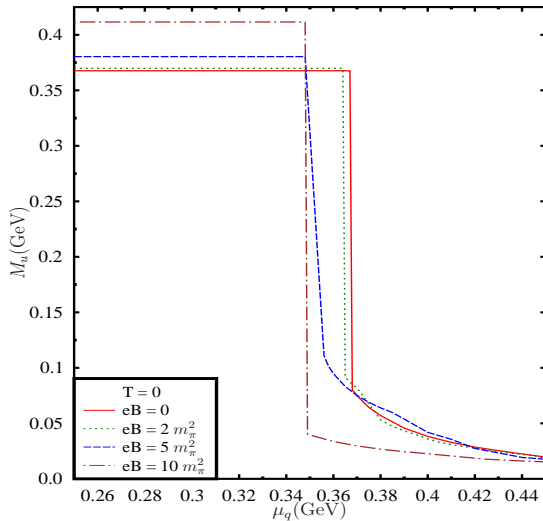


- 1 Charge neutrality demands

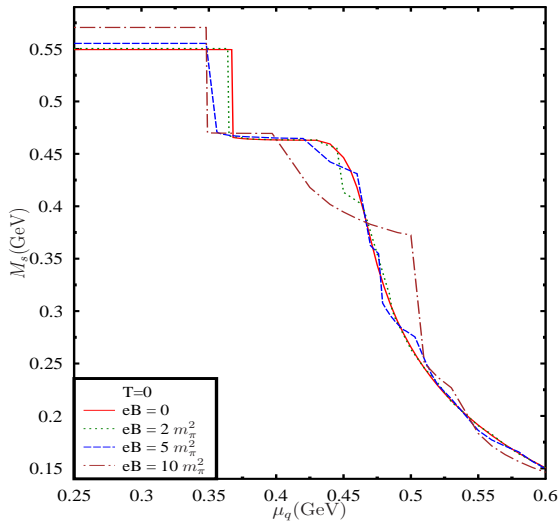
$$\mu_s = \mu_d = \mu_u + \mu_e.$$

- 2 We need 2 independent chemical potentials. $\mu_q = \frac{1}{3}\mu_B$ and μ_e .

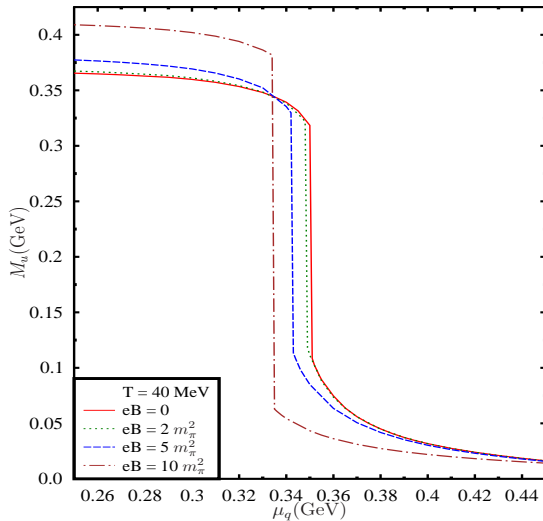
Charge neutrality at $T = 0$



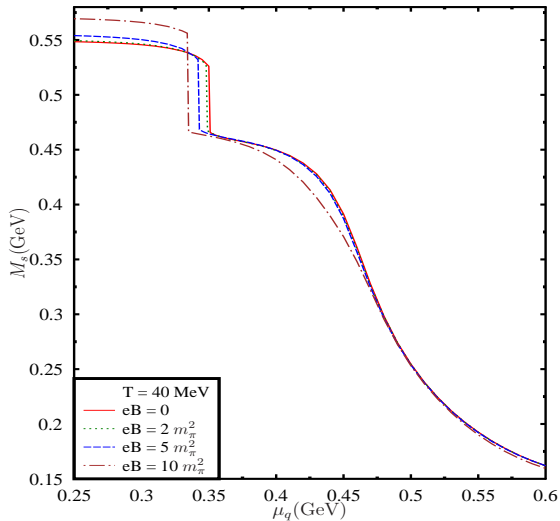
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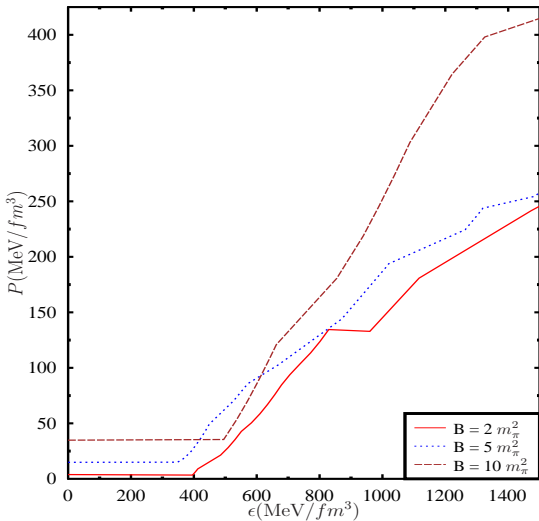
Charge neutrality at $T \neq 0$



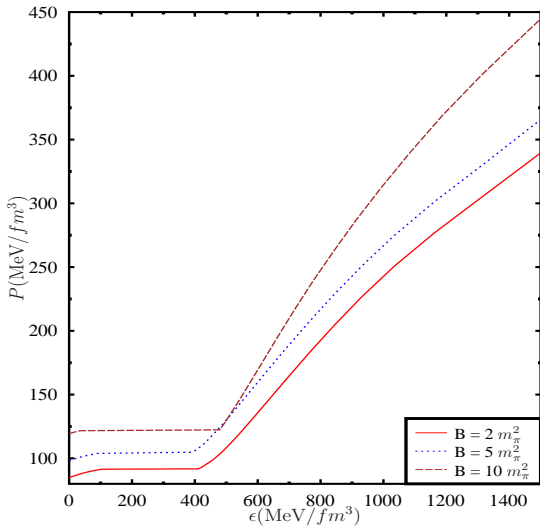
Charge neutrality at $T \neq 0$



Equation of state at $T = 0$



Equation of state at $T \neq 0$



- 1 Dr. Hiranmaya Mishra [PRL, Ahmedbad]
- 2 Dr. Amruta Mishra [IIT Delhi]

- ① Dr. Hiranmaya Mishra [PRL, Ahmedbad]
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Thank You

Dirac spinors in magnetic field

- 1 The Dirac spinors for particles with positive electrical charge are given by

$$U_{\uparrow}(\mathbf{p}_x, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} (\epsilon_n + m)l_n \\ 0 \\ p_z l_n \\ -i\sqrt{2nq_i B} l_{n-1} \end{bmatrix} e^{i\mathbf{p}_x \cdot \mathbf{x}_x} e^{-i\epsilon_n t}$$

and

$$U_{\downarrow}(\mathbf{p}_x, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} 0 \\ (\epsilon_n + m)l_{n-1} \\ i\sqrt{2nq_i B} l_n \\ -p_z l_{n-1} \end{bmatrix} e^{i\mathbf{p}_x \cdot \mathbf{x}_x} e^{-i\epsilon_n t}$$

Dirac spinors in magnetic field

- 1 The Dirac spinors for anti particles with positive electrical charge are given by

$$V_{\uparrow}(-\mathbf{p}_x, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} \sqrt{2nq_i B} l_n \\ ip_z l_{n-1} \\ 0 \\ (i\epsilon_n + m) l_{n-1} \end{bmatrix} e^{-i\mathbf{p}_x \cdot \mathbf{x}_x} e^{i\epsilon_n t}$$

and

$$V_{\downarrow}(-\mathbf{p}_x, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} ip_z l_n \\ \sqrt{2nq_i B} l_{n-1} \\ -i\epsilon_n + m) l_n \\ 0 \end{bmatrix} e^{-i\mathbf{p}_x \cdot \mathbf{x}_x} e^{i\epsilon_n t}$$

Dirac spinors in magnetic field

- 1 I_n is the solution of Hermite differential equation given by

$$I_n(\xi) = C_n e^{-\frac{\xi^2}{2}} H_n(\xi)$$

Where $H_n(\xi)$ is the Hermite polynomial and $C_n = \left[\frac{\sqrt{|q_i B|}}{n! 2^n \sqrt{\pi}} \right]^{\frac{1}{2}}$.

$\xi = \sqrt{|q_i B|} \left(x - \frac{p_y}{q_i B} \right)$ is a dimensionless variable.

- 2 I_n 's follow the following orthonormality relation

$$\int d\xi I_n(\xi) I_m(\xi) = \sqrt{|q_i B|} \delta_{n,m}$$

Important expectation values

- 1 The free particle energy term is given by

$$\begin{aligned} T + m\langle\bar{\psi}\psi\rangle &= \langle\psi^\dagger(-i\boldsymbol{\alpha}\cdot\nabla - q_i B x \alpha_2 + \beta m)\psi\rangle \\ &= -\sum_{n=0}^{\infty}\sum_i \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z \epsilon_{ni} \cos 2\theta_{\pm}^i \cos 2f_i \end{aligned}$$

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- 2 The number density term is given by

$$\langle\psi^\dagger\psi\rangle = \sum_{n=0}^{\infty}\sum_i \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z [1 - \sin^2 \theta_+^i + \sin^2 \theta_-^i]$$

$T = 0, \mu = 0$ case

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 $E = T + V.$

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- 1 Thermodynamic potential is equal to total energy,
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- 2 The kinetic energy is given by

$$T = - \sum_{n=0}^{\infty} \sum_i \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z \epsilon_{ni} \cos 2f_i$$

with

$$\epsilon_{ni} = \sqrt{m_i^2 + p_z^2 + 2n|q_i|B} = \sqrt{m_i^2 + |p_i|^2}$$

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- 3 The interaction term is given by

$$\begin{aligned} V &= - \langle G \sum_{A=0}^8 [(\bar{\psi} \lambda^A \psi)^2 + (\bar{\psi} \gamma^5 \lambda^A \psi)^2] \rangle \\ &+ K \langle [\det\{\bar{\psi}(1 + \gamma^5)\psi\} + \det\{\bar{\psi}(1 - \gamma^5)\psi\}] \rangle \\ &= V_s + V_d \end{aligned}$$

- 1 In our case

$$\langle \bar{\psi} \gamma^5 \psi \rangle = 0$$

and

$$\det(\bar{\psi}_i \psi_j) = \det \begin{bmatrix} \bar{\psi}_1 \psi_1 & \bar{\psi}_2 \psi_1 & \bar{\psi}_3 \psi_1 \\ \bar{\psi}_1 \psi_2 & \bar{\psi}_2 \psi_2 & \bar{\psi}_3 \psi_2 \\ \bar{\psi}_1 \psi_3 & \bar{\psi}_2 \psi_3 & \bar{\psi}_3 \psi_3 \end{bmatrix}$$

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Therefore

$$V_d = 2K \langle \bar{\psi}_1 \psi_1 \rangle \langle \bar{\psi}_2 \psi_2 \rangle \langle \bar{\psi}_3 \psi_3 \rangle = -2K l_1 l_2 l_3$$

All other terms will be at least $\frac{1}{N_c}$ suppressed.

- 2 We define $\frac{m_i}{\epsilon_{ni}} = \cos \phi_i^0$ and define $\phi_i = \phi_i^0 - 2f_i$

- 1 The thermodynamic potential is given by

$$\begin{aligned}\Omega &= - \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z \omega_{ni} + 2G \sum_i l_i^2 + 4K l_1 l_2 l_3 \\ &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln \{1 + e^{-\beta(\omega_i - \mu_i)}\} \\ &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln \{1 + e^{-\beta(\omega_i + \mu_i)}\}\end{aligned}$$

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Where

$$\omega_{ni} = \epsilon_{ni} = \sqrt{M_i^2 + |p_i|^2}$$

- 2 The first term is divergent.
3 For regularization, add and subtract a vacuum term

$$T_{vac} = \frac{2N_c}{(2\pi)^3} \sum_i \int d^3 p \sqrt{\mathbf{p}_i^2 + M_i^2}$$