Study of Phase Diagram and Fluctuations using PNJL model with Multi-Quark Interaction

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Plan of talk

- Relevance of the Present Study
- Nambu–Jona-Lasinio (*NJL*) Model
- Polyakov-Loop Extended Nambu–Jona-Lasinio (*PNJL*) Model
- QCD phase diagram in (*PNJL*) Model with multi-quark interactions
- Fluctuations of conserved charges in (*PNJL*) Model with multi-quark interactions
- Concluding Remarks

Restoration of Chiral Symmetry

- Restoration of chiral symmetry and deconfinement at high temperature and density are very interesting area to study.
- The phase transition associated with the chiral symmetry restoration in the vanishing quark mass limit is commonly referred to as the chiral phase transition.
- Symmetry broken in QCD vacuum; $<\Psi\bar{\Psi}>$ condensate is generated.
- Finite value of quark mass breaks the chiral symmetry explicitly.

Confinement

- At high temperature quarks and gluons are active degrees of freedom.
- In pure gauge sector Polyakov loop serves as an order parameter for the low temperature Z_{N_c} symmetric confined phase to the high temperature deconfined phase, characterised by the spontaneous breaking of Z_{N_c} symmetry. Polyakov loop measures breaking of the centre symmetry.

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- The Polyakov loop

$$\Phi = (Tr_c L)/N_c, \qquad \bar{\Phi} = (Tr_c L^\dagger)/N_c$$

$$L(\bar{x}) = \mathcal{P}exp[i\int_0^\beta d\tau A_4(\bar{x}, \tau)]$$

- Φ can be rewritten in terms of free energy

$$\Phi = e^{-\beta \Delta F_q(x)}$$

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- In the Z_{N_C} symmetric phase, $\Phi = 0$, an infinte amount of free energy is required to add an isolated heavy quark to the system. So color is Confined.
- For QCD at finite temperature and finite quark masses the global centre symmetry broken explicitely, implying a deconfinement crossover instead of the First order phase transition.

NJL model

- *NJL* model has been widely used to study Chiral Phase Transition in QCD. Here due to four fermion interaction between the quarks, the chiral symmetry is spontaneously broken : $SU_L(N_f) \times SU_R(N_f) \to SU_V(N_f)$ to give rise the constituent quark mass.
- The gap equation in the thermal medium is

$$M = m + 4 N_f N_c g_s \int \frac{d^3 p}{(2\pi)^3} M/E_p (1 - n_p(T,\mu) - \bar{n}_p(T,\mu))$$

Continued. . .

- The thermodynamic potential at finite temperature is given by

$$\Omega = 2g_S \sum_{f=u,d} \sigma_f^2 - 6 \sum_{f=u,d} \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|)$$
$$- 2T \sum_{f=u,d} \int_0^{\infty} \frac{d^3p}{(2\pi)^3} \ln \left[1 + e^{-\frac{(E_f - \mu)}{T}} \right]$$
$$- 2T \sum_{f=u,d} \int_0^{\infty} \frac{d^3p}{(2\pi)^3} \ln \left[1 + e^{-\frac{(E_f + \mu)}{T}} \right]$$

- lack of Confinement in the *NJL* model

2+1 flavour PNJL model

$$\Omega = \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2}\sigma_u \sigma_d \sigma_s + 3\frac{g_1}{2}(\sum_{f=u,d,s} {\sigma_f}^2)^2$$

$$+ 3g_2 \sum_{f=u,d,s} \sigma_f^4 - 6 \sum_{f=u,d,s} \int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|)$$

$$- 2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3(\Phi + \bar{\Phi} e^{-\frac{(E_f-\mu)}{T}}) e^{-\frac{(E_f-\mu)}{T}} + e^{-3\frac{(E_f-\mu)}{T}}\right]$$

$$- 2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3(\bar{\Phi} + \Phi e^{-\frac{(E_f+\mu)}{T}}) e^{-\frac{(E_f+\mu)}{T}} + e^{-3\frac{(E_f+\mu)}{T}}\right]$$

Polyakov loop Potential

The potential we considered here is $\mathcal{U}'$ with the Vandermonde term:

$$\frac{\mathcal{U}'(\Phi[A],\bar{\Phi}[A],T)}{T^4} = \frac{\mathcal{U}(\Phi[A],\bar{\Phi}[A],T)}{T^4} - \kappa \ln[J(\Phi,\bar{\Phi})]$$

where

$$\frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

$$b_2(T) = a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3,$$

b_3 and b_4 being constants.

————————-Pisarski *et.al*, PRD 62, 111501 (R), (2000).

Polyakov loop Potential

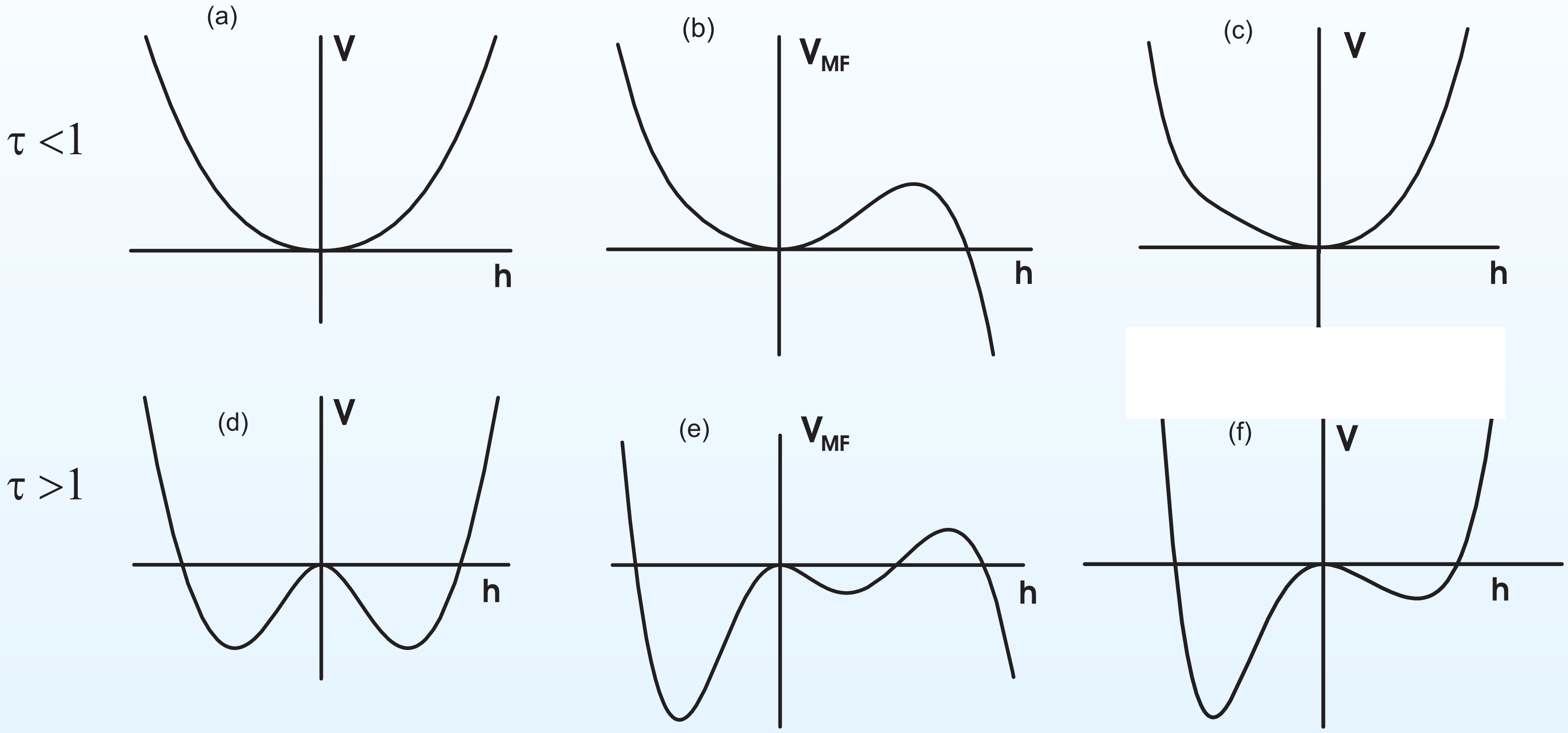
$J(\Phi, \bar{\Phi})$ is the Jacobian of transformation from Wilson line L to $(\Phi, \bar{\Phi})$ written as

$$J[\Phi, \bar{\Phi}] = (27/24\pi^2)(1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2$$

$J(\Phi, \bar{\Phi})$ is known as Vandermonde determinant.

Ghosh *et.al*, PRD 77, 094024, (2008)

Formulation with Multi-quark interaction



Osipov *et.al* PLB 634: 48-54, (2006).

Parameters

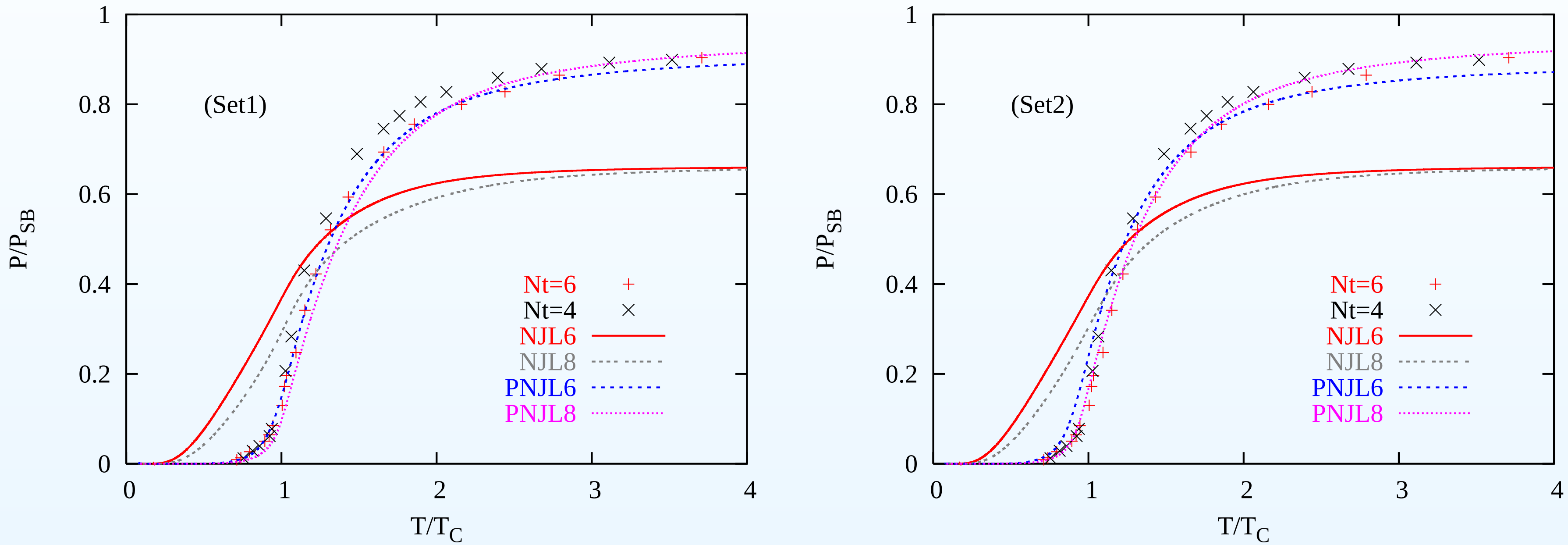
- We determine the input parameters of the NJL part of the Lagrangian by reproducing the physical variables. Two sets of input parameters with and without eight-quark interaction are given below.

set	m_u	m_s	Λ	$g_S\Lambda^2$	$g_D\Lambda^5$	$g_1 \times 10^{-21}$	$g_2 \times 10^{-22}$
a	5.5	134.758	631.357	3.664	74.636	0.0	0.0
b	5.406	133.227	641.357	3.717	61.309	0.0	0.0
c	5.5	183.468	637.720	2.914	75.968	2.193	-5.890
d	12.509	181.863	628.933	2.986	75.444	2.007	-4.538

(a,c) are taken as (set1) and (b,d) as (set2).

- Bhattacharyya *et.al*; Phys. Rev D 82, 014021, (2010).

Thermodynamic properties



The pressure of the strongly interacting matter-

$$P(T, \mu_q, \mu_Q, \mu_S) = -\Omega(T, \mu_q, \mu_Q, \mu_S),$$

Near T_C pressure decreases due to introduction of eight-quark interaction.

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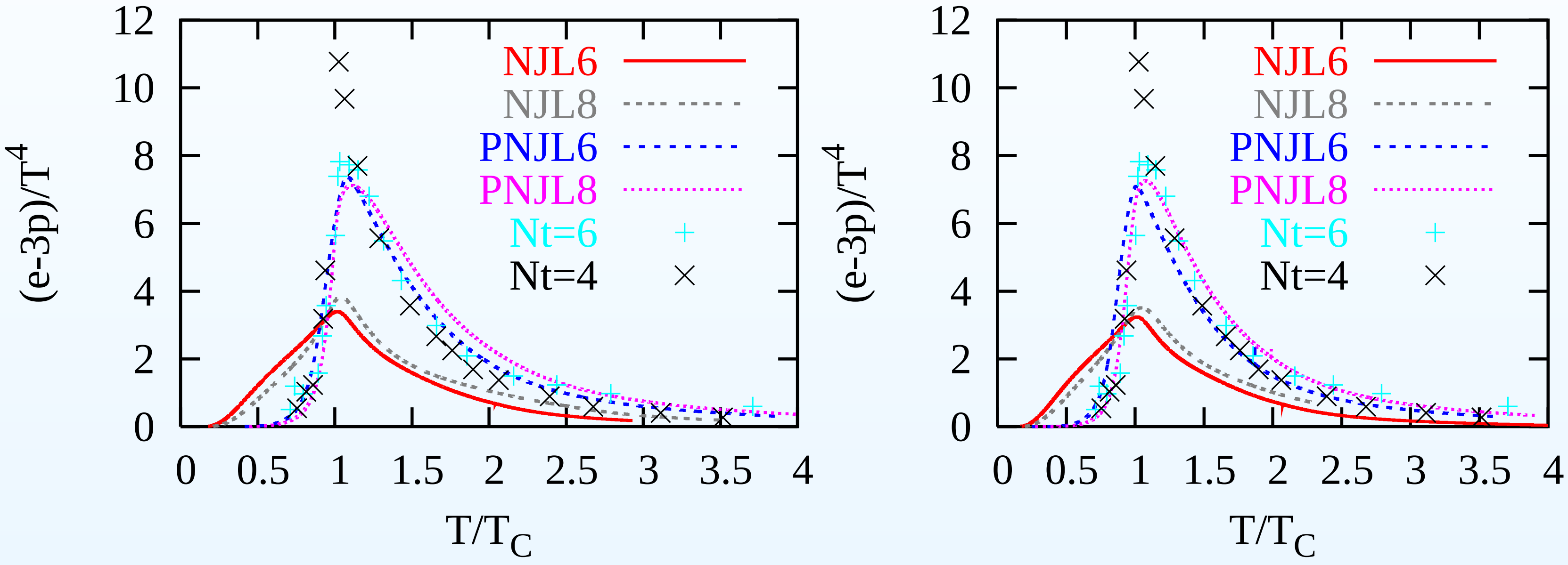


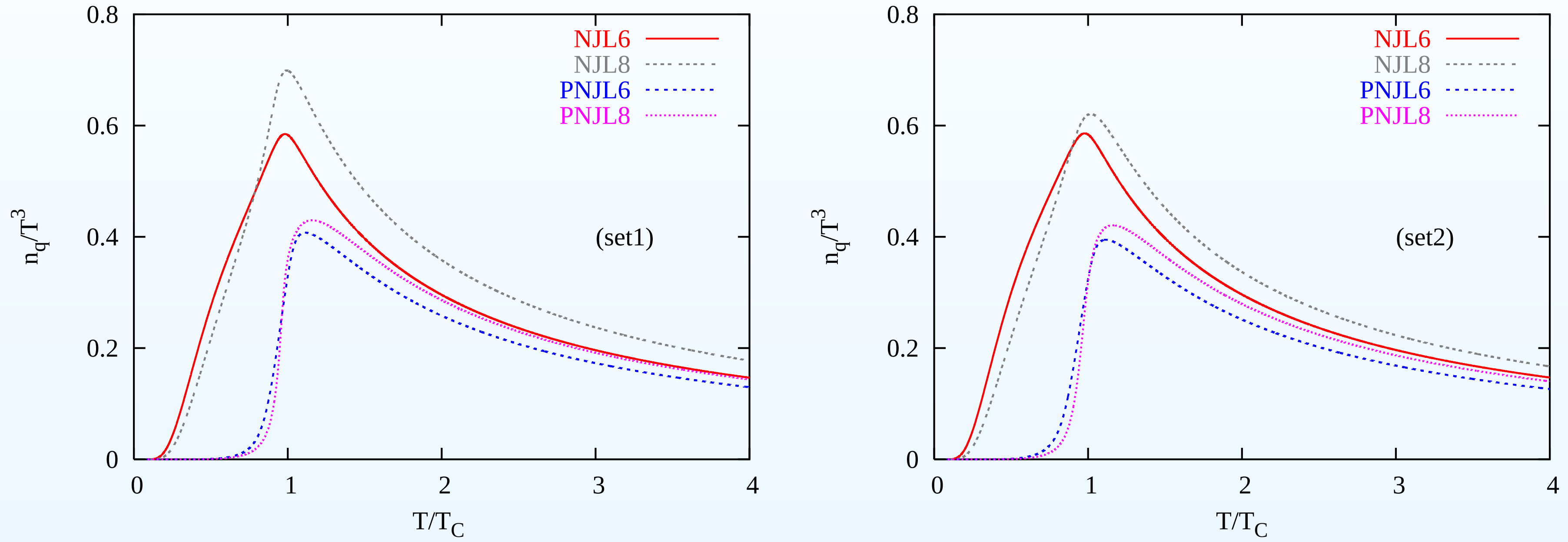
Bhattacharyya *et.al*; Phys. Rev D 82, 014021, (2010).

M. Cheng *et.al* PRD 77, 014511, (2008).

Continue...

- In conformally symmetric theory $\theta_{\mu\mu} = \epsilon - 3p = 0$
- For pure guage theory $(\epsilon - 3P) \sim T^2$. —(R. Pisarski, Phys. Rev. D 74, 121703 (2006).)
- The plots show a peak due to largest deviation of $(\epsilon - 3P)$ from the conformal limit. Introduction of eight-quark interaction slightly lowers the peak position.
- The NJL model shows constant value of much smaller magnitude. Thus $(\epsilon - 3p)$ above T_d is dominated by the Polyakov Loop.

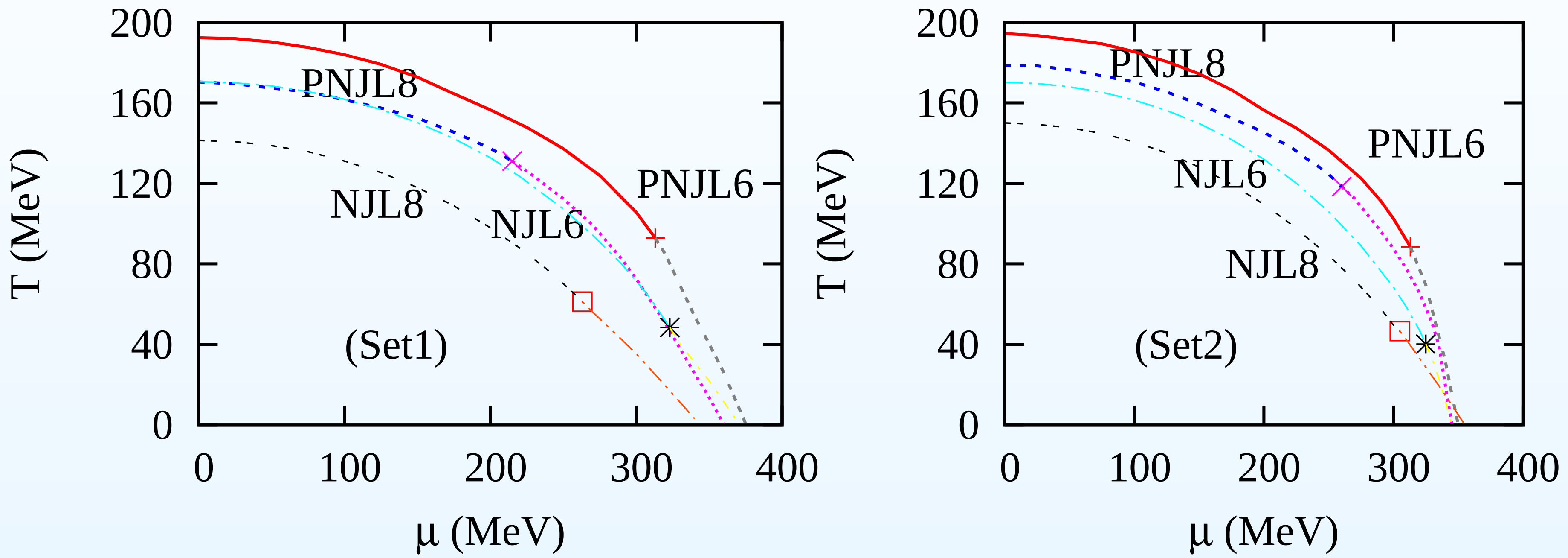
Number Density



$$n = \partial P / \partial \mu$$

The coupling of quark quasiparticle to the Φ reduces the weight of n_q as thermodynamically active degrees of freedom below T_c. The effect of Confinement is evident.

Phase diagram



The addition of eight-quark interaction shifts the CEP towards lower μ and higher T value, which is closer to the Lattice result.

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ModelIndex	$(\mu_C, T_C)_{(set1)}$MeV	$(\mu_C, T_C)_{(set2)}$MeV
NJL(Unbound)	$(323, 48.5)$	$(325, 40.15)$
NJL(Bound)	$(263, 61.2)$	$(305, 46.55)$
PNJL(Unbound)	$(313, 92.85)$	$(313, 88.5)$
PNJL(Bound)	$(260, 118.5)$	$(237, 122.05)$

Pressure and Fluctuations

- Flavour chemical potentials μ_u, μ_d, μ_s are related to μ_q, μ_Q, μ_S by,

$$\mu_u = \mu_q + \frac{2}{3}\mu_Q, \quad \mu_d = \mu_q - \frac{1}{3}\mu_Q, \quad \mu_s = \mu_q - \frac{1}{3}\mu_Q - \mu_S$$

- Expanding the scaled pressure in a Taylor series around the zero chemical potentials μ_q, μ_Q, μ_S as

$$\frac{p(T,\mu_q,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} c_{i,j,k}^{q,Q,S} \left(\frac{\mu_q}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$$c_{i,j,k}^{q,Q,S}(T) = \frac{1}{i!j!k!} \frac{\partial^i}{\partial(\frac{\mu_q}{T})^i} \frac{\partial^j}{\partial(\frac{\mu_Q}{T})^j} \frac{\partial^k (P/T^4)}{\partial(\frac{\mu_S}{T})^k}\Big|_{\mu_{q,Q,S}=0}$$

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For diagonal Taylor coefficient we have used,

$$c_n^X = \frac{1}{n!}\frac{\partial^n (P/T^4)}{\partial(\mu_X/T)^n}; n = i + j$$

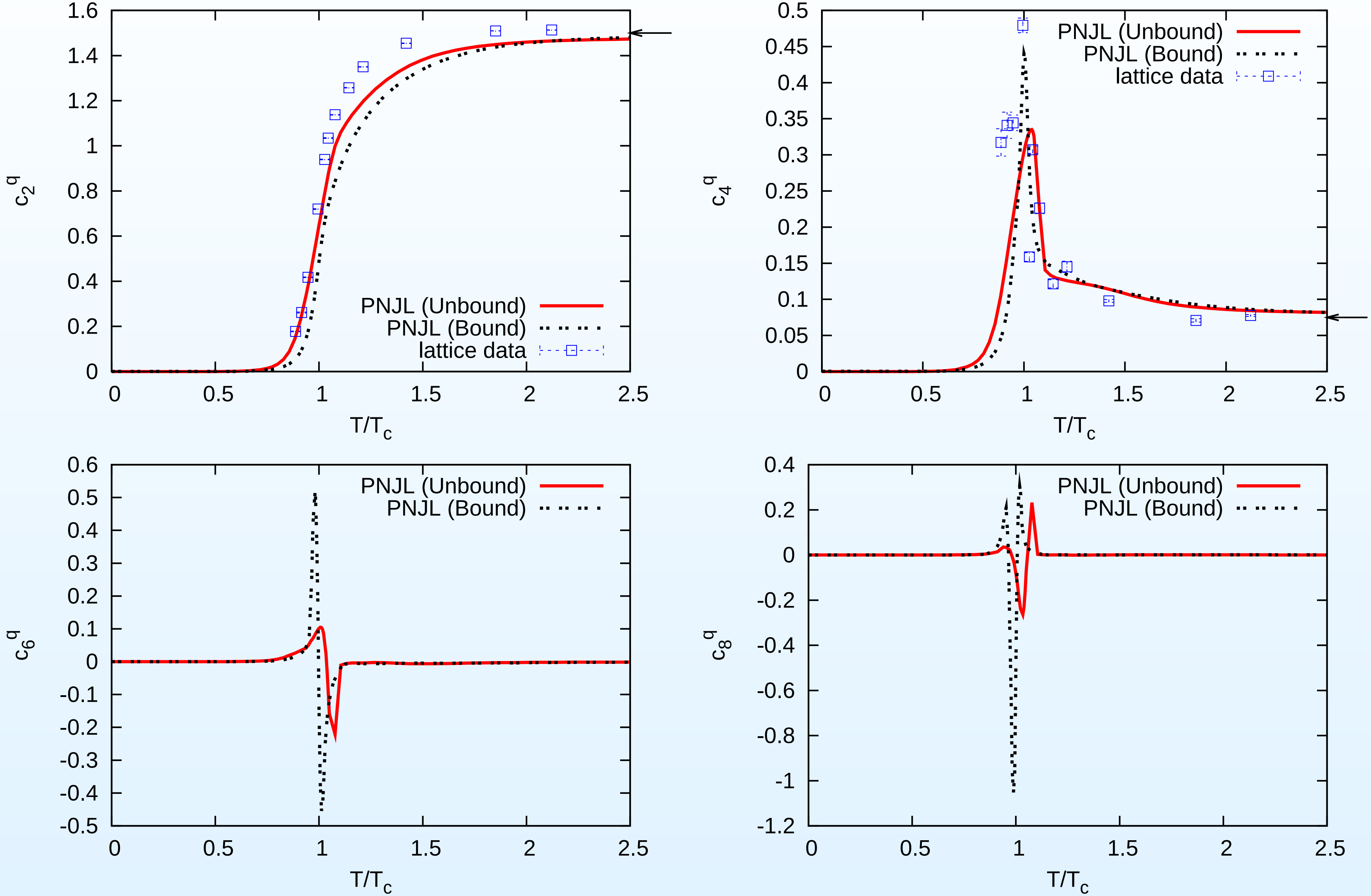
For off-diagonal Taylor coefficient we have used,

$$c_{i,j}^{X,Y} = \frac{1}{i!j!}\frac{\partial^{i+j}(P/T^4)}{\partial(\mu_X/T)^i \partial(\mu_Y/T)^j}$$

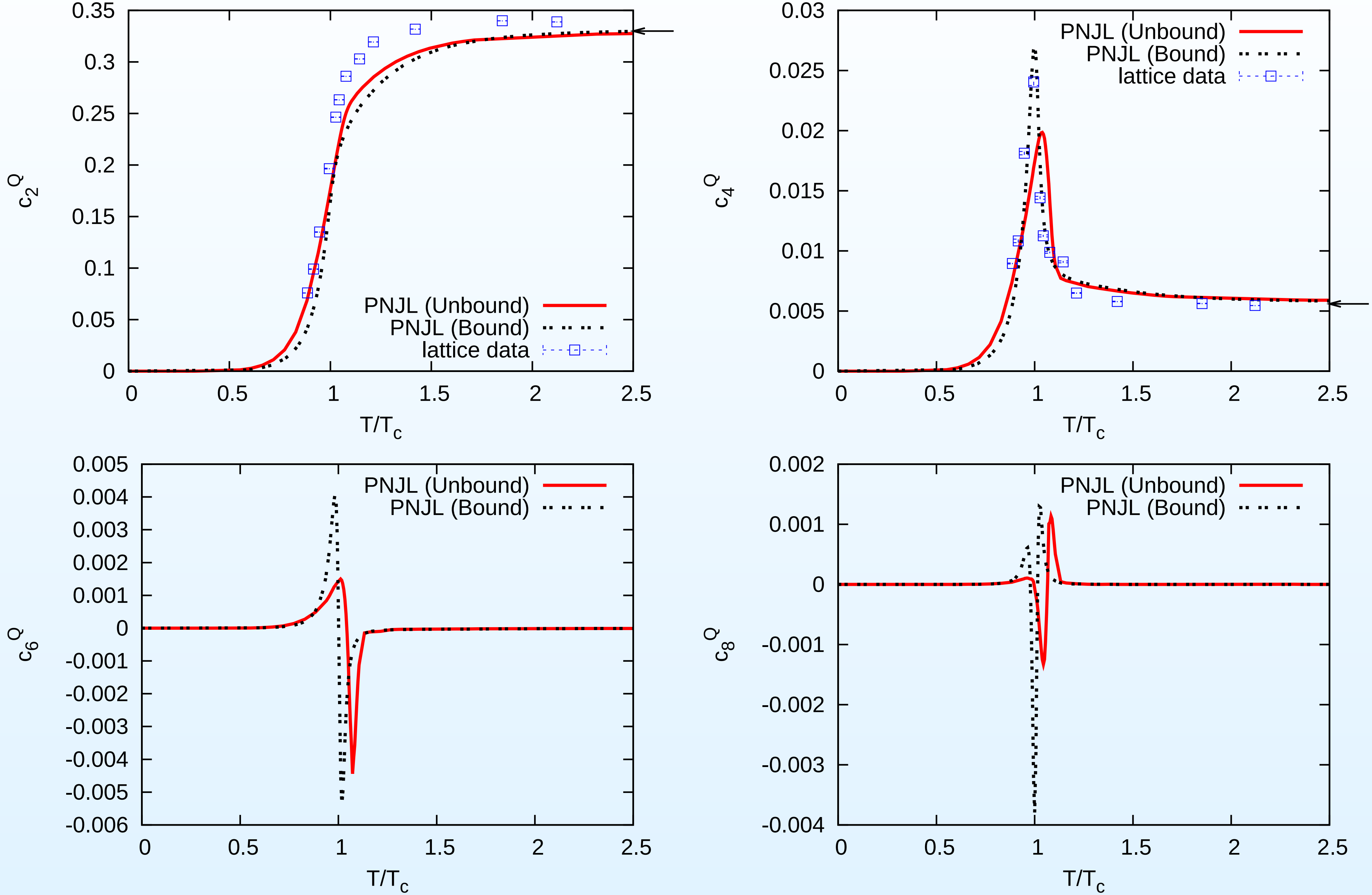
For off-diagonal and diagonal susceptibilities we have used the notation;

$$\chi_{XY} = \frac{\partial^2 P}{\partial\mu_X \partial\mu_Y}; \chi_{XX} = \frac{\partial^2 P}{\partial\mu_X^2}$$

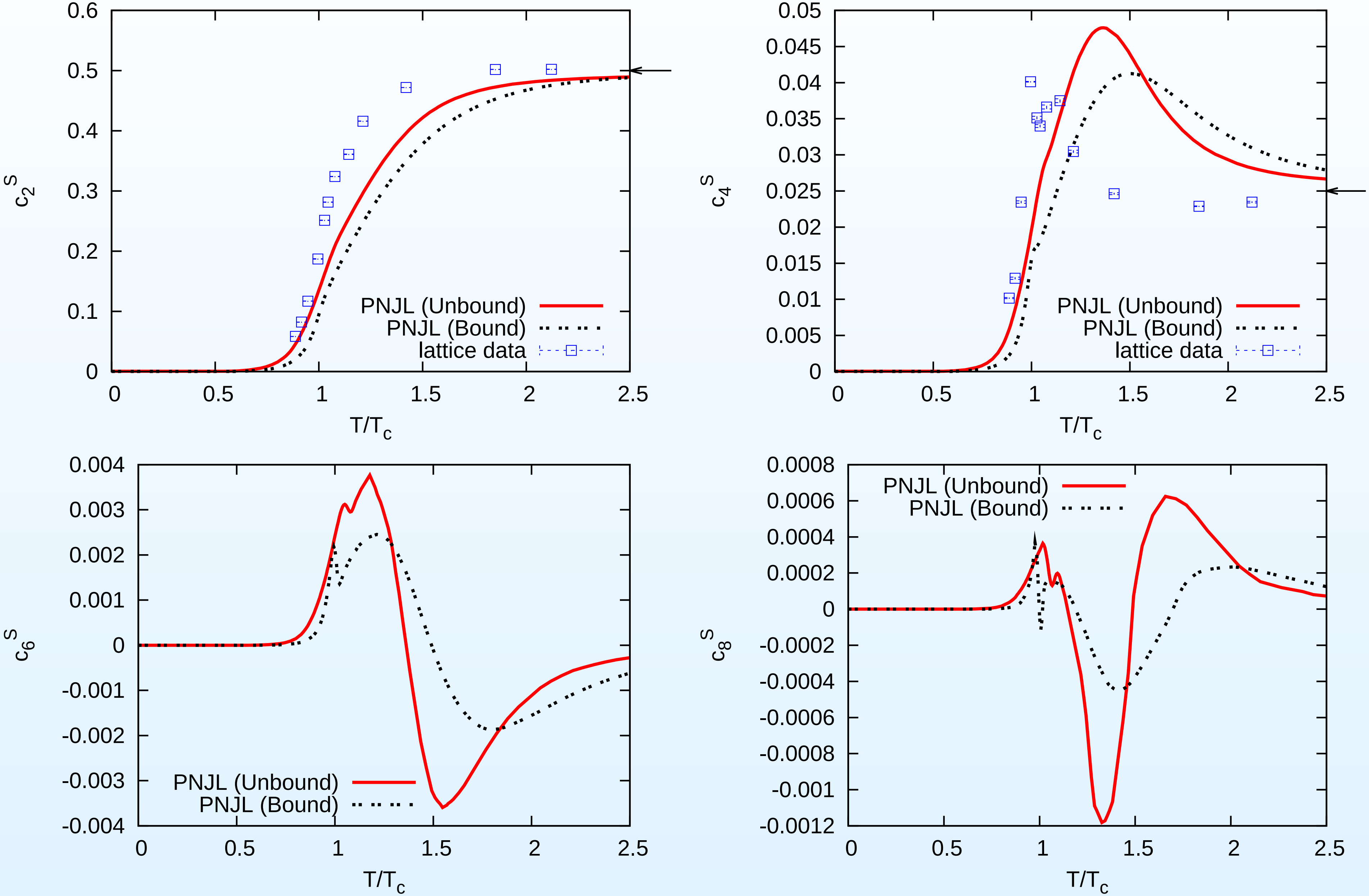
Baryonic Susceptibility (2+1 flavor)



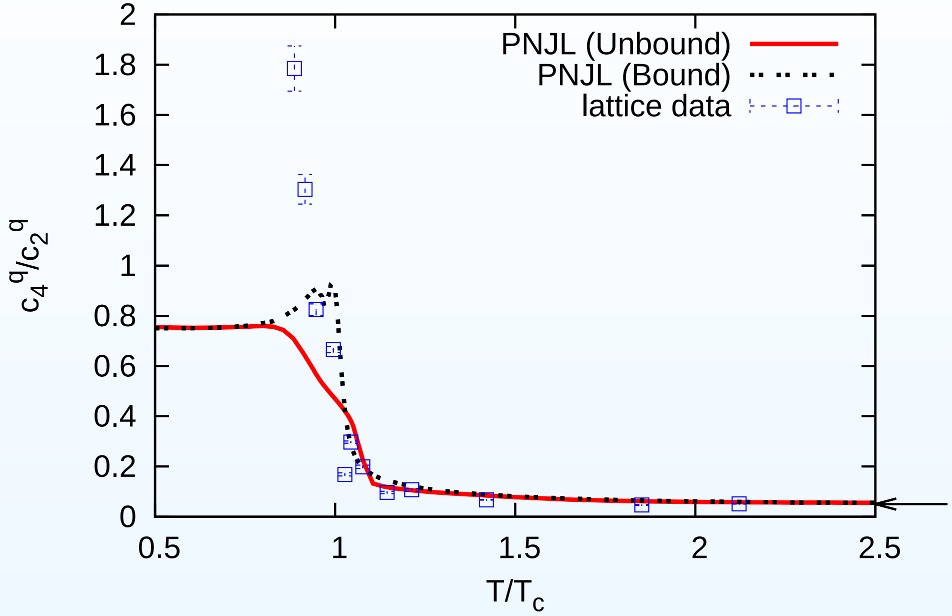
Charge Susceptibility (2+1 flavor)

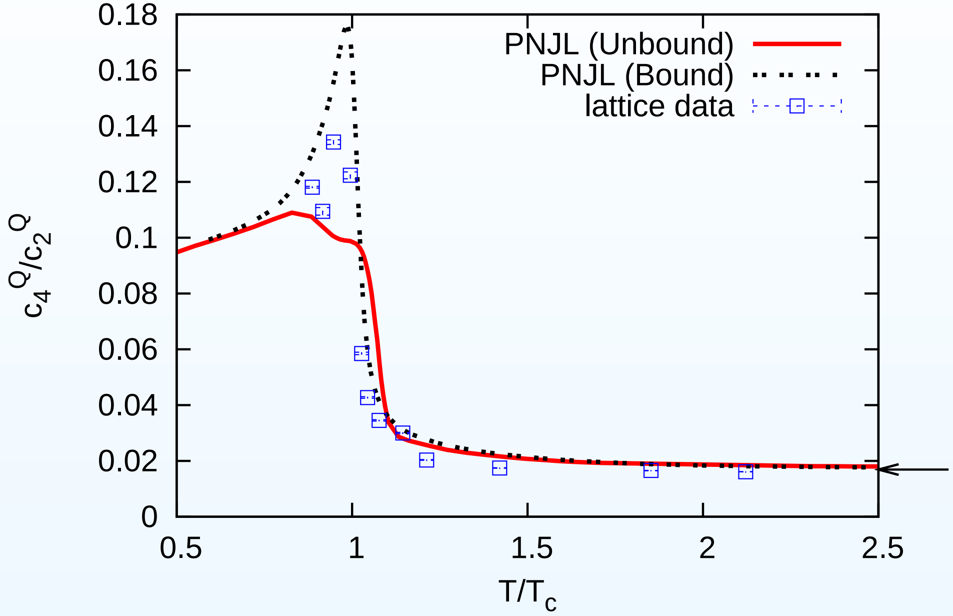


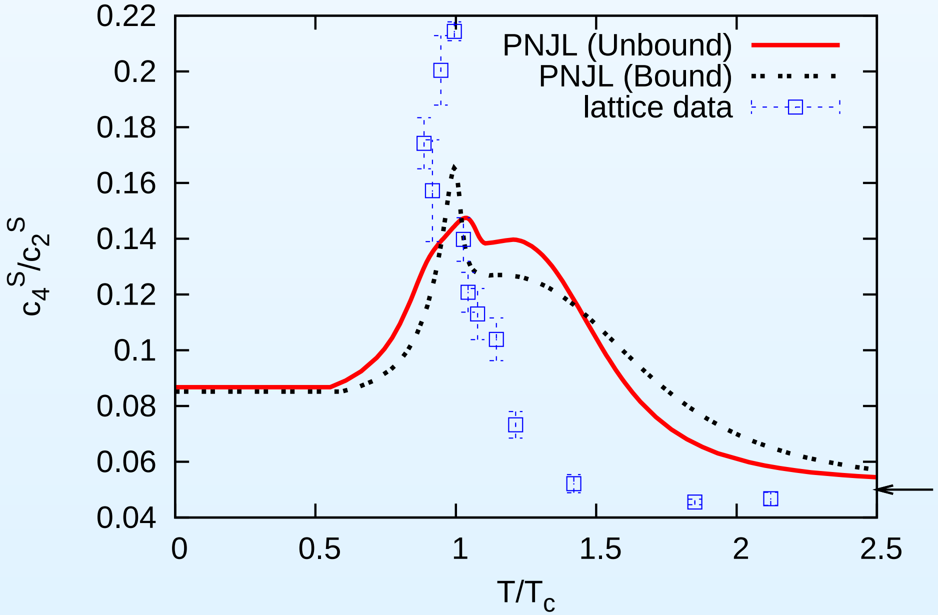
Strangeness Susceptibility (2+1 flavor)



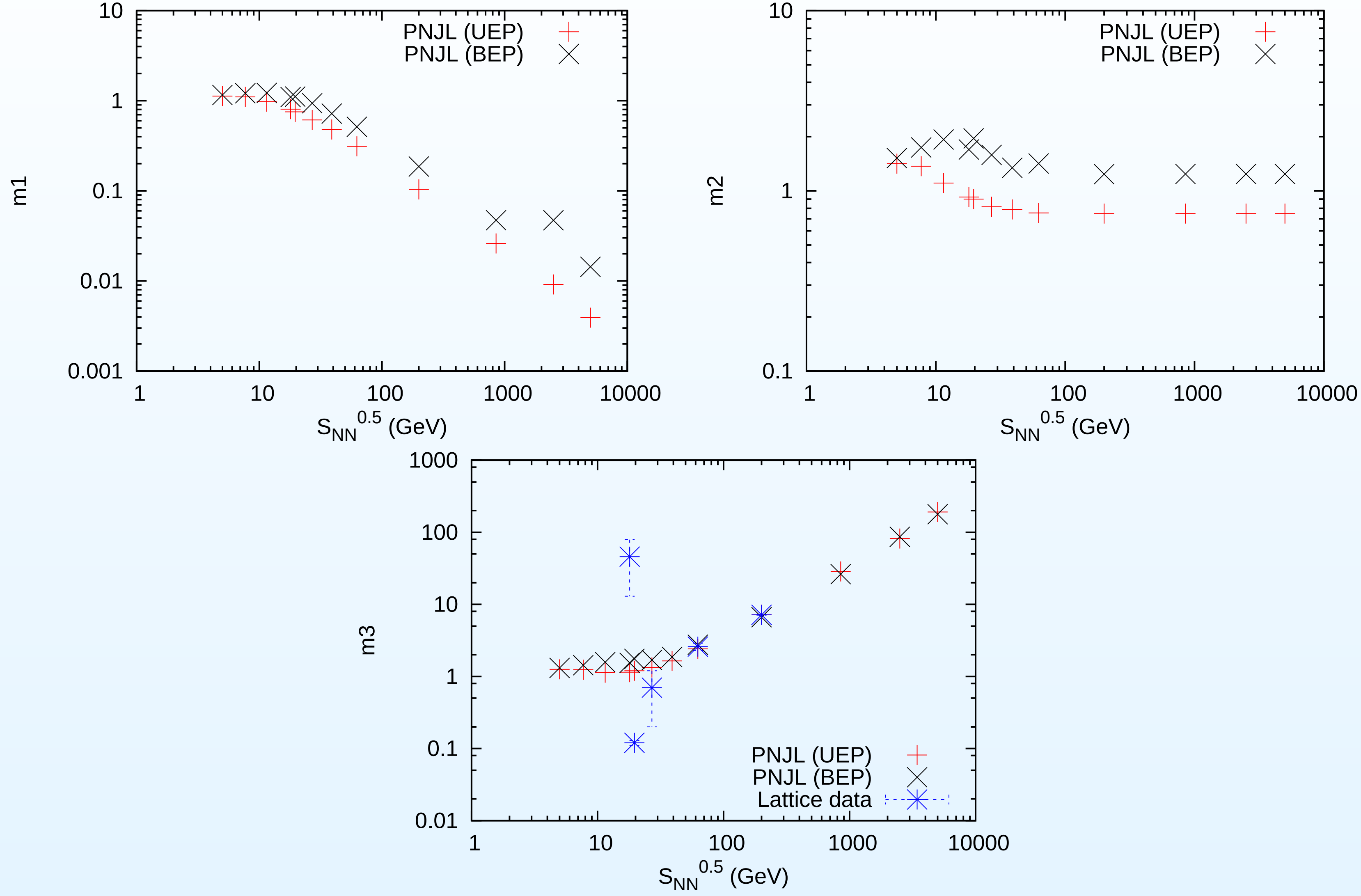
Kurtosis (2+1 flavour)







Moments (2+1 flavor)



For Lattice data Gavai, Gupta; arxiv:1001.3796v1

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$$m1 = \frac{B^3}{B^2} = \frac{\chi^{(3)}/T}{\chi^{(2)}/T^2}$$

$$m2 = \frac{[B^4]}{[B^2]} = \frac{\chi^{(4)}}{\chi^{(2)}/T^2}$$

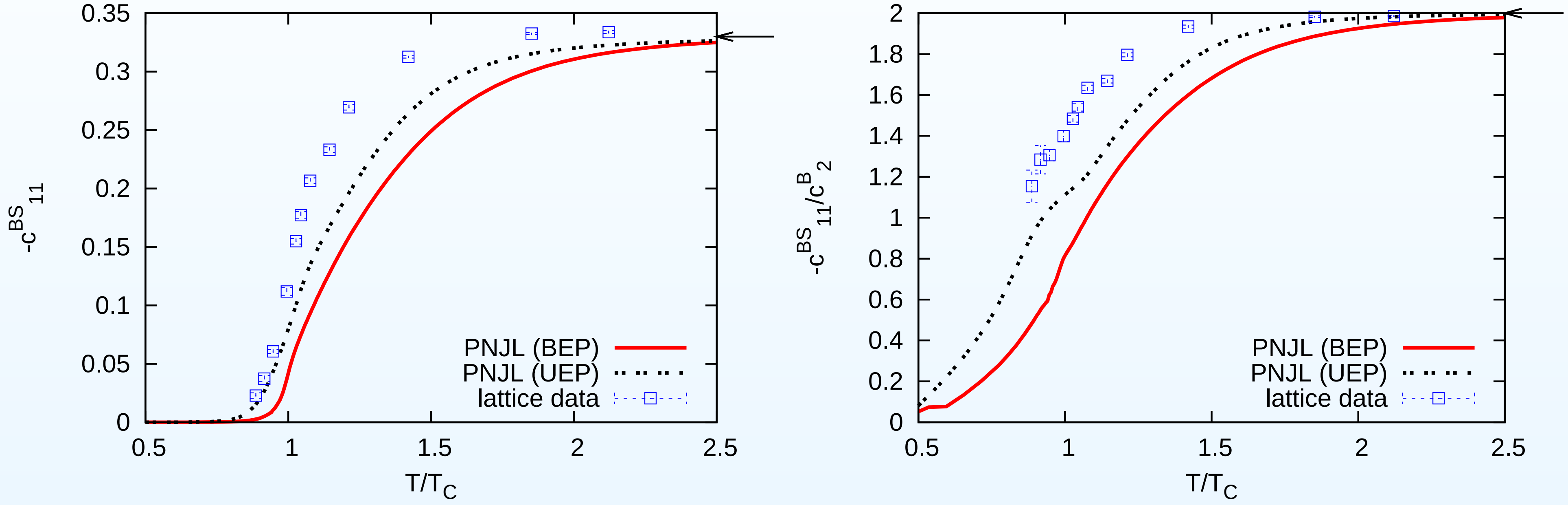
$$m3 = \frac{[B^4]}{[B^3]} = \frac{\chi^{(4)}}{\chi^{(3)}/T}$$

$$[B^2] = T^3 V(\chi^{(2)}/T^2)$$

$$[B^3] = T^3 V(\chi^{(3)}/T)$$

$$[B^4] = T^3 V(\chi^{(4)})$$

BS correlation



Lattice data taken from Phys. Rev. D79, 074505 (2009). Characteristic crossover of c_{11}^{qS} implies QCD phase transition liberates quarks. lightest baryons carry no strangeness ; so $\frac{c_{11}^{qS}}{c_2^q}$ tends to zero at low T.

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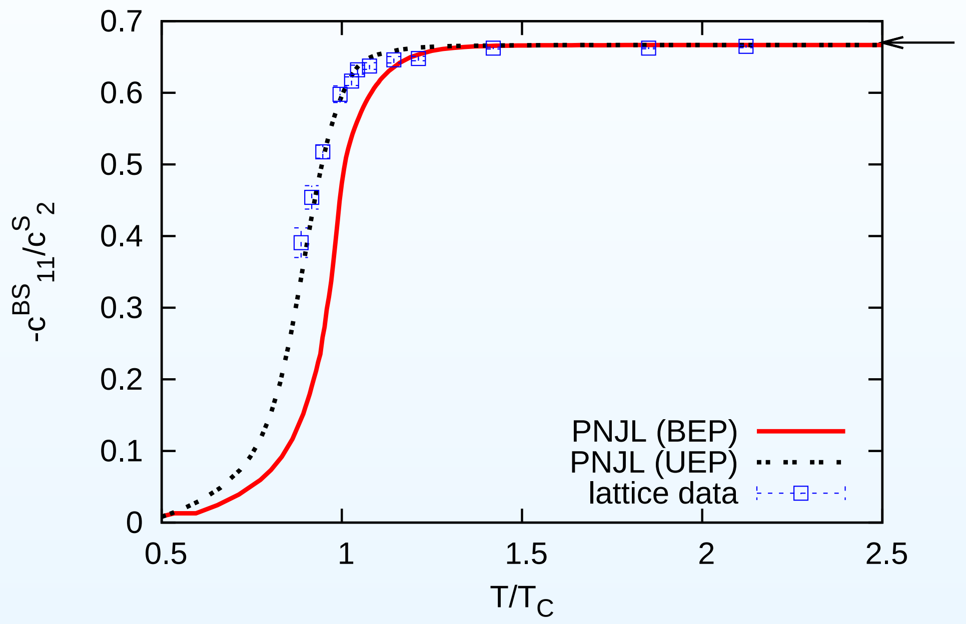
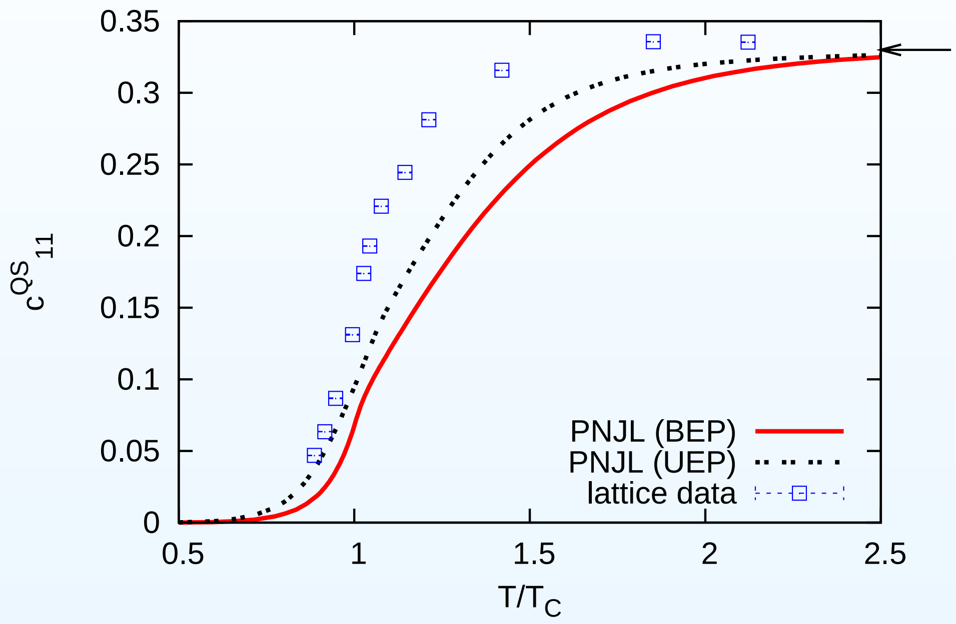
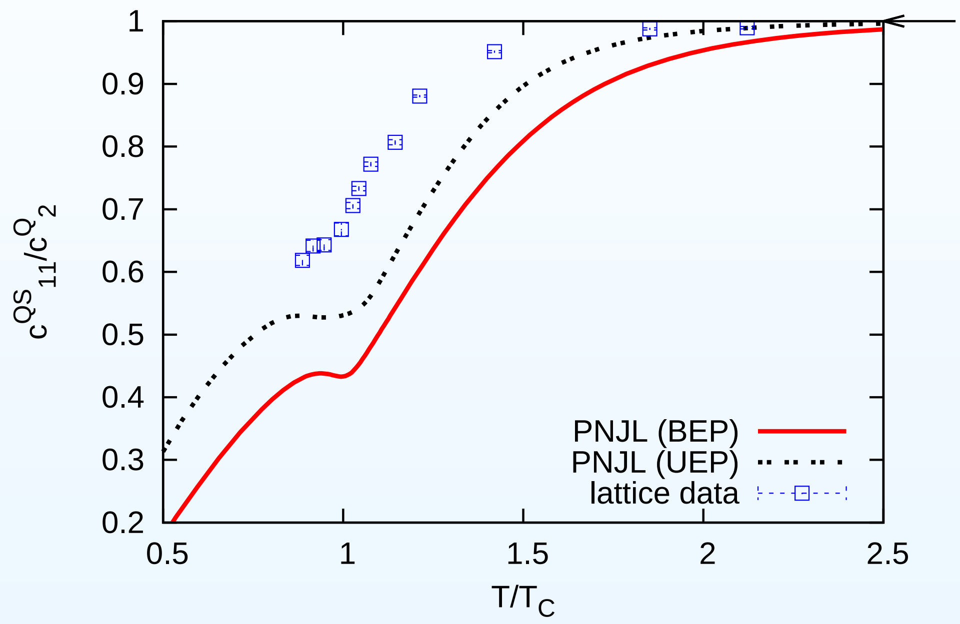


$$C_{BS} = -3\frac{\chi_{BS}}{\chi_{SS}} = -\frac{1}{2}\frac{c_{11}^{qS}}{c_2^S}$$

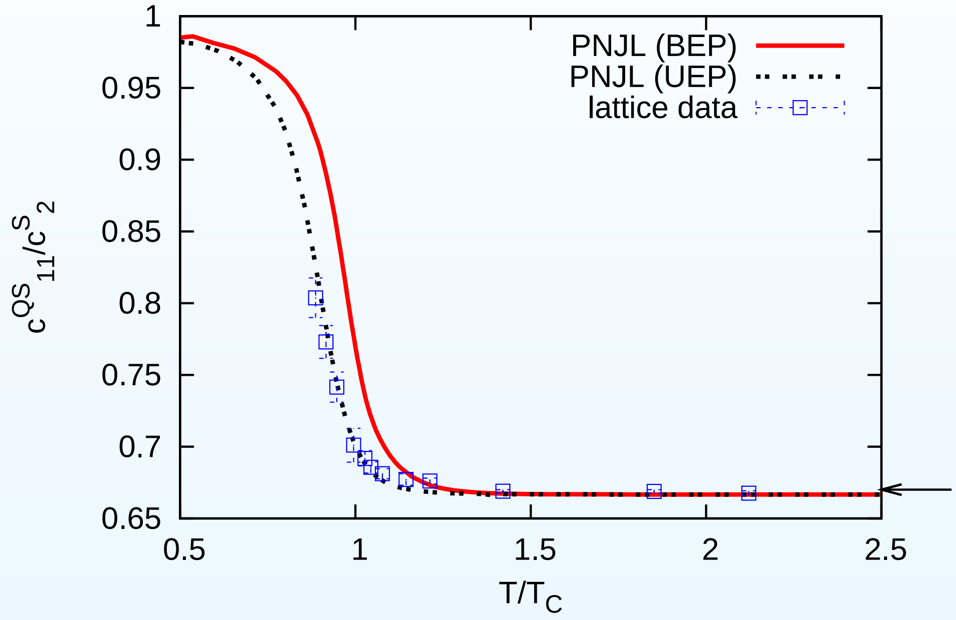
QS correlation





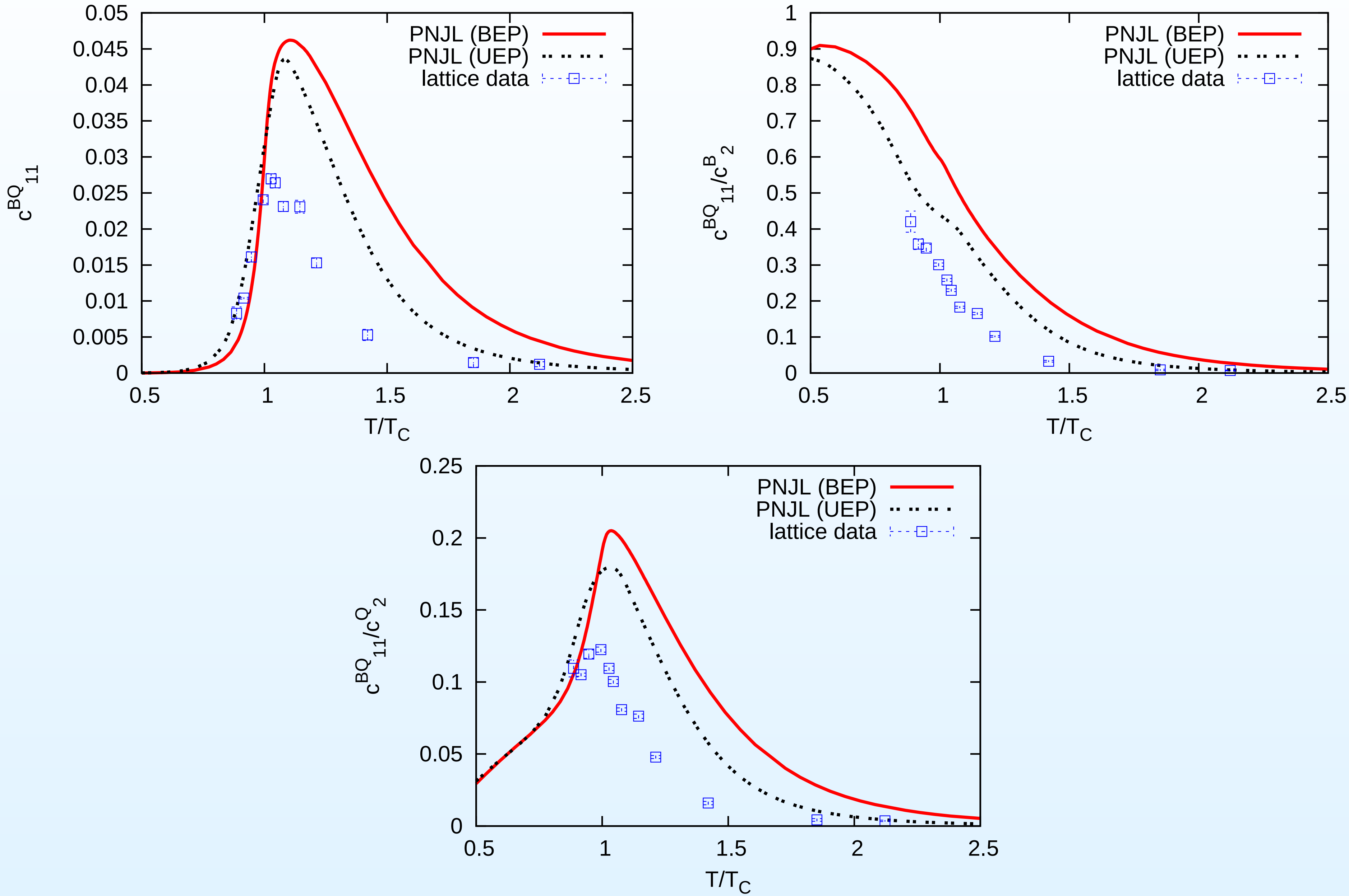
c_{11}^{QS} also shows crossover feature. $\frac{c_{11}^{QS}}{c_2^Q}$ tends to zero at low T because lightest charged particles do not carry strangeness.

Continue . . .

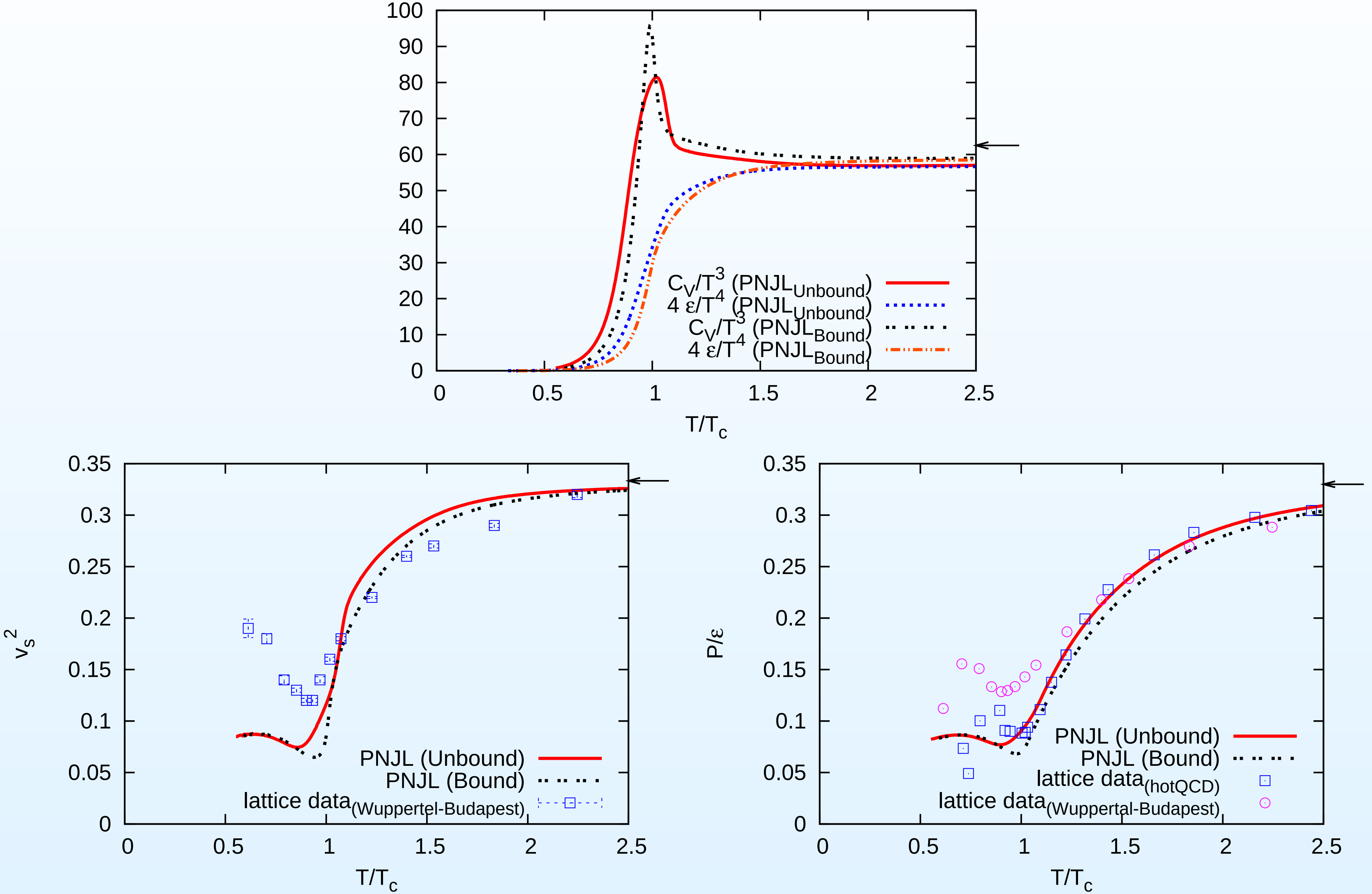


$$C_{QS} = 3\frac{\chi^{QS}_{11}}{\chi_{SS}} = \frac{3}{2}\frac{c^{QS}_{11}}{c^{S}_{2}}$$

BQ correlation



Specific heat and Speed of sound



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- The specific heat is the rate of change of energy density with temperature at constant volume–

$$C_V = \frac{\partial \epsilon}{\partial T}\Big|_V = -T\frac{\partial^2 \Omega}{\partial T^2}\Big|_V .$$

- The square of speed of sound at constant entropy S –

$$v_s^2 = \frac{\partial P}{\partial \epsilon}\Big|_S = \frac{\partial P}{\partial T}\Big|_V \Big/ \frac{\partial \epsilon}{\partial T}\Big|_V = \frac{\partial \Omega}{\partial T}\Big|_V \Big/ T\frac{\partial^2 \Omega}{\partial T^2}\Big|_V \quad . \tag{1}$$

- v_s^2 is related to elliptic flow and the rapidity distribution.
- $\Delta/\epsilon(\Delta = \epsilon - 3P)$ measures the similarity between long distance QCD and the conformal theories.
- The softest point of P/ϵ is 0.07 for 6q, 0.06 for 8q and 0.08 for the Hot-QCD group.

—Bhattacharyya *et.al*; arxiv:1008.0768 [hep-ph].; To appear in Phys. Rev. D.

For Lattice data— M. Cheng *et.al*, PRD 79, 074505, (2009).

Concluding Remarks. . .

- Introduction of eight-quark interaction term stabilizes the vacuum.
- Eight-quark interaction affects the bulk thermodynamic properties at zero density.
- Eight-quark interaction shifts the CEP to the low μ and high T value.
- Fluctuations of different conserved charges show significant behavior near the phase transition temparature.

List of Collaborators

- Abhijit Bhattacharyya
  Sanjay K. Ghosh
  Rajarshi Ray
  Anirban Lahiri

THANK YOU !!