Screening Effects and Vortex Configurations in Dual QCD

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Monopoles in QCD

- Dual superconductor scenario requires the condensation of magnetic monopoles as a key ingredient, but the QCD as such does not have the magnetic charge as an elementary degrees of freedom (DOF).
- Such dual picture is based on the Abelian gauge theory, but QCD is essentially a non-Abelian gauge theory.
- There exists the gauge fixing techniques which reduce the QCD as an Abelian gauge theory with the monopoles as essential DOF's.
- Some Examples: 't Hooft's Abelian gauge fixing, Magnetic gauge fixing and Maximally Abelian gauge fixing in lattice QCD.

(G Ripka, arXiv : hep-ph/0310102 and references therein.)

Attributes: Conventional Superconductor $\Longleftrightarrow$ Dual QCD

Type of Charges: Electric $\Longleftrightarrow$ Magnetic

Condensation: Cooper Pairs $\Longleftrightarrow$ Monopole/Dyon Pairs

Net Effect: Meissner $\Longleftrightarrow$ Dual Meissner

Confinement: Magnetic Flux $\Longleftrightarrow$ Electric Flux

Type of States: $m\bar{m}$ Bound $\Longleftrightarrow$ $q\bar{q}$ Bound

S Mandelstam, Phys. Rep. **C23** (1976) 245 and references therein.

Present Formulation

- Lagrangian

$$\pounds = -\frac{1}{4}\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu} + \left|(\partial_\mu + i\,\tilde{g}\,\tilde{C}_\mu)\,\phi\,\right|^2 + \bar{\psi}\,\gamma^\mu(i\,\partial_\mu + \tilde{g}\,\tilde{C}_\mu)\,\psi - m\,\psi\bar{\psi} - V(\phi^*\phi) \quad (1)$$

where $\tilde{G}_{\mu\nu} = \partial_\mu \tilde{C}_\nu - \partial_\nu \tilde{C}_\mu$, $\psi \rightarrow$ quark field and $\phi \rightarrow$ monopole field.

- Effective potential

$$V(\phi^*\phi) = \Omega\,(\,\phi^*\phi - \eta^2)^2, \quad (2)$$

where $\eta^2 = < \phi^*\phi >_0$ and $\Omega = 3\lambda/\alpha_s^2$.

- Field equations

$$(i\gamma^\mu\,\partial_\mu + \tilde{g}\gamma^\mu\tilde{C}_\mu - m\,)\,\psi = 0\,, \quad (3)$$

$$(\partial_\mu + i\,\tilde{g}\tilde{C}_\mu)^2\phi + 2\Omega\,(\,|\phi|^2 - \eta^2)\,\phi = 0\,, \quad (4)$$

$$\partial^\nu\tilde{G}_{\mu\nu} + i\tilde{g}(\phi^*\overleftrightarrow{\partial_\mu}\phi) - \tilde{g}\bar{\psi}\gamma_\mu\psi - \tilde{m}^2\,\tilde{C}_\mu = 0\,;\,(\tilde{m} = (8\pi/\,\alpha_s)^{1/2})\,\eta \quad (5)$$

K Shima, IL Nuovo Cimento **44A** (1978)163.

H Nandan et.al, Euro Phy. Lett.**67**(2004)746.

Total Charge

- Colour charge density $j_c = \tilde{m}^2\, \tilde{C}_0 \, + \tilde{g}\, \psi^\dagger \psi.$ (6)
- Total colour charge of the system

$$Q_c \equiv \int d^3x j_c = \tilde{m}^2 \int d^3x\, \tilde{C}_0 \, + \tilde{g} \int d^3x\, \psi^\dagger \psi. \tag{7}$$

- Since observed hadrons are colour singlet so

$$Q_c \equiv \int d^3x\, j_c = 0. \tag{8}$$

- It means the colour charge of quark must be totally screened.
- For N_q quark state ($\int d^3x\, \psi^\dagger \psi = N_q$)

$$N_q = -\tilde{m}^2/\tilde{g} \int d^3x\, \tilde{C}_0 \tag{9}$$

- For one quark system $1 + \tilde{m}^2/\tilde{g} \int d^3x\, \tilde{C}_0 = 0$ (10)

Existence of a screening potential $\tilde{C}_0$ in strong coupling regime!

Condition for Charge Screening

- Let us consider cylindrically symmetric ansatz

$$\tilde{\mathbf{C}} = -\, e_\theta\, \tilde{C}(\rho),\ \ \tilde{C}_0 = C_0(\rho),\ \ \text{and}\ \ \phi(\rho) = \chi(\rho)\, \exp\,(i\, n\theta) \tag{11}$$

- ψ for the stationary solution to the bound quark

$$\psi = \exp(-i\epsilon t) \begin{Bmatrix} u\,(\rho, \theta, z) \\ v\,(\rho, \theta, z) \end{Bmatrix}, \tag{12}$$

  where $u(\rho)$ and $v(\rho) \to 0$ as $\rho \to \infty$.

- In order to guarantee the total charge screening, $\tilde{C}_0$ must fall off rapidly than ρ^{-1} from the Gauss law.

- For such $\tilde{C}_0$, from field equation $\tilde{C}_0$ for $\rho \to \infty$

$$\tilde{m}^2\, \tilde{C}_0 \ \gg\ \tilde{g}\, \psi^\dagger \psi, \tag{13}$$

  acts as a necessary condition for total charge screening.

DME and Colour Flux Screening

- Field equations
$$\tilde{C}_0'' + \frac{1}{\rho}\,\tilde{C}_0' - \,\tilde{m}^2\,\tilde{C}_0 \;= 0\,, \tag{14}$$
$$\tilde{C}'' + \frac{\tilde{C}'}{\rho} - \frac{\tilde{C}}{\rho^2} - \frac{2n\tilde{g}}{\rho}\,\chi^2 \,-\, \tilde{m}^2\,\tilde{C} = 0\,, \tag{15}$$
$$\chi'' + \frac{\chi'}{\rho} - \left(\frac{n}{\rho} + \tilde{g}\,\tilde{C}\right)^2 \chi + \tilde{g}^2\tilde{C}_0^2\chi + 2\Omega\,(\chi^2 - \eta^2)\,\chi = 0, \tag{16}$$
with the following boundary condition for $\tilde{C}$ in addition to (13).
$$\tilde{m}^2\,\rho\,\tilde{C} + 2n\,\tilde{g}\,\zeta^2 \;\gg\; \rho\,(\psi^\dagger\,\boldsymbol{\gamma}\,\psi). \tag{17}$$

- Electric and magnetic fields $\tilde{E} = -\,\frac{1}{\rho}\,(\,\rho\,\tilde{C})'$ and $\tilde{H} \,= \tilde{C}_0'$

- For instance, at large distances
$$\tilde{E} \;\rightarrow\; C\,\sqrt{\frac{\eta}{\rho}}\;e^{-\,\tilde{m}\rho} \;+\; \text{non-leading terms}\,, \tag{18}$$
which signifies DME and $\tilde{E}$ screened up to a distance $\tilde{m}^{-1}$.

Flux Quantisation

- Flux quantisation can be seen though the kinetic energy (K) term $|\mathfrak{D}_\mu \phi|^2$ in the Lagrangian (1) where $\mathfrak{D}_\mu \equiv \partial_\mu + i\,\tilde{g}\tilde{C}_\mu$.

- It leads to
$$K = \int_0^\infty \rho\, d\rho \int_0^{2\pi} d\theta \left(\frac{1}{\rho}\,\frac{d\phi}{d\theta} - \tilde{g}\,\tilde{C} \right)^2 \eta^2 \,. \qquad (19)$$

- Minimisation of the K in superconducting phase i.e. $\eta \neq 0$ leads to $2\pi n/\tilde{g} = \int \tilde{C}\rho\, d\theta = \oint \tilde{\mathbf{C}} \cdot d\mathbf{l}$.

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- Using the Stokes theorem then leads
$$\Phi_{\tilde{E}} = \int (\nabla \times \tilde{\mathbf{C}}) \cdot d\mathbf{S} = \int \tilde{\mathbf{E}} \cdot d\mathbf{S} = n\, Q_e \,, \qquad (20)$$
where $Q_e = g/2$ acts as the colour electric charge of a quark.

- Condition (20) is valid with the requirement that ϕ be continuous along any closed path in dual QCD vacuum which encircles $\tilde{\mathbf{E}}$.

Energy Configuration in DGL Model

- Lagrangian $$\mathcal{L} = -\frac{1}{4}\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu} + |\mathfrak{D}_\mu \phi|^2 + V(\phi\phi^*). \quad (21)$$
- Lagrangian (21) ≡ Lagrangian (1) with charge screening effect.
- Vortices are invariant under translations along any fixed axis and therefore may be viewed as finite energy solutions in two dimensions.
- With vanishing temporal degrees of freedom of $\tilde{C}_\mu$ (i.e. $\tilde{C}_0 = 0$)
$$E = \int d^2x \left[\frac{1}{4}\tilde{G}_{ij}^2 + |\mathfrak{D}_k \phi|^2 + \Omega(\phi\phi^* - \eta^2)^2\right]. \quad (22)$$
- Energy (22) contains a term $\tilde{m}^2\tilde{C}_i^2 \longrightarrow$ DME.
- Energy expression can be rewritten as (22)

$$E = \int d^2x\mathcal{E} = \int d^2x \left[|\mathfrak{D}_1\phi|^2 + |\mathfrak{D}_2\phi|^2 + \frac{1}{2}\tilde{G}_{12}^2 + \Omega(\phi\phi^* - \eta^2)^2\right]. \quad (23)$$

Bogomol'nyi Bound

- Restructured energy functional

$$\mathcal{E} = \Big[|(\mathfrak{D}_1 + i\mathfrak{D}_2)\phi|^2 + \frac{1}{2}\{[\tilde{G}_{12} + \tilde{g}(\phi\phi^* - \eta^2)]^2 + (2\Omega - \tilde{g}^2)(\phi\phi^* - \eta^2)^2\} + \tilde{g}\eta^2\,\tilde{G}_{12}\Big].$$

- For $2\Omega = \tilde{g}^2$ (i.e. $g^2 = 6\lambda$) $\Longrightarrow \kappa = \left[\frac{3\lambda}{2\pi\alpha_s}\right]^{1/2} = \frac{\sqrt{6\lambda}}{g} = 1$, energy

$$E = \int d^2x \left[\,|\,(\mathfrak{D}_1 + i\mathfrak{D}_2)\phi\,|^2 + \frac{1}{2}[\,\tilde{G}_{12} + \tilde{g}(\phi\,\phi^* - \eta^2)\,]^2\right] + \tilde{g}\,\eta^2\,\Phi_{\tilde{E}}.$$

- With the vanishing of the squared entities i.e.,

$$(\mathfrak{D}_1 + i\mathfrak{D}_2)\,\phi = 0, \qquad \tilde{G}_{12} + \tilde{g}\,(\phi\,\phi^* - \eta^2)^2 = 0. \tag{24}$$

- The minimum energy

$$E_n = \,2n\,\pi\,\eta^2. \tag{25}$$

- E_n can be re-casted as $E_n \geq \,2n\,\pi\,\eta^2$ whether n is positive or negative.

Conclusions and Future Possibilities

- QCD vacuum with magnetic condensation with total colour charge screening necessarily give rise to DME leading to quark bound states.

- For the transition from type-II to type-I (for $\kappa = 1$, $\lambda = 1$; $\alpha_s \simeq 0 \cdot 0.5$) dual QCD vacuum, n-vortex solutions exist with BPS conditions.

- It remains to see the evolution pattern of the interaction energy among vortices in type-I ($\kappa < 1$) and type-II ($\kappa > 1$) superconducting dual QCD vacuum with numerical estimations.

- It would also be meaningful to investigate the exact process of how the vortices attract/repel each other in different coupling regimes.

THANKS