Gauge Free Electroweak Theory: Radiative Effects

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S. Bhattacharjee and P. Majumdar, arXiv:1006.1712v1
(Theory Group, SINP, Kolkata-64)

Motivation

- Gauge transformations always acts on unphysical d.o.f. Natural phenomena should be described in terms of physical variables.
- Vacuum Electrodynamics reformulated in gauge free framework in terms of physical vector potential which is space-time transverse.

$$
\begin{aligned}
Z[\mathbf{J}] &= \int \mathcal{D}\mathbf{A}_{\mathcal{P}} \; \exp i \left(\frac{1}{2} A^{\mu}_{\mathcal{P}} \Box A_{\mathcal{P}\mu} + \int d^4x \; \mathbf{J} \cdot \mathbf{A}_{\mathcal{P}} \right) \; \delta[\partial_\mu \, \mathbf{A}^{\mu}_{\mathcal{P}}] \\
&= \int \mathcal{D}\mathcal{S} \; \mathcal{D}\mathbf{A}_{\mathcal{P}} \; \exp i \int d^4x \left[\frac{1}{2} A^{\mu}_{\mathcal{P}} \Box A_{\mathcal{P}\mu} + (J_\mu - \partial_\mu \mathcal{S}) A^{\mu}_{\mathcal{P}} \right] .
\end{aligned}
$$

- All charged matter fields are complex Φ can be 'radially' decomposed : $\Phi = \rho \exp i\theta$. $\rightarrow$ spin-charge separation.
- Upshot: Abelian Higgs model is a gauge-free mechanism of mass generation. Phase part attaches as a physical longitudinal d.o.f. of vector boson. [P. Majumdar and S. Bhattacharjee, arXiv:0903.4340, 2009] (Elitzur's Theorem)
- Can this be extended to non-abelian theory?

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- Can this be extended to non-abelian theory?
- Yes!

AIM Of Talk

- A new-variable approach for Salam-Weinberg theory $\Rightarrow$ completely inert under $SU(2)$ gauge transformations.

  [M. N. Chernodub, L. D. Faddeev and A. J. Niemi, JHEP, 014 (2008)], [L. D. Faddeev, arXiv:0811.3311]

- Residual theory still has a $U(1)_{em}$ gauge invariance. We made all the fields manifestly $U(1)$ invariant following our earlier work $\Rightarrow$ no need of any gauge fixing at all.

  [P. Majumdar and S. Bhattacharjee, arXiv:0903.4340, 2009]

- F. C. N. proposed an alternative mass generation scheme of the gauge bosons without any tree level Higgs potential! $\Rightarrow$ Mass generation of gauge bosons without any symmetry breaking!

- We have shown radiative effects generate an effective Higgs potential$\Rightarrow$ minimum away from origin $\Rightarrow$ another radiative mass generation scheme for the weak gauge bosons as well as Higgs boson.

AIM Of Talk

- Higgs field enters in the functional integral as a local factor $\Rightarrow$ F.C.N. interpreted Higgs boson as a conformal degrees of freedom in an equivalent background conformal gravity theory where the VEV of Higgs field is fixed by it's asymptotic behaviour.
- Genuine one loop radiative effects cancel the local functional measure of the Higgs field.

[S. Bhattacharjee and P. Majumdar, arXiv:1006.1712]

Plan Of Talk

- New variable form of $SU(2) \times U(1)$ theory
- Effective Potential for the New variable Electroweak Theory
- Discussions
- Summary and Scope

- Lagrangian for Bosonic sector of the standard electroweak theory:

$$\mathcal{L} = (\nabla_\mu \Phi, \nabla^\mu \Phi) + \frac{1}{4g^2} tr\, \mathbf{B}^2_{\mu\nu} + \frac{1}{4g'^2} Y^2_{\mu\nu}$$

- with

$$\begin{aligned}
\nabla_\mu \Phi =& \partial_\mu \Phi + \frac{i}{2} Y_\mu \Phi + B^a_\mu t^a \Phi \\
B^a_{\mu\nu} =& \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + \epsilon_{abc} B^b_\mu B^c_\nu \\
Y_{\mu\nu} =& \partial_\mu Y_\nu - \partial_\nu Y_\mu
\end{aligned}$$

- with $t^a = \frac{i}{2}\tau^a$, τ^a Pauli matrices, g and g' are the coupling constants.

Introducing radial decomposition

- $SU(2)$ and $U(1)$ transformations of different gauge fields and the Higgs $SU(2)$ doublet:-

$$\begin{aligned} \mathbf{B}_\mu \to \mathbf{B}_\mu^{(\Omega)} &= \Omega \mathbf{B}_\mu \Omega^{-1} - \partial_\mu \Omega \Omega^{-1} \\ Y_\mu \to Y_\mu^{(\omega)} &= Y_\mu - 2\partial_\mu \omega \\ \Phi \to \Phi^{(\Omega)} = \Phi\Omega \quad &, \quad \Phi \to \Phi^{(\omega)} = \Phi \exp i\omega. \end{aligned}$$

- The essential features behind achieving the $SU(2)$ invariant Lagrangian are the polar decomposition of scalar doublet

$$\Phi = \frac{1}{\sqrt{2}} \rho \chi$$

and finding a unitary unimodular matrix

$$g = \begin{pmatrix} \chi_1 & -\bar{\chi}_2 \\ \chi_2 & \bar{\chi}_1 \end{pmatrix}$$

Gauge transformations of new variables

- It is clear that $g \in SU(2)$ and it has a similar transformation property under $SU(2)$ as Φ

$$g \to g^{(\Omega)} = \Omega g$$

- But g has a different abelian transformation since χ_i and $\bar{\chi}_i$ have different weak hypercharges

$$g^{\omega} = g \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{-i\omega} \end{pmatrix} = g e^{i\omega\tau_3}.$$

- The covariant derivative of g :

$$\nabla_\mu g = \partial_\mu g + \frac{i}{2} Y_\mu g \tau_3 + \mathbf{B}_\mu g \ .$$

$SU(2)$ gauge invariant vector boson

- Defining an $SU(2)$ gauge invariant vector boson $\mathbf{W}_\mu = W_\mu^a t^a$ as

$$\mathbf{W}_\mu \equiv g^\dagger \left(\mathbf{B}_\mu + \partial_\mu\right) g$$

- Gauge transformations of $\mathbf{W}$:

$$\begin{aligned} \mathbf{W}_\mu^{(\Omega)} &= \mathbf{W}_\mu \\ \mathbf{W}_\mu^{(\omega)} &= e^{-i\omega\tau_3}\mathbf{W}_\mu e^{i\omega\tau_3} + i\tau_3\partial_\mu\omega. \end{aligned}$$

- Introducing the components of field W_μ :

$$\begin{aligned} W_\mu =& \frac{i}{2}(W_\mu^1\tau_1 + W_\mu^2\tau_2 + W_\mu^3\tau_3) \\ W_\mu^\pm =& W_\mu^1 \pm W_\mu^2. \end{aligned}$$

With the abelian transformations of the components :

$$(W_\mu^\pm)^\omega = e^{\pm 2i\omega} W_\mu^\pm, \quad (W_\mu^3)^\omega = W_\mu^3 + 2\partial_\mu\omega$$

Chargeless bosons Z and A

- Introducing a chargeless vector field

$$Z_\mu = Y_\mu + W_\mu^3$$

$|\nabla_\mu \Phi|^2$ is rewritten as :

$$\frac{\rho^2}{8}(Z_\mu^2 + W_\mu^+ W_\mu^-) + \frac{1}{2}\partial_\mu \rho \partial_\mu \rho$$

- The remaining vector part is rewritten in terms of neutral fields Z, A and charged fields $W^\pm$ by introducing an abelian vector field A_μ,

$$A_\mu \equiv \frac{1}{g^2 + g'^2}\,(g'^2 W_\mu^3 - g^2 Y_\mu)$$

- The abelian vector field A_μ transforms in the same way as W_μ^3

$$A_\mu^\omega = A_\mu - 2\partial_\mu \omega$$

Lagrangian in terms of new variables

- The full Lagrangian in terms of new variables:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{\rho^2}{8}(Z_\mu^2 + W_\mu^+ W^{\mu,-}) + \frac{1}{4g^2}(\nabla_{[\mu}W_{\nu]}^+)(\nabla^{[\mu}W^{\nu]-})$$
$$+\frac{1}{4(g^2+g'^2)}Z_{\mu\nu}^2 + \frac{1}{4e^2}A_{\mu\nu}^2 + \frac{2}{4g^2}\,H_{\mu\nu}(A^{\mu\nu} + e^2 Z^{\mu\nu}) + \frac{1}{4g^2}H_{\mu\nu}^2,$$

- where

$$\begin{aligned} Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ W^3_{\mu\nu} &= \partial_\mu W^3_\nu - \partial_\nu W^3_\mu \\ H_{\mu\nu} &= \frac{1}{2i}(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^-) \\ \nabla_\mu W_\nu^\pm &= \partial_\mu W_\nu^\pm \pm i W_\mu^3 W_\nu^\pm . \end{aligned}$$

Removing residual $U(1)_{em}$ degrees of freedom

- Radial decomposition of charged vector bosons :

$$W^{\pm}_{\mu} = w_{\mu} \exp \pm i\theta$$

- 'separation of the charge and spin modes' :

$$[w_{\mu}]^{(\omega)} = w_{\mu} \ , \ [\theta]^{(\omega)} = \theta + 2\omega \ .$$

- Field redefinitions :- $\Theta \equiv \theta - 2\mathrm{a} \ , \ \mathbf{A}_{\mu} \equiv A_{\mu} - 2\partial_{\mu}\mathrm{a}$
- Gauge-free construction :

$$\begin{aligned} \nabla_{[\mu}W^{+}_{\nu]}\nabla^{[\mu}W^{\nu]-} &= w^2_{\mu\nu} + \frac{1}{2}w^2\left(\mathbf{A} + \partial\Theta + \frac{e^2}{g'^2}Z\right)^2 \\ &- \frac{1}{2}\left[w\cdot\left(\mathbf{A} + \partial\Theta + \frac{e^2}{g'^2}Z\right)\right]^2 , \end{aligned}$$

where,

$$w_{\mu\nu} \equiv 2\partial_{[\mu}w_{\nu]}$$

Interpretation of ρ^2 factor in measure

- The functional measure in new variables :-

$$d\mu = \prod_x \rho^2 d\rho^2 \, dZ_\mu dW_\mu^+ \, dW_\mu^- \, dA \, dg$$

- F.C.N. interpreted ρ^2 field as a dilaton in a conformally flat background gravity

$$\mathcal{G}_{\mu\nu} = \left(\frac{\rho}{\kappa}\right)^2 \eta_{\mu\nu}$$

with a Lagrangian in manifestly generally covariant form:

$$\mathcal{L}_{\mathrm{WS}} = \sqrt{-\mathcal{G}} \left\{ \frac{1}{16\pi G}(R - 2\Lambda) + \mathcal{L}_M \right\}$$

- Vacuum value of ρ is determined by the requirement

$$\rho^2|_{r\to\infty} \to \Lambda^2$$

- The Lagrangian relevant for one-loop effective potential :

$$\mathcal{L}_{trun} = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho + \frac{\rho^2}{8}(Z_\mu^2 + w_\mu^2) - \frac{1}{4g^2} w_{\mu\nu}^2 - \frac{1}{4e^2}\mathbf{A}_{\mu\nu}^2 - \frac{1}{4(g^2 + g'^2)} Z_{\mu\nu}^2$$

- The generating functional of all Feynman graphs is given by

$$Z[J, \mathbf{J}_A, \mathbf{J}_Z, \mathbf{J}^+, \mathbf{J}^-] = \int d\mu \; \delta[\partial \cdot \mathbf{A}] \; \exp i \int d^4x \, \mathcal{L}_{trun} \; . \exp i \int d^4x (J\rho + \mathbf{J_A}.\mathbf{A} + \mathbf{J}_Z.Z + \mathbf{J}_w.w + J_\Theta \Theta)$$

- The measure

$$d\mu = \rho^2 \mathcal{D}\rho^2 \mathcal{D}\mathbf{A} \mathcal{D}a \mathcal{D}Z \mathcal{D}w \mathcal{D}\Theta \mathcal{D}g$$

where $\mathcal{D}g$ is the $SU(2)$ group volume and $\mathcal{D}a$ is that of the $U(1)_{em}$.

- The upshot is group volumes are factored out without gauge fixing!

Cancellation of the local factor

- One-loop EP :-

$$
\begin{aligned}
V_{eff}(\rho_0) = & \quad -3\int \frac{d^4k}{(2\pi)^4}\ln(\rho_0) + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\ln(k^2) \\
+ & \quad 3\int \frac{d^4k}{(2\pi)^4}\ln\left(k^2 + g^2\frac{\rho_0^2}{4}\right) + \int \frac{d^4k}{(2\pi)^4}\ln(\rho_0) \\
+ & \quad \frac{3}{2}\int \frac{d^4k}{(2\pi)^4}\ln\left(k^2 + (g^2 + g'^2)\frac{\rho_0^2}{4}\right) + 2\int \frac{d^4k}{(2\pi)^4}\ln\rho_0 \, .
\end{aligned}
$$

- The Jacobian contribution from the functional measure is exactly cancelled by two terms coming from the neutral and charged vector boson operators
- Photon part of the Lagrangian doesn't contribute to the one-loop effective potential since it is not coupled to any other field of the theory.

One-loop EP and Mass spectrum

- The renormalized effective one loop Higgs potential :

$$V_{eff}(\rho_0) = \frac{27(g^2+g'^2)^2M^2\rho_0^2}{512\pi^2} + \frac{27g^4M^2\rho_0^2}{256\pi^2} + \left(\frac{3(g^2+g'^2)^2\rho_0^4}{1024\pi^2} + \frac{3g^4\rho_0^4}{512\pi^2}\right)\left(\ln\frac{\rho_0^2}{M^2} - \frac{25}{6}\right)$$

- The mass generated for the charged and uncharged gauge bosons :

$$m_{W^\pm} = \frac{1}{2}g\langle\rho\rangle$$

and

$$m_Z = \frac{1}{2}(g^2+g'^2)^{1/2}\langle\rho\rangle$$

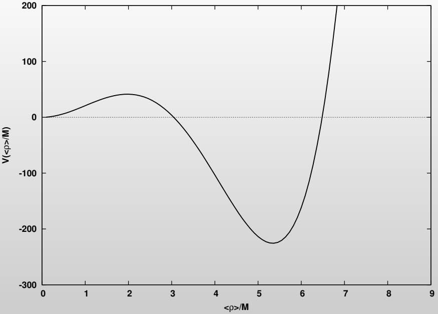

Figure: Plot of the Effective Potential as a function of $\frac{\langle\rho\rangle}{M}$.

- Since $\langle\rho\rangle = 246 Gev$ reproduces the observed W and Z boson mass spectrum, this implies that one must make the choice $M \sim 46$ Gev
- ρ boson mass :- $m_H^2 = 9\frac{(g^2+g'^2)^2+2g^4}{256\pi^2}\left(3M^2 + \langle\rho\rangle^2 \, ln\frac{(\langle\rho\rangle)^2}{M^2} - 3\langle\rho\rangle^2\right)$
- The mass of the ρ field is computed to be 6.9 Gev : perhaps too light to be phenomenologically relevant as a standard Higgs field.

Discussions

- In case of F.C.N. the Higgs field remains massless even after generating the masses of the weak gauge bosons at the tree level but here we have shown that Higgs gets a radiative mass.
- The Higgs mass is determined by the renormalization scale and gauge couplings without the application of 'Dimensional Transmutation'. In fact due to absence of any self-coupling we could not have any 'D T' like in the standard C-W mechanism here.
- The Higgs mass is completely determined by the gauge couplings which are well determined experimentally $\Rightarrow$ the scale of the theory M can't slide arbitrarily to the GUT or the Planck scale $\Rightarrow$ no problem of fine tuning of dimensionless parameters.
- The mass of the Higgs field is too light $(\ 7Gev)$ to be of any phenomenological interest.

Summary and Scope

- $SU(2) \times U(1)$ theory can be made *completely* gauge-free with the help of new variable formulation by F.C.N..
- We have illustrated an alternative scheme of generating masses to the gauge bosons in Electroweak theory. The masses are generated radiatively without using any self coupling of the Higgs field. Vector bosons get correct masses through Higgs Mechanism without any symmetry breaking!
- The novel interpretation of the Higgs field as a conformal degrees of freedom in F.C.N. doesn't survive upon quantization, as genuine radiative effects cancel it exactly.
- Fermion sector : Chiral symmetry prevents to generate masses of fermions dynamically at the quantum level. Source of fermion masses may require nonperturbative mechanism.

Thank You