

# Neutrino Masses and Mixing: Current Theoretical Status

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## INTRODUCTION

Results of solar and atmospheric experiments have

- Revealed the presence of new sub-eV scale which cannot be accommodated in SM characterized by the weak scale.
- Led to revision of our ideas on flavour structure due to two reasons:
  - Flavour mixing among neutrinos is large
  - Neutrino mass hierarchies are milder compared to the charged fermion mass hierarchies with all neutrinos being degenerate as an allowed possibility

We will review:

- Proposed mechanisms for neutrino mass generation
- Theoretical ways of obtaining different types of neutrino mass hierarchies
- Old and new ideas on understanding large neutrino mixing angles
- Simultaneous understanding of quark and leptonic mixing patterns in GUT

## PLAN

- Present experimental knowledge on neutrinos
- Mechanisms for neutrino mass generation
- Symmetries of  $\mathcal{M}_{\nu f}$
- Discrete flavour symmetries
- Dynamical mechanisms for generating neutrino mixing angles
- Generating neutrino mass hierarchies
- Ways to “predict”  $\theta_{13}$  and CP violating phase

## What we know about Neutrinos

### Masses:

$$\Delta_{\odot} \approx 7.6 \times 10^{-5} \text{ eV}^2$$

$$|\Delta_{atm}| \approx 2.4 \times 10^{-3} \text{ eV}^2$$

$$0.3 \text{ eV} \leq \sum m_{\nu_i} \leq 2 \text{ eV}$$

This information allows two mass hierarchies:

- Normal: Solar pair is lighter than the heaviest state
- Inverted: Solar pair is heavier than the lightest state

## Mixing:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

We know,

$$|\sin^2 \theta_{23} - 1/2| \leq 0.12$$

$$|\sin^2 \theta_{13} - 1/3| \leq 0.04$$

$$|\sin^2 \theta_{13}| \leq 0.04$$

Some indication of a non-zero  $\sin^2 \theta_{13} \sim 0.016 \pm 0.01$  from global fits

Some indication of non-zero  $\delta \sim 220^\circ$  from atmospheric data

## Mechanisms for neutrino mass generation

Different mechanisms aim at explaining

- Smallness of neutrino mass
- Origin of Lepton number violation if it is violated
- If not then origin of a Dirac neutrino with a mass at least six orders of magnitude smaller than the lightest charged fermion

Since the direct coupling of  $\nu_L$  with itself is not possible one can generate it effectively through indirect coupling with

- Singlet fermion (Type-I seesaw)
- Triplet Higgs (Type-II seesaw)
- Triplet Fermion (Type-III seesaw)

## Type I+II seesaw

Obtained by coupling  $\nu_L$  with  $\nu_R$ :

$$M_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} \quad \Rightarrow \quad M_\nu \approx m_L - m_D M_R^{-1} m_D^T$$

In left right theories

$$m_L M_R \sim M_W^2$$

and largeness of  $M_R$  explains smallness of  $M_\nu$ .

## Type-III mechanism

Most well-motivated example of type-III mechanism is

Supersymmetry with R-parity violation

$$\nu_L \lambda \langle \tilde{\nu} \rangle$$

Smallness of neutrino masses is linked to suppression of the sneutrino vev and one can get small neutrino masses even with weak seesaw scale

One can consider non-susy models with a triplet fermion.

- Light triplet fermions help gauge coupling unification
- By adding 24-plet of  $SU(5)$  one can obtain both type-I and III mechanisms together
- No specific reason for the lightness of fermions and no specific mechanism for suppression of the triplet fermion coupling

## Variants of seesaw: Double seesaw, Inverse seesaw

Add more singlets  $S$  in addition to  $\nu_R$

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu \end{pmatrix}$$

- Lepton number is conserved in the limit  $\mu = 0$ . Hence smallness of neutrino mass linked to small  $\mu$  [Schechter and Valle; Valle]
- allows low seesaw scale since

$$M_\nu \sim m_D M^{-1} \mu M^{T^{-1}} m_D^T$$

## THE OBJECT OF MAIN INTEREST: $\mathcal{M}_{\nu f}$

- $\mathcal{M}_{\nu f}$  can be reconstructed from experiments since

$$\mathcal{M}_{\nu f} \equiv U_{PMNS}^* \text{Diag.}(m_1, m_2, m_3) U_{PMNS}^\dagger$$

- $\mathcal{M}_{\nu f}$  is known only partially at present. The gap can be filled by
  - Educated guesses on its possible structure
  - Studying symmetries of the partially known  $\mathcal{M}_{\nu f}$
  - Starting with a well-defined theoretical framework e.g. GUTs and use it to 'predict' complete  $\mathcal{M}_{\nu f}$

## Symmetries of $\mathcal{M}_{\nu f}$

$\mathcal{M}_{\nu f}$  is invariant under two types of symmetries:

For any given mixing patterns,  $\mathcal{M}_{\nu f}$  is always invariant under a  $Z_2 \times Z_2$  symmetry. This symmetry is a “kinemetical” symmetry. [Lam; Grimus Lavoura]

$\mathcal{M}_{\nu f}$  can be invariant under more dynamical symmetries which can be used to restrict other properties like CP violation and neutrino mass hierarchies

## $Z_2 \times Z_2$ symmetries

Let  $|\psi_i\rangle$  ( $i = 1, 2, 3$ ) be columns of the PMNS matrix. Then

$$\mathcal{M}_{\nu f} = m_i |\psi_i\rangle \langle \psi_i|$$

Let

$$S_i = 2|\psi_i\rangle \langle \psi_i| - 1 \quad ; \quad S_i^2 = 1 \quad \text{and} \quad S_1 + S_2 + S_3 = -1$$

$S_i$  define two independent  $Z_2$  symmetries satisfying

$$S_i^T \mathcal{M}_{\nu f} S_i = \mathcal{M}_{\nu f}$$

Thus given any specific mixing matrix one can always construct  $Z_2 \times Z_2$  symmetries which leave  $\mathcal{M}_{\nu f}$  invariant.

## Special $Z_2 \times Z_2$ symmetry

We know to a good approximation

$$\theta_{13} = 0, \theta_{23} = \frac{\pi}{4}$$

. This  $\Rightarrow$

$$\psi_3 = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

is one of the column of the  $U_{PMNS}$ . This  $\Rightarrow$

$$S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$S_3$  is the well-known  $\mu - \tau$  symmetry.

We know that to a good approximation  $\sin^2 \theta_{12} = \frac{1}{3}$  also. This leads to another  $Z_2$ :

$$\psi_2 = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$$

corresponding to

$$S_2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

and

$$S_2^T \mathcal{M}_{\nu f} S_2 = \mathcal{M}_{\nu f}$$

$S_2, S_3$  define special  $Z_2 \times Z_2$  symmetry under which any successful  $\mathcal{M}_{\nu f}$  should be invariant at least approximately.

## From symmetry of $\mathcal{M}_{\nu f}$ to theory for $\mathcal{M}_{\nu f}$

It is possible that  $Z_2 \times Z_2$  invariance of  $\mathcal{M}_{\nu f}$  is

- an (approximate) symmetry imposed at the Lagrangian level
- a consequence of some more fundamental symmetry imposed on the theory
- may arise from some dynamical mechanism which leads to

$$\theta_{23} \approx \frac{\pi}{4}, \theta_{13} \approx 0, \sin \theta_{12} \approx \frac{1}{\sqrt{3}}.$$

## Approximate $\mu$ - $\tau$ symmetry at the Lagrangian level

Exact  $\mu$ - $\tau$  symmetry at the Lagrangian level implies  $\theta_{23} = 0$

**BUT**

Even a tiny breaking of the symmetry can lead to large mixing angle thanks to the seesaw mechanism [AsJ]

$$M_l = \frac{m_\tau}{2} \begin{pmatrix} 1 & 1 + \lambda_l \\ 1 + \lambda_l & 1 \end{pmatrix} \quad m_D = \frac{m_{3D}}{2} \begin{pmatrix} 1 - \epsilon_D & 1 + \lambda_D \\ 1 + \lambda_D & 1 + \epsilon_D \end{pmatrix}$$

$$M_R = \frac{M_3}{2} \begin{pmatrix} 1 & 1 + \lambda_R \\ 1 + \lambda_R & 1 \end{pmatrix}$$

$$\lambda_f \approx 2 \frac{m_{2f}}{m_{3f}} \quad f = l, D, R$$

and only source of the  $\mu$ - $\tau$  breaking is

$$\epsilon_D \sim \frac{m_{2D}}{m_{3D}} \ll 1$$

. This implies after seesaw

$$M_\nu \approx \begin{pmatrix} B(1 - \epsilon_\nu) & C \\ C & B(1 + \epsilon_\nu) \end{pmatrix}$$

with

$$\epsilon_\nu \approx -\frac{2\epsilon_D \lambda_D}{\epsilon_D^2 + \lambda_D^2} \approx 1$$

As a result, mixing is suppressed in  $M_\nu$ ;  $M_l$  leads to nearly maximal  $\rightarrow \theta_{23} \approx \frac{\pi}{4}$

- The above example illustrates direct but approximate realization of the  $\mu$ - $\tau$  symmetry at the Lagrangian level
- There are indirect ways to obtain exact  $\mu$ - $\tau$  symmetry for  $\mathcal{M}_{\nu f}$ . Start with entirely different symmetry imposed on the Lagrangian namely  $D_4$ . Its judicious breaking then leads to the exact  $\mu$ - $\tau$  symmetry [Grimus Lavoura]

## ADDING ANOTHER $Z_2$ : TRI-BIMAXIMAL MIXING

- Assume that the charged lepton mass matrix is invariant under a  $Z_3$  symmetry

$$S_L^\dagger M_l S_R = M_l$$

defined by

$$e_{L,R} \rightarrow S_{L,R} e_{L,R}$$

with

$$S_L = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad S_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \quad \omega^3 = 1$$

This implies

$$M_l = U D_l$$

,

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

- Also assume that

$$\nu_L \rightarrow S_{\nu L} \nu_L$$

with

$$S_{\nu L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- The  $Z_3$  symmetry of  $\epsilon_L$  and and  $Z_2$  of  $\nu_L$  do not commute but they are sub-groups of a group  $A_4$  whose breaking leads

to these discrete subgroups after the SM symmetry breaking  
and to the tri-bimaximal mixing.

## DYNAMICAL MECHANISMS

We discuss dynamical mechanisms and models which lead to

- Large  $\theta_{23}$  and  $\theta_{12}$  but different from their magic values
- Non-zero and generically large  $\theta_{13}$

Many of them do not use flavour symmetry but finally lead to approximate  $Z_2 \times Z_2$  symmetry!

## $\theta_{13}$

- Generic expectation relatively large  $\theta_{13} \sim 0.05$  unless one has specific symmetry broken in a specific manner
- $\theta_{13}$  obtains contribution from two sources:

$$\sin \theta_{13} \sim s_{13\nu} - s_{12l} \sin \theta_A$$

In the absence of cancellations,

$$\sin \theta_{13} \sim \frac{1}{\sqrt{2}} s_{12l} \approx \mathcal{O}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}}\right) \sim \mathcal{O}(0.05)$$

- Start with a specific texture

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & s^2 & sc \\ 0 & sc & c^2 \end{pmatrix} \quad \begin{pmatrix} 0 & c & s \\ c & 0 & 0 \\ s & 0 & 0 \end{pmatrix}$$

Perturbation  $\Rightarrow$

$$\begin{pmatrix} 0 & 0 & \lambda \\ 0 & s^2 & sc \\ \lambda & sc & c^2 \end{pmatrix}$$

This leads to

$$|\theta_{13}| \approx \frac{\tan 2\theta_{\odot}}{2 \tan \theta_A} \left( \frac{\Delta_{\odot}}{\Delta_A \cos 2\theta_{\odot}} \right)^{\frac{1}{2}} \approx 0.13$$

- The above structure gets realized in several scenarios, e.g. single RH nu dominance [King],  $SO(10)$  model with type-II seesaw [Goh, Mohapatra and Ng]

## Broken discrete symmetry versus dynamical mechanisms

Breaking of  $Z_2 \times Z_2$  symmetry leads to non-zero  $\theta_{13}$  and departures from magic values. Special perturbations exist which can distinguish broken discrete symmetries from the models which have small  $\theta_{13}$  without specific flavour symmetries.

- One can consider arbitrary but small breaking of the  $\mu$ - $\tau$  symmetry or the tri-bimaximal structure and study its consequences [Grimus *et al*; Mohapatra; Albright and Rodejohann]

Normal Hierarchy:

$$\sin \theta_{13} \approx c_{12} s_{12} \sqrt{\frac{\Delta_{sun}}{\Delta_A}} \approx 0.1(\epsilon + \epsilon'/2) \quad \cos 2\theta_{23} \approx \epsilon'$$

Inverted hierarchy:

$$\sin \theta_{13} \approx \frac{1}{4} \sin 2\theta_{12}(\epsilon - \epsilon'/2) \approx 0.1(\epsilon - \epsilon'/2) \quad \cos 2\theta_{23} \approx (\epsilon + \epsilon'/2)$$

Radiatively generated  $\theta_{13}$

In this case,

$$\epsilon = \epsilon'/2$$

This  $\Rightarrow \theta_{13} = 0$  for inverted hierarchy at 1-loop level [Ray, Rodejohann and Schmidt].

Very small contribution is generated at two loop and typical values  $(1 - 5) \times 10^{-4}$  generated by the Planck scale effect [Berezinsky, Narayan and Vissani] may be regarded as a hallmark of such schemes.

- Typical symmetries which lead to tri-bimaximal mixing also dictate the structure of its breaking by the non-renormalizable

interactions. They lead in a specific instance [Altarelli and Feruglio] to

$$\sin^2 \theta_{12} = \frac{1}{3} + \mathcal{O}(\epsilon) \quad ; \quad \sin^2 \theta_{23} = \frac{1}{2} + \mathcal{O}(\epsilon) \quad ; \quad \sin \theta_{13} = \mathcal{O}(\epsilon)$$

$\epsilon$  is restricted to be  $\sim 0.04$  implying relatively small  $\theta_{13}$ .

## DYNAMICAL MECHANISMS

- Seesaw mechanisms
- Grand Unified Theories
- Radiative enhancement
- Quasi Degeneracy of neutrinos

## Grand Unified Theories

$SU(5)$  prediction

$$M_d = M_l^T \Rightarrow$$

”lopsided texture“

$$M_l \approx \begin{pmatrix} 0 & 0 & C_1 \\ 0 & \epsilon & C_2 \\ \epsilon_1 & \epsilon_2 & 1 \end{pmatrix}$$

Minimal  $SO(10)$  with fermions coupling to  $10 + \overline{126}$  of Higgs + type-II seesaw mechanism [Bajc, Senjanovic, Vissani; Aulakh; Garg and Aulakh]

$$M_\nu = M_d - M_l \approx \begin{pmatrix} m_s + m_b \lambda^2 - m_\mu & (m_b - m_s) \lambda \\ (m_b - m_s) \lambda & m_b - m_\tau \end{pmatrix}$$

$b - \tau$  unification leads to large mixing.

- This mechanism also explains why  $\theta_{13}$  and  $\frac{\Delta_{sol}}{\Delta_A}$  are small.
- Predicts  $\theta_{13}$  near the present limit
- Requires intermediate scale and is uncomfortable with the gauge coupling unification
- Many variants with additional 120 and type-I seesaw mechanism are analyzed.

## Large mixing through type-I seesaw

Minimal non-supersymmetric model with  $10+126$  Higgs leads to excellent fits for fermion masses and mixing angles.

All fermionic mass matrices exhibit identical structure [K. M. Patel and ASJ]

$$M_f \approx \begin{pmatrix} c_{11}\lambda^4 & c_{12}\lambda^3 & c_{13}\lambda^2 \\ c_{12}\lambda^3 & c_{22}\lambda^2 & c_{23}\lambda \\ c_{13}\lambda^2 & c_{23}\lambda & c_{33} \end{pmatrix}$$

- $c_{ij} \sim \mathcal{O}(1)$  ,  $\lambda \sim$  Cabibbo angle
- Nearly singular  $M_R$  after the seesaw mechanism leads to large solar and atmospheric mixing angles [Smirnov]
- One predicts large  $\theta_{13} \sim 0.1$

## Large mixing through quasi degeneracy

- Hierarchical in fermion masses  $\rightarrow$  small mixing  $\tan \theta = \sqrt{\frac{m_1}{m_2}}$
- Quasi degeneracy  $\rightarrow$  implies large mixing

$$M_\nu = \begin{pmatrix} \epsilon_1 & 1 \\ 1 & \epsilon_2 \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Quasi Degeneracy also explains smallness of  $\theta_{13}$ [Branco et al]

Quasi Degenerate structure

$$\mathcal{M}_{\nu f} = m_0 U = \text{Symmetric unitary matrix}$$

An Example

$$M_l = U_{lL} D_l U_{lR} \quad \text{and} \quad 'M_\nu = m_0 I$$

. This leads to above  $\mathcal{M}_{\nu f}$  with  $U = U_{lL}^T U_{lL}$   $\mathcal{M}_{\nu f}$  is diagonalized by

$$V = U_{23}(\theta_{23}) U_{12}(\theta_{12})$$

leading to  $\theta_{13} = 0$  and unconstrained  $\theta_{12}, \theta_{23}$  at the tree level. If perturbations which lift degeneracy do not change the mixing angles appreciably then correct mixing patterns are dictated by QDG. This happens in several schemes leading to QDG neutrinos in type-I seesaw model [Patel, Vempati, ASJ; Patel, ASJ]

## Inverted hierarchy in type-I seesaw

Assume a symmetry [Rodejohann and ASJ]

$$\nu_L \rightarrow S\nu_L \quad \text{Det}S \neq 1$$

This implies  $m_D = Sm_D$  ;  $S^T M_\nu S = M_\nu$  and  $\text{Det}m_D = 0$

If in addition,  $(0, s, c)^T$  is an eigenvector corresponding to zero eigenvalue then one gets inverted hierarchy. The most general matrix satisfying these requirements have scaling form [Mohapatra and Rodejohann]:

$$M_\nu = \begin{pmatrix} a & b & rb \\ b & c & rc \\ rb & rc & r^2c \end{pmatrix}$$

## Type-I seesaw model with quasi degeneracy

Type-I neutrino mass matrix is given by

$$M_\nu = m_D M_R^{-1} m_D^T$$

- Degeneracy follows if  $m_D \sim M_R \sim I$  Examples are  $O(3)$  or more economic  $A_4$  symmetry [Ma and Rajasekaran; Babu Ma and Valle]
- More general way of obtaining quasi degeneracy follows from application of Minimal Flavour Violation hypothesis to leptonic sector. Specific symmetry of Yukawa interaction  $y_L, y_D$  when used to determine structure of  $M_R$  [Patel, Vempati and ASJ] implies

$$M_R \sim m_D^T m_D$$

and hence the degeneracy. Departures from degeneracy dictated by symmetry:

$$\mathcal{M}_{\nu f} \approx m_0(I - p y_l y_l^T) \quad y_l = D_l V_R \text{ in flavour basis}$$

- Realized through ideas of "Dirac Screening" [Schmidt, Lindner and Smirnov] and in specific  $SO(10)$  model which provide a unified description of all fermion masses along with quasi degenerate neutrinos

## Predicting CP violating phase $\delta$

Generic expectation:

Large  $\delta$ !

- Presence of CP violation in theory
- Smallness of  $\theta_{13}$

Models with vanishing  $\theta_{13}$  and non-zero Majorana phases when perturbed by a small amount  $\epsilon$  lead to large  $\delta$  independent of the strength of perturbation. Examples:

- broken  $\mu$ - $\tau$  symmetry

$$\tan \delta \approx \frac{m_1 m_2 \sin 2(\rho - \sigma) - m_3 m_1 \sin 2\rho + m_2 m_3 \sin 2\sigma}{m_1 c_{12}^2 - m_2^2 s_{12}^2 - m_1 m_2 \cos 2\theta_{12} \cos 2(\rho - \sigma) + m_3 m_1 \cos 2\rho - m_2 m_3 \cos 2\sigma}$$

- $\delta$  is large irrespective of the neutrino mass hierarchies if  $\rho, \sigma$  are large and not fine-tuned.
- Models with radiatively generated  $\theta_{13}$  also lead to  $\delta$  related to the original Majorana phases [Dighe *et al*; Rindani, Singh and ASJ]

## Large $\delta$ from symmetry

Harrison-Scott structure:

$$|U_{\alpha 2}| = |U_{\alpha 3}| \quad \alpha = e, \mu, \tau$$

- This leads to  $\theta_{23} = \frac{\pi}{4}$  [Grimus and Lavoura]
- Follows as a consequence of generalized  $\mu$ - $\tau$  symmetry

$$S_{\mu\tau}^T \mathcal{M}_{\nu f} S_{\mu\tau} = \mathcal{M}_{\nu f}^*$$

- $\Rightarrow s_{13} \cos \delta = 0 \quad \delta = \frac{\pi}{2}$  if  $\theta_{13} \neq 0$ .

Realized in an example based on  $A_4$  [Babu, Ma, Valle]

There exist other models/scenario which lead to correlation between  $\theta_{13}$  and  $\cos \delta$ :

$$\sin \theta_{13} \cos \delta = C$$

$C$  is a known constant. Examples

- Lopsided  $SO(10)$ :  $C \approx \frac{1}{4} \sin \theta_{12}$  [Barr and Khan]

- $|U_{\mu 2}|^2 = |U_{\tau 2}|^2$   $C = \frac{c_{12}^2 - s_{12}^2 s_{13}^2}{\tan 2\theta_{23} \sin 2\theta_{12}}$

$\Delta(27)$  symmetry [Grimus, Lavoura]

More examples in [(Albright, Dueck, Rodejohann)]

- Different  $SO(10)$  models lead to predictions of "best fit" values for  $\delta$ , e.g.

$\delta \sim 227^\circ$  [Chen-Mohantappa]

$\delta \sim 54^\circ$   $SO(10) + QDG$  neutrinos

and several other models.....

## SUMMARY:

- The presently available information on neutrino mixing angles point to “magic values” of mixing angles
- They may arise from special leptonic symmetries
- Breaking of this symmetry may leave its traces:  
Small  $\theta_{13} \leq 0.02$  for normal or inverted hierarchy  
Relatively large  $\theta_{13}$  for quasi degenerate spectrum
- Mixing angles may arise due to underlying dynamics: Grand Unified theories and seesaw mechanism can lead to this dynamics
- Generic expectations in large class of models:  $\theta_{13}$  near its present value and large  $\tan \delta$