

FLAVOR UNIFICATION, DARK MATTER, PROTON  
DECAY AND OTHER OBSERVABLE PREDICTIONS  
WITH LOW-SCALE  $S_4$  SYMMETRY

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## PLAN OF THE TALK

- 1 Introduction
- 2 The standard gauge theory with low-scale  $S_4$  symmetry
- 3 Unification with single-step breaking
- 4 Unification with  $SU(2)_L \times SU(2)_R \times SU(4)_C \times S_4$  ( $g_{2L} \neq g_{2R}$ ) intermediate symmetry
- 5 Predictions with  $G_{224D} \times S_4$  ( $g_{2L} = g_{2R}$ ) intermediate symmetry
- 6 Unification with  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times S_4$  intermediate symmetry
- 7 Unification with light fermions and dark matter
- 8 Summary and Conclusions

## 1. INTRODUCTION

- The standard model gauge theory based upon  $SU(2)_L \times U(1)_Y \times SU(3)_C (\equiv G_{213})$  has enjoyed tremendous success, though it has several shortcomings
- Some of which are circumvented when the model emerges from a GUT
- Predictions of gauge boson mediated proton decays in non-SUSY GUTs are neat and robust
- Possible elegant solutions to the fermion flavor problem might need flavor symmetries on which considerable attention has been devoted. Experimental evidences of masses and large mixings of neutrinos which renewed interests to understand flavor puzzle have led to the suggestions of grand unification symmetry of flavor including  $SO(10) \times G_f (G_f = S_4, SO(3)_f, SU(3)_f)$

## INTRODUCTION CNTD. . .

- Recently Hagedorn, Lindner, and Mohapatra [HLM, JHEP **6**, 42 (2006)] have examined an interesting model based upon the non-SUSY SM gauge structure as a possible solution to the fermion flavor problem with surviving  $G_{213} \times S_4$  symmetry near the electroweak scale.
- It predicts a rich structure of neutral and charged Higgs scalars near the TeV scale accessible to Tevatron, LHC, and ILC. It embodies unification as well as an ansatz for all fermion masses and mixings.
- The problem of embedding R-parity conserving SUSY left-right gauge symmetry and  $S_4$  family symmetry as an intermediate symmetry,  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c \times S_4 (g_{2L} = g_{2R})$  in SUSY  $SO(10) \times S_4$  has been successfully solved recently with a high intermediate scale [M K Parida, PRD **78**, 53004 (2008)],  $M_R \simeq 10^{13}$  GeV, where all fermion masses and mixings are adequately well represented by the model parameters.

## INTRODUCTION CNTD...

- However, in this model there is no signature of the underlying flavor symmetry near the TeV scales.
- Gauge hierarchy certainly prefers SUSY  $SO(10) \times G_f$  model, in the absence of any evidence of SUSY at low energies and for the sake of simplicity alone, however we believe that prospects of minimal non-SUSY  $SO(10) \times G_f$  should be thoroughly explored and confronted with experiments.

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### In the present work

we investigate the prospects of GUT of the low-scale flavor symmetric theory of HLM (2006) through the popular unifying groups  $GU \times G_f$  where the GUT gauge theory  $GU = SU(5) SO(10)$  and the flavor group is  $G_f$  and examine if there are any other possible signatures at low energies through proton decay and accelerator searches.

- We also estimate threshold effects for GUT scale and proton lifetime.

## 2. THE STANDARD GAUGE THEORY WITH LOW-SCALE $S_4$ SYMMETRY

- First, we discuss salient features of the  $G_{213} \times S_4$  model of HLM (2006) relevant for the present work.
- The non-abelian discrete symmetry group  $S_4$  has two types of triplet representations,  $\mathbf{3}_1$  and  $\mathbf{3}_2$ , two types of singlet representations,  $\mathbf{1}_1$  and  $\mathbf{1}_2$ , but only one type of doublet representation,  $\mathbf{2}$ .
- Three generations of standard fermions are to transform as  $\mathbf{3}_2$ .

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- Three generations of standard fermions are to transform as  $\mathbf{3}_2$ .

The proposal gives a very interesting possibility of embedding in the most attractive grand unified theory like  $SO(10)$  by appending it with the continuous flavor group  $G_f = \mathbf{SO}(3)_f$  or  $\mathbf{SU}(3)_f$ . The six plets of  $S_4$  doublet Higgs representations are appropriately fitted into those of  $G_f$  by using six 10-plets of  $\mathbf{SO}(10)$  with transformation property  $(\mathbf{10}, \mathbf{3}_1 + \mathbf{2} + \mathbf{1}_1)$  under  $\mathbf{SO}(10) \times S_4$ .

## THE STANDARD GAUGE THEORY CNTD...

The particle content is summarized as

Particle	$SU(2)_L \times U(1)_Y \times SU(3)_c$	$S_4$
Quarks $Q$	$(2, +\frac{1}{3}, 3)$	$3_2$
Anti quarks $u^c$	$(1, -\frac{4}{3}, \bar{3})$	$3_2$
Anti quarks $d^c$	$(1, +\frac{2}{3}, \bar{3})$	$3_2$
Leptons $L$	$(2, -1, 1)$	$3_2$
Anti leptons $e^c$	$(1, +2, 1)$	$3_2$
Right-handed $\nu$ 's $\nu^c$	$(1, 0, 1)$	$3_2$
Doublet Higgs $\phi_0$	$(2, -1, 1)$	$1_1$
Doublet Higgs $(\phi_1, \phi_2)$	$(2, -1, 1)$	$2$
Doublet Higgs $(\sigma_1, \sigma_2, \sigma_3)$	$(2, -1, 1)$	$3_1$

### 3. UNIFICATION WITH SINGLE-STEP BREAKING

#### Absence of unification in the minimal model

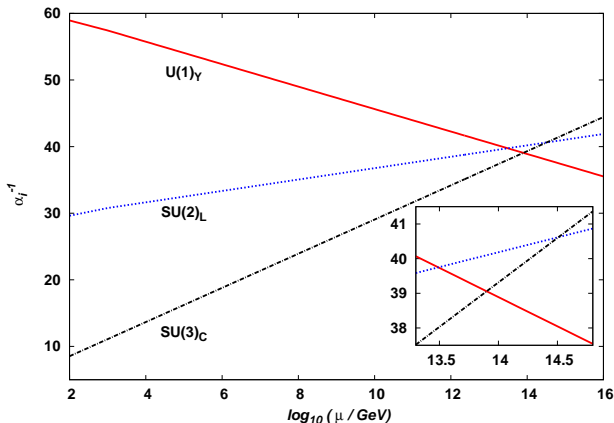
We assume the symmetry  $\mathbf{G}_{213} \times \mathbf{S}_4$  to be restored at a low scale  $\mu = M_S \simeq 300 \text{ GeV} - 500 \text{ GeV}$ . We explore the possibility of grand unification in the single-step breaking to  $\mathbf{G}_{213} \times \mathbf{S}_4$ ,

$$\mathbf{GU} \times \mathbf{G}_f \xrightarrow{M_U} \mathbf{G}_{213} \times \mathbf{S}_4 \xrightarrow{M_S} \mathbf{G}_{213}$$

For the evolution of gauge couplings, we utilize the two-loop renormalization group equations

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

RG evolutions of the three gauge couplings from  $\mu = M_Z$  to  $\mu = M_{Planck}$  showing absence of unification in the minimal  $G_{213} \times S_4$  model.



### Unification with light scalars

It has been shown that in the absence of flavor symmetry in SUSY as well as non-SUSY models, the three gauge couplings of the SM may unify at  $M_U \geq 10^{15}$  GeV provided there are additional particle degrees of freedom (fermions or scalars) at lower scales.

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

- First, we attempt to unify couplings in the presence of flavor symmetry, by minimally populating the non-SUSY grand desert by the scalar  $\xi(3, 0, 8)$  where the quantum numbers are under  $G_{213}$  [Kynshi and Parida, PRD **47**, R4830 (1993)].

- Model I(a)

$$M_C(1, 0, 8) = M_X = 10^5 \text{ GeV}$$

$$M_\xi(3, 0, 8) = M_I = 10^{13} \text{ GeV}$$

Both scalars are contained in  $(210, 1)$  of  $SO(10) \times S_4$

- Model I(b)

$$M_\sigma(3, 0, 1) = M_X = 10^3 \text{ GeV}$$

$$M_C(1, 0, 8) = M_I = 10^{3.5} \text{ GeV}$$

$2C(1, 0, 8)$  and  $2\sigma(3, 0, 1)$  can be embedded into  $(210, 2)$ , or  $(45, 2)$  under  $SO(10) \times S_4$

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

- $M_Z—M_S$

$$a_i = \left( \frac{41}{10}, \frac{-19}{6}, -7 \right)$$

$$b_{ij} = \left( \frac{122}{25}, \frac{36}{5}, \frac{44}{5} \right), \left( \frac{12}{5}, \frac{50}{3}, 12 \right), \left( \frac{11}{10}, \frac{9}{2}, -26 \right)$$

- $M_S—M_X$

$$a_i = \left( \frac{23}{5}, \frac{-7}{3}, -7 \right)$$

$$b_{ij} = \left( \frac{122}{25}, \frac{36}{5}, \frac{44}{5} \right), \left( \frac{12}{5}, \frac{50}{3}, 12 \right), \left( \frac{11}{10}, \frac{9}{2}, 16 \right)$$

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

For Model-I(a), we find almost exact unification of gauge couplings for which we have

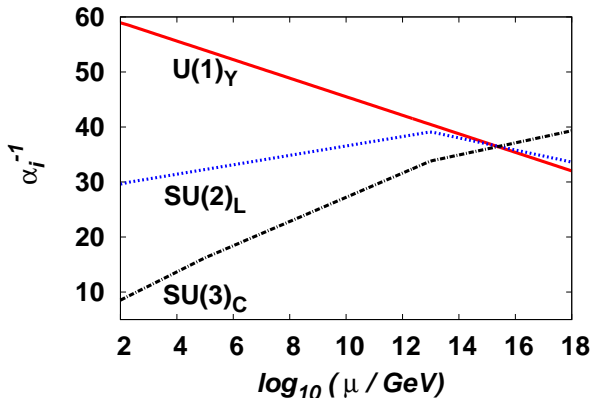
$$M_S = 10^{2.3} \text{ GeV}, \quad M_X = M_C(1, 0, 8) = 10^5 \text{ GeV}$$

$$M_I = M_\xi(3, 0, 8) = 10^{13} \text{ GeV}$$

$$M_U = 10^{15.46} \text{ GeV}, \quad 1/\alpha_G = 36.4$$

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

Unification of gauge couplings in the single-step breaking Model-I(a) with scalars  $\xi(3, 0, 8)$  and  $C(1, 0, 8)$  at  $M_\xi = 10^{13}$  GeV and  $M_C = 10^5$  GeV, respectively.



## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

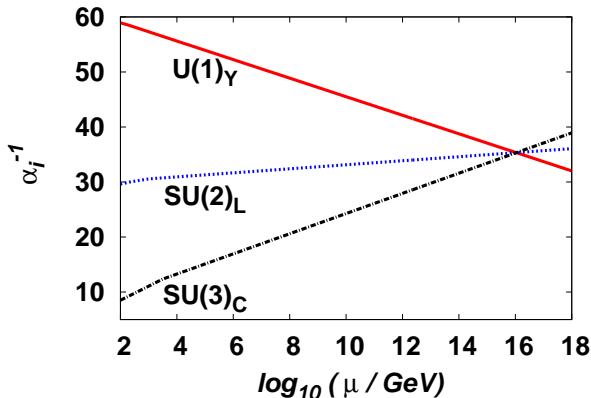
In Model-I(b), up to two-loop level the unification requirement gives

$$M_S = 10^{2.3} \text{ GeV}, \quad M_X = M_\sigma(3, 0, 1) = 10^3 \text{ GeV},$$

$$M_I = M_C(1, 0, 8) = 10^{3.5} \text{ GeV}, \quad M_U = 10^{16} \text{ GeV}, \quad 1/\alpha_G = 35.3$$

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

Unification of gauge couplings in single-step breaking Model-I(b) with scalars  $\sigma(3, 0, 1)$  and  $C(1, 0, 8)$  at  $M_\sigma \simeq 1$  TeV and  $M_C \simeq 3$  TeV, respectively



## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

Neutrino masses and mixings being governed by the type-I see-saw mechanism, the RH neutrino mass matrix is proportional to a diagonal matrix. As the Higgs  $(\overline{126}, 1)$  generates the RH neutrino mass via its coupling  $f 16_f 16_f \overline{126}$ , and the vacuum expectation value  $\langle \overline{126} \rangle \sim M_{GUT}$ , it is necessary that the Majorana type Yukawa coupling  $f \simeq 10^{-2}$  in Model I(a), but  $f \simeq 10^{-3}$  in Model I(b).

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

### Predictions on proton lifetime

Up to a good approximation, the decay width for  $p \rightarrow e^+\pi^0$  is

$$\Gamma(p \rightarrow e^+\pi^0) = \frac{m_p^5}{64\pi f_\pi^2} \left(\frac{g_G^4}{M_U^4}\right) A_L^2 \bar{\alpha}_H^2 (1 + D + F)^2 [(A_{SR}^2 + A_{SL}^2) \times (1 + |V_{ud}|^2)^2]$$

- $\bar{\alpha}_H$  = hadronic matrix elements,  $m_p$  = proton mass = 939.3 MeV, and  $f_\pi$  = pion decay constant = 139 MeV
- Chiral Lagrangian parameters are  $D=0.81$  and  $F=0.47$ ,  $V_{ud}$  represents the CKM- matrix element ( $V_{CKM}$ )<sub>12</sub> for quark mixings
- $A_{SL} \simeq A_{SR} \simeq A_{SD} \simeq 2.34(2.566)$  for model I(a)(I(b)), the long distance renormalization factor is  $A_L \simeq 1.25$ .
- $A_R = A_L A_{SD} \simeq 2.93$ ,  $F_q = 2(1 + |V_{ud}|^2) \simeq 7.6$ , The two-loop value of Model I(a),  $M_U = M_U^0 = 10^{15.46}$  GeV

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

$$\tau_p^0(p \rightarrow e^+\pi^0) = 1.365 \times 10^{33} \text{ yrs.}$$

which is about 88% shorter than the experimental limit and is found to increase due to threshold corrections

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

Maximal GUT-threshold uncertainties on unification mass and proton lifetime in the single-step breaking of  $SO(10) \times G_f$  model for two different types of superheavy masses ( $\mathcal{M}$ )

		Model I(a)		Model I(b)	
$\mathcal{M}$	$ \eta $	$\frac{M_U}{M_U^0}$	$\frac{\tau_p}{\tau_p^0}$	$\frac{M_U}{M_U^0}$	$\frac{\tau_p}{\tau_p^0}$
Degen	0.693	$10^{\pm 0.0162}$	$10^{\pm 0.0648}$	$10^{\pm 0.275}$	$10^{\pm 1.103}$
	1.0	$10^{\pm 0.0234}$	$10^{\pm 0.0939}$	$10^{\pm 0.398}$	$10^{\pm 1.592}$
	2.302	$10^{\pm 0.0538}$	$10^{\pm 0.2153}$		
	3.312	$10^{\pm 0.0772}$	$10^{\pm 0.3088}$		
Non-degen	0.693	$10^{\pm 0.0655}$	$10^{\pm 0.2620}$	$10^{\pm 0.748}$	$10^{\pm 2.993}$
	1.0	$10^{\pm 0.0946}$	$10^{\pm 0.3780}$	$10^{\pm 1.08}$	$10^{\pm 4.32}$
	2.302	$10^{\pm 0.2175}$	$10^{\pm 0.8703}$		
	3.312	$10^{\pm 0.3133}$	$10^{\pm 1.254}$		

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

The Model I(b) has the additional testable prediction that both the Higgs scalar components  $\sigma(3, 0, 1)$  and  $C(1, 0, 8)$  being in the mass range (1-3) TeV are accessible for detection within the LHC energy ranges. Because of low scale  $S_4$  symmetry, all Higgs scalars emerging from six doublets should also be detected at LHC or ILC energies. This latter feature is common to all models discussed here.

#### 4. UNIFICATION WITH $SU(2)_L \times SU(2)_R \times SU(4)_C \times S_4$ ( $g_{2L} \neq g_{2R}$ ) INTERMEDIATE SYMMETRY

In the absence of flavor symmetry whereas the single-step breaking minimal grand desert models have been found to be ruled out, GUTs such as  $SO(10)$  with one or more intermediate symmetries have been found to be consistent with  $\sin^2 \theta_W$  and proton lifetime constraints. In addition, the left-right symmetry breaking intermediate scale has been identified with type-I see-saw scale in a number of models.

## UNIFICATION WITH $SU(2)_L \times SU(2)_R \times SU(4)_C$ CNTD...

*In this section using flavor unification through  $SO(10) \times S_4$ , we explore the possibility of intermediate Pati-Salam gauge theory with or without D-Parity at the intermediate scale*

$$SO(10) \times S_4 \xrightarrow{M_U} (G_{224} \text{ or } G_{224D}) \times S_4 \xrightarrow{M_R} G_{213} \times S_4$$

*where we have defined  $SU(2)_L \times SU(2)_R \times SU(4)_C (g_{2L} \neq g_{2R}) \equiv G_{224}$ , which has broken D-parity and  $G_{224D}$  is the same gauge theory with unbroken D-parity ( $g_{2L} = g_{2R}$ )*

## UNIFICATION WITH $SU(2)_L \times SU(2)_R \times SU(4)_C$ CNTD...

### Model II

At two-loop level for this model we obtain

$$\begin{aligned}M_S &= 10^{2.5} \text{ GeV}, \quad M_X = M_C(1, 0, 8) = 10^{2.7} \text{ GeV}, \text{ GeV} \\M_R^0 &= M_{C'}(1, 1, 15) = 10^{10.5} \text{ GeV}, \quad M_U^0 = 10^{15.7} \\ \alpha_G^{-1} &= 39.62\end{aligned}$$

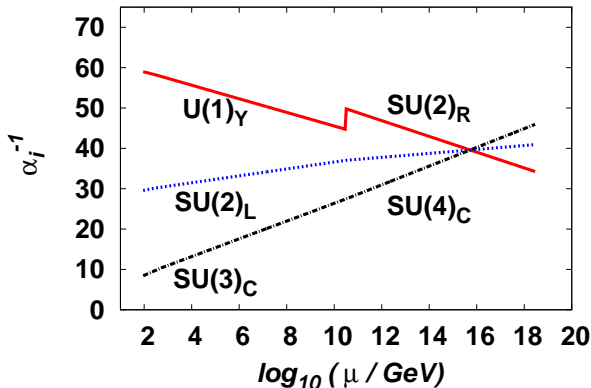
With  $M_U^0$  slightly larger than the minimal value ( $M_U^{\min} = 10^{15.668} \text{ GeV}$ ), the two-loop solution predicts proton lifetime about 19% more than the current experimental limit

$$\tau_p^0 \simeq 1.385 \times 10^{34} \text{ yrs.}$$

which is clearly accessible to ongoing searches for  $p \rightarrow e^+ \pi^0$

## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

Unification of gauge couplings in Model-II with  $G_{224} \times S_4$  intermediate symmetry where  $g_{2L} \neq g_{2R}$



## UNIFICATION WITH SINGLE-STEP BREAKING CNTD...

Intermediate  $G_{224} \times S_4$  symmetry (Model II) or  $G_{224D} \times S_4$  (Model III(a))

		Model II		Model III(a)	
$\mathcal{M}$	$ \eta $	$\frac{M_U}{M_U^0}$	$\frac{\tau_p}{\tau_p^0}$	$\frac{M_U}{M_U^0}$	$\frac{\tau_p}{\tau_p^0}$
Degen	0.693	$10^{\pm 0.0335}$	$10^{\pm 0.0672}$	$10^{\pm 0.0457}$	$10^{\pm 0.184}$
	1.0	$10^{\pm 0.0424}$	$10^{\pm 0.097}$	$10^{\pm 0.066}$	$10^{\pm 0.266}$
	2.302	$10^{\pm 0.976}$	$10^{\pm 0.223}$	$10^{\pm 0.1519}$	$10^{\pm 0.6125}$
	3.312	$10^{\pm 0.1404}$	$10^{\pm 0.321}$	$10^{\pm 0.218}$	$10^{\pm 0.881}$
Non-degen	0.693	$10^{\pm 0.2086}$	$10^{\pm 0.410}$	$10^{\pm 0.139}$	$10^{\pm 0.556}$
	1.0	$10^{\pm 0.301}$	$10^{\pm 0.592}$	$10^{\pm 0.2004}$	$10^{0.8016}$
	2.302	$10^{\pm 0.6929}$	$10^{\pm 1.362}$	$10^{\pm 0.4613}$	$10^{\pm 1.845}$
	3.312	$10^{\pm 0.997}$	$10^{\pm 1.961}$	$10^{\pm 0.664}$	$10^{\pm 2.655}$

## UNIFICATION WITH $SU(2)_L \times SU(2)_R \times SU(4)_C$ CNTD...

We find that the predicted lifetime is accessible to ongoing searches for proton decay even in the non-degenerate case for superheavy masses few (2-3) times heavier or lighter than the GUT scale. An additional advantage of this model is that the color-octet Higgs scalar with mass  $M_{C(1, 0, 8)} = M_X = 500$  GeV is clearly accessible to LHC and ILC

## 5. PREDICTIONS WITH $G_{224D} \times S_4$ ( $g_{2L} = g_{2R}$ ) INTERMEDIATE SYMMETRY

Pati-Salam intermediate symmetry with D-parity has the additional advantage over the one without D-parity in that in the presence of D-parity theorems on vanishing corrections on  $\sin^2 \theta_W$  and intermediate scale ( $M_R$ ) lead to the stability of  $M_R$

PREDICTIONS WITH  $G_{224D} \times S_4$  ( $g_{2L} = g_{2R}$ ) CNTD...

$G_{224D}$  intermediate scale models with light color-octets

Model III(a)

$$\begin{aligned}M_S &= 10^{2.5} \text{ GeV}, \quad M_X = M_C(1, 0, 8) = 10^{2.7} \text{ GeV} \\M_R^0 &= M_{C'}(1, 1, 15) = 10^{13.5} \text{ GeV}, \quad M_U^0 = 10^{15.26} \text{ GeV} \\ \alpha_G^{-1} &= 36.2\end{aligned}$$

Like Model II,  $A_R \simeq 2.347$  in Model III(a). Since the unification mass  $M_U^0 \sim 10^{-0.4} M_U^{min}$ , the proton lifetime predicted at two-loop level is At two-loop level for this model we obtain

$$\tau_p^0 \simeq 10^{-1.6} \times (\tau_p)_{\text{expt.}}$$

But we note that the deficit in the predicted  $\tau_p^0$  is easily compensated by the GUT- threshold

## PREDICTIONS WITH $G_{224D} \times S_4$ ( $g_{2L} = g_{2R}$ ) CNTD...

We now examine the unification and model predictions when two or three color-octet scalars,  $C(1, 0, 8)$ , transforming as a doublet or triplet under  $S_4$  are in the accessible range of accelerator energies. Our results are summarized below as Model-III(b) and Model-III(c) for  $n_C = 2$  and  $n_C = 3$ , respectively

## PREDICTIONS WITH $G_{224D} \times S_4$ ( $g_{2L} = g_{2R}$ ) CNTD...

- Mass ranges

- $M_X - M_R$

$$n_C = 2, \quad a_i = \left( \frac{23}{5}, \frac{-7}{3}, -5 \right)$$

$$n_C = 3, \quad a_i = \left( \frac{23}{5}, \frac{-7}{3}, -4 \right)$$

- $M_R - M_U$

$$n_C = 2, \quad a_i = \left( \frac{16}{3}, \frac{16}{3}, -2 \right)$$

$$n_C = 3, \quad a_i = \left( \frac{16}{3}, \frac{16}{3}, \frac{-2}{3} \right)$$

## PREDICTIONS WITH $G_{224D} \times S_4$ ( $g_{2L} = g_{2R}$ ) CNTD...

- At one-loop level unification solutions give the following masses and the GUT couplings
  - Model-III(b)

$$M_S^0 = 10^{2.5} \text{ GeV}, \quad M_X^0 = M_C^0 = 10^{2.7} \text{ GeV}$$
$$M_R^0 = 10^{13.5} \text{ GeV}, \quad M_U^0 = 10^{17} \text{ GeV}$$
$$\alpha_G^{-1} = 32.7$$

- Model-III(c)

$$M_S^0 = 10^{2.5} \text{ GeV}, \quad M_X^0 = M_C^0 = 10^{2.7} \text{ GeV}$$
$$M_R^0 = 10^{13.5} \text{ GeV}, \quad M_U^0 = 10^{19.6} \text{ GeV}$$
$$\alpha_G^{-1} = 27.6$$

## 6. UNIFICATION WITH $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times S_4$ INTERMEDIATE SYMMETRY

We explore the possibility of flavor unification through  $SO(10) \times S_4$  with left-right intermediate gauge symmetry  $SU(2)_L \times SU(2)_R \times U(1)_R \times SU(3)_C \times S_4 (= G_{2213D} \times S_4)$  with D-parity ( $g_{2L} = g_{2R}$ ) which we designated as Model V. As in the case of Model II and Model III(a)-III(c), in this case also we find that an additional scalar  $C(1, 1, 0, 8)$  under  $G_{2213}$  is needed beyond the minimal Higgs representations of the model

## UNIFICATION WITH $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ CNTD...

$$\mu = M_R - M_U:$$

The Higgs content of the model under  $G_{2213}$  are  $6\Phi(2, 2, 0, 1) + \Delta_L(3, 1, 0, 1) + \Delta_R(1, 3, -1, 1) + C(1, 1, 0, 8)$ . The RG coefficients are

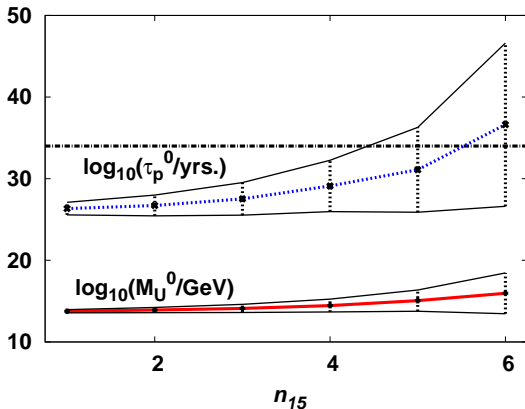
$$a''_{2L} = a''_{2R} = -2/3, \quad a''_{BL} = 7, \quad a''_{3C} = -6,$$

$$b_{ij} = \left\{ \frac{145}{3}, 18, \frac{315}{8}, 24 \right\}, \left\{ 18, \frac{145}{3}, \frac{315}{8}, 0 \right\},$$

$$\left\{ \frac{81}{2}, \frac{81}{2}, \frac{115}{2}, 4 \right\}, \left\{ \frac{27}{2}, \frac{27}{2}, \frac{9}{2}, \frac{-146}{9} \right\}$$

## UNIFICATION WITH $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ CNTD...

Unification of gauge couplings of Model V with left-right symmetric  $G_{2213D} \times S_4 (g_{2L} = g_{2R})$  intermediate symmetry



$$M_X = 10^{2.5} \text{ GeV}, M_R = 10^{13} \text{ GeV}, M_U = 10^{14.16} \text{ GeV}, \alpha_G^{-1} = 39.4$$

## 7. UNIFICATION WITH LIGHT FERMIONS AND DARK MATTER

- These models need additional fine tunings to keep the additional scalars light. On the other hand if the unification is achieved with light fermions instead of light scalars, the model may not need the corresponding fine tuning
- Some of these fermions are the well known candidates of dark matter identified so far in GUTs without flavor symmetry
- $2\sigma(3, 0, 1)$  are replaced by a single Weyl fermion  $F_\sigma(3, 0, 1)$  and similarly,  $2C(1, 0, 8)$  can be replaced by a fermion  $F_C(1, 0, 8)$

## 8. SUMMARY AND CONCLUSIONS

- In this work we have attempted to implement unification of gauge couplings with low-scale flavor symmetry  $G_{213} \times S_4$  through the unifying symmetry like non-supersymmetric  $SO(10) \times G_f$ , where  $G_f = S_4, SO(3)_f, SU(3)_f$ .
- Although no unification is possible through the minimal single-step breaking model, additional scalars like  $C(1, 0, 8)$ ,  $\xi(3, 0, 8)$  or  $\sigma(3, 0, 1)$ ,  $C(1, 0, 8) \subset 75$  of  $SU(5)$  or 210 of  $SO(10)$  are needed to achieve unification.
- With intermediate gauge symmetries like  $G_{224} \times S_4$  or  $G_{224D} \times S_4$  we have found that a low-mass color-octet scalar  $C(1, 0, 8)$  under  $G_{213}$  and  $C'(1, 1, 15)$  under  $G_{224}$  at intermediate scale are needed to achieve unification. In these models the predicted proton lifetime for  $p \rightarrow e^+ \pi^0$  are accessible to either ongoing or planned searches, or next-generation experiments.

## SUMMARY AND CONCLUSIONS CNTD. . .

- If a pair (triplet) of  $C(1, 0, 8)$  transforming as  $S_4$ -doublet ( $S_4$ -triplet) is light, proton stability increases beyond the observable limit.
- In the case of  $G_{2213D} \times S_4$  intermediate symmetry we find that trification with  $SU(3)^3 \times G_f$  at  $M_U^0 \simeq 3 \times 10^{14}$  GeV requires the presence of  $C(1, 0, 0, 8)$  at high intermediate scale  $M_R \simeq 10^{13}$  GeV. In this model proton is stable..
- As another interesting aspect of the present investigation we identify one single-step breaking model and another two-step breaking model with only additional light fermions but without any additional light scalars. One such additional fermionic multiplet is the well known dark matter particle  $F_\sigma(3, 0, 1)$  with mass in the energy range accessible to LHC or ILC.

## SUMMARY AND CONCLUSIONS CNTD. . .

- We have thus implemented the unification program through the non-supersymmetric  $SO(10) \times G_f$  (or  $SU(5) \times G_f$ ) model with interesting predictions on proton decay, dark matter having a rich structure of light particles falsifiable by accelerator searches.
- We conclude that the low scale  $G_{213} \times S_4$  model emerges from an underlying theory of unity of three forces of nature and all fermion masses and mixings