Calculation of the Quarkonia decays in Non-relativistic QCD

Ajay Kumar Rai, N. D. DEVLANI

Applied Physics Department, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat, India

J. N. PANDYA and P. C. VINODKUMAR

Applied Physics Department, M S University of Baroda, Gujarat

Department of Physics, S P University, VVNagar, Anand, Gujarat

Decay Rates of Heavy Qurkonia in NRQCD Formalism

- The new role of the heavy flavour studies as the testing ground for the non-perturbative aspects of QCD, demands extension of earlier phenomenological potential model studies on quarkonium masses to their predictions of decay widths with the non-perturbative approaches like NRQCD. The decay rates of the heavy-quarkonium states into photons and pairs of leptons are among the earliest applications of perturbative quantum chromodynamics (QCD) [Appelquist T *et al.* 1975, Barbieri R *et al.* 1976].

- NRQCD consists of a nonrelativistic Schrodinger field theory for the heavy quark and antiquark that is coupled to the usual relativistic field theory for light quarks and gluons.

- NRQCD not only organize calculation of all orders in α_s, but also elaborate systematically the relativistic corrections to the conventional formula.

- Furthermore, it also provides nonperturbative definitions of the long-distance factors in terms of matrix elements of NRQCD, making it possible to evaluate them in the numerical lattice calculations.

Here, we study the di-gamma and di-lepton decay widths based on the NRQCD formalism [Bodwin *et al.* 1995]. It is expected that the NRQCD formalism has all the corrective contributions for the right predictions of the decay rates. NRQCD factorization expressions for the decay widths of quarkonia are given by [Ajay Kumar Rai *et al.* 2008, Bodwin *et al.* 2002]

$$
\begin{aligned}
\Gamma({}^1S_0 \to \gamma\gamma) \;=\; & \frac{F_{\gamma\gamma}({}^1S_0)}{m_Q^2}\left|\left\langle 0|\chi^{\dagger}\psi|{}^1S_0\right\rangle\right|^2 \\
& + \frac{G_{\gamma\gamma}({}^1S_0)}{m_Q^4} Re\left[\left\langle {}^1S_0|\psi^{\dagger}\chi|0\right\rangle \left\langle 0|\chi^{\dagger}\left(-\frac{i}{2}\overleftrightarrow{D}\right)^2 \psi|{}^1S_0\right\rangle\right] \\
& + \frac{H^1_{\gamma\gamma}({}^1S_0)}{m_Q^6}\left\langle {}^1S_0|\psi^{\dagger}\left(-\frac{i}{2}\overleftrightarrow{D}\right)^2 \chi|0\right\rangle \left\langle 0|\chi^{\dagger}\left(-\frac{i}{2}\overleftrightarrow{D}\right)^2 \psi|{}^1S_0\right\rangle \\
& \frac{H^2_{\gamma\gamma}({}^1S_0)}{m_Q^6}\; Re\left[\left\langle {}^1S_0|\psi^{\dagger}\chi|0\right\rangle \left\langle 0|\chi^{\dagger}\left(-\frac{i}{2}\overleftrightarrow{D}\right)^4 \psi|{}^1S_0\right\rangle\right]
\end{aligned}
$$

$$
\begin{aligned}
\Gamma({}^3S_1 \to e^+e^-) \;=\; & \frac{F_{ee}({}^3S_1)}{m_Q^2}\left|\left\langle 0|\chi^{\dagger}\sigma\psi|{}^3S_1\right\rangle\right|^2 \\
& + \frac{G_{ee}({}^3S_1)}{m_Q^4} Re\left[\left\langle {}^3S_1|\psi^{\dagger}\sigma\chi|0\right\rangle \left\langle 0|\chi^{\dagger}\sigma\left(-\frac{i}{2}\overleftrightarrow{D}\right)^2 \psi|{}^3S_1\right\rangle\right] \\
& + \frac{H^1_{ee}({}^1S_0)}{m_Q^6}\left\langle {}^3S_1|\psi^{\dagger}\sigma\left(-\frac{i}{2}\overleftrightarrow{D}\right)^2 \chi|0\right\rangle \left\langle 0|\chi^{\dagger}\sigma\left(-\frac{i}{2}\overleftrightarrow{D}\right)^2 \psi|{}^3S_1\right\rangle \\
& + \frac{H^2_{ee}({}^1S_0)}{m_Q^6}\; Re\left[\left\langle {}^3S_1|\psi^{\dagger}\sigma\chi|0\right\rangle \left\langle 0|\chi^{\dagger}\sigma\left(-\frac{i}{2}\overleftrightarrow{D}\right)^4 \psi|{}^3S_1\right\rangle\right]
\end{aligned}
$$

▶ The short distance coefficients F's and G's of the order of α_s^2 and α_s^3 are given by [Bodwin *et al.* 2002]

$$F_{\gamma\gamma}(^1S_0) = 2\pi Q^4\alpha^2\left[1 + \left(\frac{\pi^2}{4} - 5\right)C_F\frac{\alpha_s}{\pi}\right]$$

$$G_{\gamma\gamma}(^1S_0) = -\frac{8\pi Q^4}{3}\alpha^2$$

$$H^1_{\gamma\gamma}(^1S_0) + H^2_{\gamma\gamma}(^1S_0) = \frac{136\pi}{45}Q^4\alpha^2$$

$$\begin{aligned} F_{ee}(^3S_1) &= \frac{2\pi Q^2\alpha^2}{3}\{1 - 4C_F\frac{\alpha_s(m)}{\pi} \\ &+ \left[-117.46 + 0.82n_f + \frac{140\pi^2}{27}ln\left(\frac{2m}{\mu_A}\right)\right](\frac{\alpha_s}{\pi})^2\} \end{aligned}$$

$$G_{ee}(^3S_1) = -\frac{8\pi Q^2}{9}\alpha^2$$

$$H^1_{ee}(^3S_1) + H^2_{ee}(^3S_1) = \frac{58\pi}{54}Q^2\alpha^2$$

▶ The matrix elements that contributes to the decay rates of the S wave states into $\eta_Q \rightarrow \gamma\gamma$ and $\psi \rightarrow e^+e^-$ through next-to-leading order in v^2, the vacuum-saturation approximation gives [Bodwin *et al.* 1995]

$$\left\langle ^1S_0|\mathcal{O}(^1S_0)|^1S_0\right\rangle = \left|\left\langle 0|\chi^\dagger\psi|^1S_0\right\rangle\right|^2[1 + O(v^4\Gamma)]$$

$$\left\langle ^3S_1|\mathcal{O}(^3S_1)|^3S_1\right\rangle = \left|\left\langle 0|\chi^\dagger\sigma\psi|^3S_1\right\rangle\right|^2[1 + O(v^4\Gamma)]$$

$$\left\langle ^1S_0|\mathcal{P}_1(^1S_0)|^1S_0\right\rangle = Re\left[\left\langle ^1S_0|\psi^\dagger\chi|0\right\rangle\left\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{D})^2\psi|^1S_0\right\rangle\right] + O(v^4\Gamma)$$

- We have computed $\overline{\nabla^2} R_{P/V}$ term as given by [Hafsakhan 1996], Accordingly

$$\nabla^2 R = -\epsilon_B R \frac{M}{2}, \quad as \quad r \longrightarrow 0$$

where ϵ_B is the binding energy and M is the mass of the respective meson state. The binding energy is computed as $\epsilon_B = M - (2m_Q)$.

- In many cases of potential model predictions, it is observed that the radial wave function at the origin are over estimated than the required value for the correct predictions of the decay widths.

- In such cases, it is argued that the decay of $Q\bar{Q}$ occurs not at zero separation, but at some finite $Q - \bar{Q}$ radial separation rather than the arbitrary scaling of the radial wave function at zero separation to estimate the decay rates correctly [Eichten *et al.* 1978].

- In the present computation of the decay rates using the NRQCD formalism we present our results obtained by using the radial wave function and their derivatives at zero separation as well as at a finite radial separation of $r_0 = r_c$, given by [Ajay Kumar Rai *et al.* 2008]

$$r_c = \frac{N_c |e_Q|}{M_{P/V}}$$

- It is related the electromagnetic process similar to the Compton radius and we call it as color Compton radius of the $Q\bar{Q}$ system.

Di-gamma decay widths [$\Gamma(0^{-+} \to \gamma\, \gamma)$] (in keV) of $c\bar{c}$ system ($1S - 3S$ states) in NRQCD formalism.

		With m_Q				With m_Q^{eff}				
State	PI	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$	Others
$1S$	0.5	8.37	2.83	5.44	1.84	5.34	1.56	1.02	3.47	7.2±0.7 [7]
	0.7	10.91	3.80	7.09	2.47	6.96	1.99	1.29	4.52	7.5 – 10[133]
	1.0	14.10	5.32	9.17	3.46	9.04	2.65	1.72	5.88	7.14±0.95[134]
	1.1	15.02	5.79	9.76	3.77	9.65	2.86	1.86	6.27	5.5[132]
	1.3	16.64	6.76	10.82	4.39	10.74	3.29	2.14	6.98	7.46[60]
	1.5	18.02	7.62	11.71	4.95	11.69	3.67	2.39	7.60	7.18[67]
$2S$	0.5	3.92	0.46	2.55	0.30	1.95	0.33	0.21	1.27	1.3 ± 0.6 [159]
	0.7	7.48	0.65	4.86	0.42	3.07	0.44	0.29	2.00	3.5 – 4.5[133]
	1.0	17.27	1.01	11.22	0.66	5.32	0.59	0.38	3.46	4.44±0.48[134]
	1.1	22.18	1.16	14.42	0.76	6.22	0.63	0.41	4.05	1.8[132]
	1.3	35.21	1.53	22.89	0.99	8.28	0.71	0.46	5.38	1.71[67]
	1.5	53.27	1.98	34.63	1.29	10.66	0.79	0.51	6.93	
$3S$	0.5	2.59	0.22	1.68	0.15	1.17	0.15	0.10	0.76	1.21[67]
	0.7	6.04	0.33	3.93	0.22	1.99	0.21	0.14	1.29	
	1.0	20.31	0.53	13.20	0.35	3.86	0.30	0.19	2.51	
	1.1	29.70	0.61	19.30	0.40	4.71	0.33	0.21	3.06	
	1.3	60.64	0.81	39.41	0.52	6.81	0.38	0.25	4.43	
	1.5	115.97	1.07	75.38	0.69	9.47	0.43	0.28	6.15	

Di-gamma decay widths [$\Gamma(0^{-+} \rightarrow \gamma\ \gamma)$] (in keV) of $c\bar{c}$ system ($4S - 6S$ states) in NRQCD formalism.

		With m_Q				With m_Q^{eff}			
State	PI	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$
$4S$	0.5	1.96	0.13	1.27	0.09	0.84	0.08	0.05	0.55
	0.7	5.24	0.22	3.41	0.14	1.48	0.12	0.08	0.96
	1.0	23.35	0.36	15.18	0.24	3.07	0.18	0.12	1.99
	1.1	37.67	0.42	24.48	0.27	3.83	0.20	0.13	2.49
	1.3	92.81	0.55	60.32	0.36	5.81	0.24	0.16	3.77
	1.5	210.84	0.74	137.04	0.48	8.44	0.27	0.18	5.49
$5S$	0.5	1.59	0.09	1.03	0.06	0.66	0.05	0.03	0.43
	0.7	4.71	0.16	3.06	0.10	1.18	0.08	0.05	0.77
	1.0	26.40	0.28	17.16	0.18	2.55	0.13	0.08	1.66
	1.1	46.07	0.32	29.95	0.21	3.24	0.14	0.09	2.11
	1.3	131.76	0.43	85.64	0.28	5.07	0.17	0.11	3.30
	1.5	342.58	0.57	222.66	0.37	7.59	0.19	0.12	4.93
$6S$	0.5	1.34	0.07	0.87	0.04	0.54	0.04	0.02	0.35
	0.7	4.34	0.12	2.82	0.08	0.98	0.06	0.04	0.64
	1.0	29.44	0.22	19.14	0.14	2.19	0.09	0.06	1.42
	1.1	54.89	0.26	35.68	0.17	2.81	0.10	0.07	1.83
	1.3	177.54	0.35	115.40	0.23	4.50	0.12	0.08	2.93
	1.5	515.77	0.47	335.23	0.31	6.88	0.14	0.09	4.47

Di-gamma decay widths $[\Gamma(0^{-+} \rightarrow \gamma\,\gamma)]$ (in keV) of $b\bar{b}$ system ($1S - 3S$ states) in NRQCD formalism.

		With m_Q				With m_Q^{eff}				
State	PI	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$	Others
$1S$	0.5	0.943	0.180	0.717	0.137	0.757	0.161	0.122	0.575	0.56[133]
	0.7	1.300	0.221	0.988	0.168	1.039	0.196	0.149	0.790	0.384±0.047[134]
	1.0	1.792	0.273	1.362	0.208	1.430	0.241	0.183	1.087	0.35[132]
	1.1	1.941	0.289	1.476	0.219	1.549	0.254	0.193	1.178	0.444[60]
	1.3	2.217	0.316	1.686	0.240	1.769	0.278	0.211	1.345	0.23[67]
	1.5	2.465	0.341	1.874	0.259	1.967	0.299	0.227	1.495	
$2S$	0.5	0.368	0.042	0.280	0.032	0.262	0.035	0.027	0.199	0.269[133]
	0.7	0.763	0.059	0.580	0.045	0.497	0.049	0.037	0.378	0.191±0.025[134]
	1.0	1.883	0.085	1.431	0.065	1.084	0.069	0.052	0.824	0.15[132]
	1.1	2.446	0.093	1.859	0.071	1.354	0.075	0.057	1.029	0.07[67]
	1.3	3.927	0.110	2.985	0.084	2.020	0.087	0.066	1.536	
	1.5	5.960	0.126	4.531	0.096	2.864	0.099	0.075	2.177	
$3S$	0.5	0.225	0.018	0.171	0.014	0.153	0.015	0.011	0.117	0.208[133]
	0.7	0.567	0.029	0.431	0.022	0.332	0.023	0.017	0.252	0.040[67]
	1.0	1.933	0.047	1.469	0.036	0.895	0.035	0.027	0.680	0.1[132]
	1.1	2.792	0.054	2.122	0.041	1.198	0.039	0.030	0.910	
	1.3	5.492	0.067	4.175	0.051	2.033	0.048	0.036	1.546	
	1.5	10.067	0.080	7.652	0.061	3.237	0.055	0.042	2.461	

Di-gamma decay widths [$\Gamma(0^{-+} \rightarrow \gamma\,\gamma)$] (in keV) of $b\bar{b}$ ($4S-6S$ states) system in NRQCD formalism.

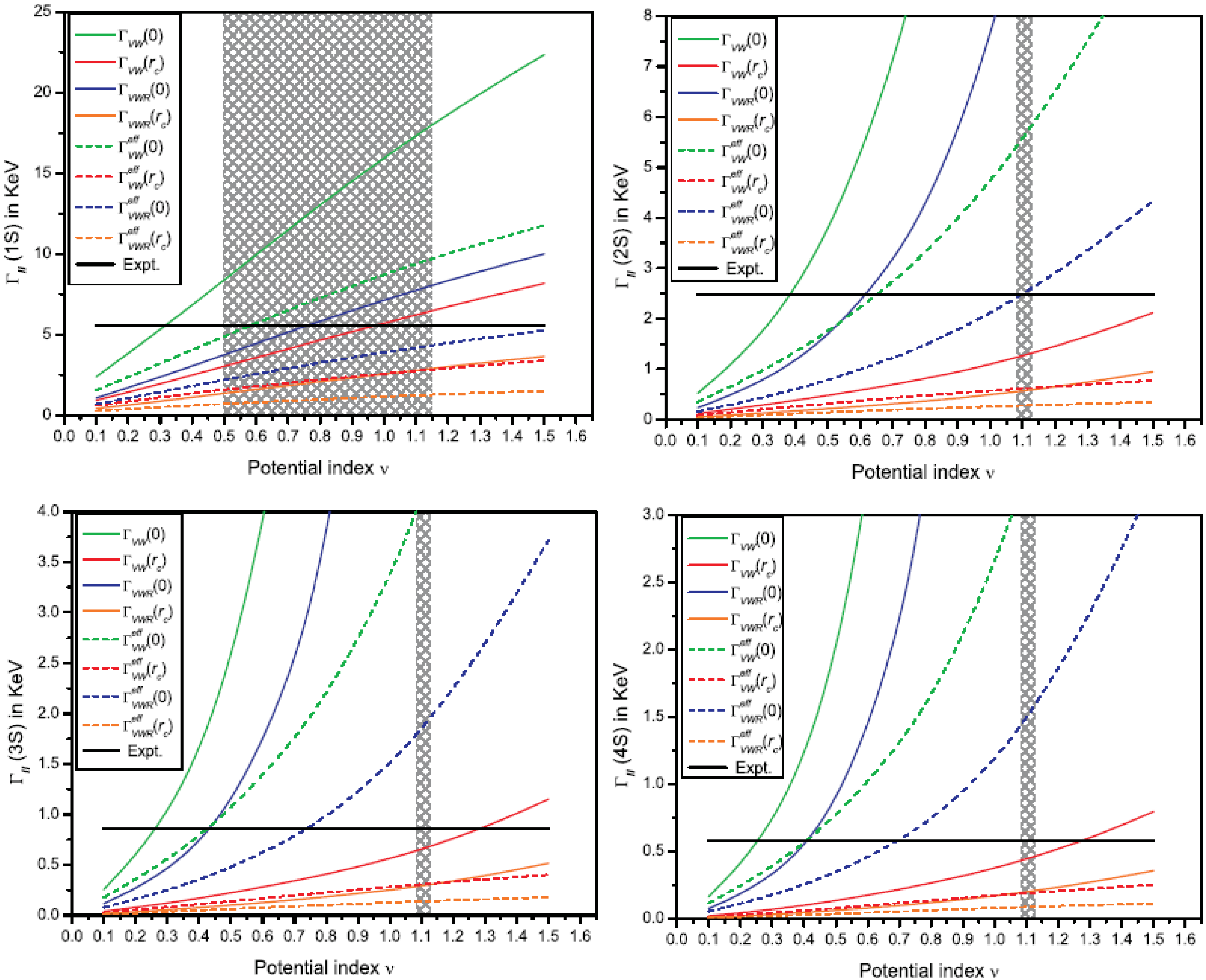
		With m_Q				With m_Q^{eff}			
State	PI	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$	$\Gamma_0(0)$	$\Gamma_0(r_c)$	$\Gamma_{0R}(0)$	$\Gamma_{0R}(r_c)$
$4S$	0.5	0.162	0.010	0.123	0.008	0.108	0.008	0.006	0.082
	0.7	0.464	0.018	0.353	0.014	0.252	0.013	0.010	0.191
	1.0	1.997	0.032	1.518	0.024	0.778	0.022	0.017	0.591
	1.1	3.118	0.037	2.370	0.028	1.088	0.025	0.019	0.827
	1.3	7.130	0.048	5.420	0.037	2.005	0.031	0.024	1.524
	1.5	15.042	0.060	11.434	0.046	3.431	0.037	0.028	2.608
$5S$	0.5	0.128	0.007	0.097	0.005	0.083	0.005	0.004	0.063
	0.7	0.400	0.012	0.304	0.009	0.204	0.009	0.007	0.155
	1.0	2.068	0.023	1.572	0.018	0.696	0.015	0.012	0.529
	1.1	3.438	0.028	2.614	0.021	1.005	0.018	0.013	0.764
	1.3	8.876	0.038	6.747	0.029	1.963	0.022	0.017	1.492
	1.5	20.973	0.048	15.943	0.037	3.530	0.027	0.020	2.683
$6S$	0.5	0.105	0.005	0.080	0.004	0.068	0.004	0.003	0.052
	0.7	0.356	0.009	0.271	0.007	0.173	0.006	0.005	0.131
	1.0	2.143	0.018	1.629	0.014	0.634	0.011	0.009	0.482
	1.1	3.757	0.022	2.856	0.017	0.938	0.013	0.010	0.713
	1.3	10.742	0.031	8.165	0.023	1.915	0.017	0.013	1.456
	1.5	27.931	0.040	21.232	0.031	3.573	0.021	0.016	2.716

Di-lepton decay width $[\Gamma(1^{--} \to \ell^+ \ell^-)]$ (in keV) of $c\bar{c}$ ($1S-3S$ states) system in NRQCD formalism.

		With m_Q				With m_Q^{eff}				
State	PI	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	Others
$1S$	0.5	8.38	3.05	3.75	1.36	4.89	1.60	2.19	0.71	5.55±0.14 [7]
	0.7	11.50	4.09	5.14	1.83	6.50	1.99	2.91	0.89	5.47[55]
	1.0	15.95	5.71	7.13	2.55	8.71	2.57	3.90	1.15	2.94[129]
	1.1	17.34	6.22	7.75	2.78	9.39	2.74	4.20	1.23	
	1.3	19.96	7.24	8.92	3.24	10.63	3.09	4.76	1.38	
	1.5	22.34	8.16	9.99	3.65	11.76	3.38	5.26	1.51	
$2S$	0.5	3.55	0.47	1.59	0.21	1.72	0.32	0.77	0.14	2.48±0.06 [7]
	0.7	6.94	0.69	3.10	0.31	2.71	0.43	1.21	0.19	2.14[55]
	1.0	16.95	1.10	7.58	0.49	4.72	0.57	2.11	0.26	1.22[129]
	1.1	22.25	1.26	9.95	0.56	5.55	0.62	2.48	0.28	
	1.3	36.96	1.65	16.53	0.74	7.45	0.70	3.33	0.31	
	1.5	58.53	2.12	26.17	0.95	9.69	0.77	4.33	0.35	
$3S$	0.5	2.30	0.22	1.03	0.10	1.04	0.14	0.46	0.06	0.86±0.07 [7]
	0.7	5.42	0.34	2.42	0.15	1.74	0.20	0.78	0.09	0.796[55]
	1.0	18.97	0.56	8.48	0.25	3.36	0.28	1.50	0.13	0.76[129]
	1.1	28.27	0.65	12.64	0.29	4.10	0.31	1.83	0.14	
	1.3	60.10	0.87	26.88	0.39	5.94	0.36	2.66	0.16	
	1.5	119.90	1.15	53.62	0.51	8.31	0.40	3.72	0.18	

Di-lepton decay width [$\Gamma(1^{--} \to \ell^+ \ell^-)$] (in keV) of $c\bar{c}$ ($4S - 6S$ states) system in NRQCD formalism.

		With m_Q				With m_Q^{eff}				
State	PI	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	Others
$4S$	0.5	1.73	0.13	0.77	0.06	0.74	0.08	0.33	0.03	0.58±0.07 [7]
	0.7	4.62	0.21	2.07	0.10	1.29	0.11	0.58	0.05	0.288[55]
	1.0	21.22	0.37	9.49	0.17	2.64	0.17	1.18	0.08	0.43[129]
	1.1	34.80	0.44	15.56	0.20	3.29	0.19	1.47	0.08	
	1.3	88.94	0.59	39.77	0.26	4.99	0.22	2.23	0.10	
	1.5	210.04	0.79	93.92	0.36	7.28	0.25	3.26	0.11	
$5S$	0.5	1.40	0.09	0.62	0.04	0.58	0.05	0.26	0.02	
	0.7	4.11	0.15	1.84	0.07	1.03	0.08	0.46	0.03	0.27[129]
	1.0	23.56	0.28	10.53	0.12	2.19	0.11	0.98	0.05	
	1.1	41.73	0.33	18.66	0.15	2.77	0.13	1.24	0.06	
	1.3	123.42	0.45	55.19	0.20	4.31	0.15	1.93	0.07	
	1.5	332.65	0.61	148.75	0.27	6.47	0.17	2.89	0.08	
$6S$	0.5	1.18	0.06	0.53	0.03	0.48	0.03	0.21	0.02	
	0.7	3.76	0.11	1.68	0.05	0.86	0.05	0.38	0.02	
	1.0	25.94	0.22	11.60	0.10	1.87	0.08	0.84	0.04	
	1.1	49.02	0.27	21.92	0.12	2.39	0.09	1.07	0.04	
	1.3	163.58	0.37	73.15	0.16	3.80	0.11	1.70	0.05	
	1.5	491.43	0.50	219.75	0.22	5.81	0.12	2.60	0.06	



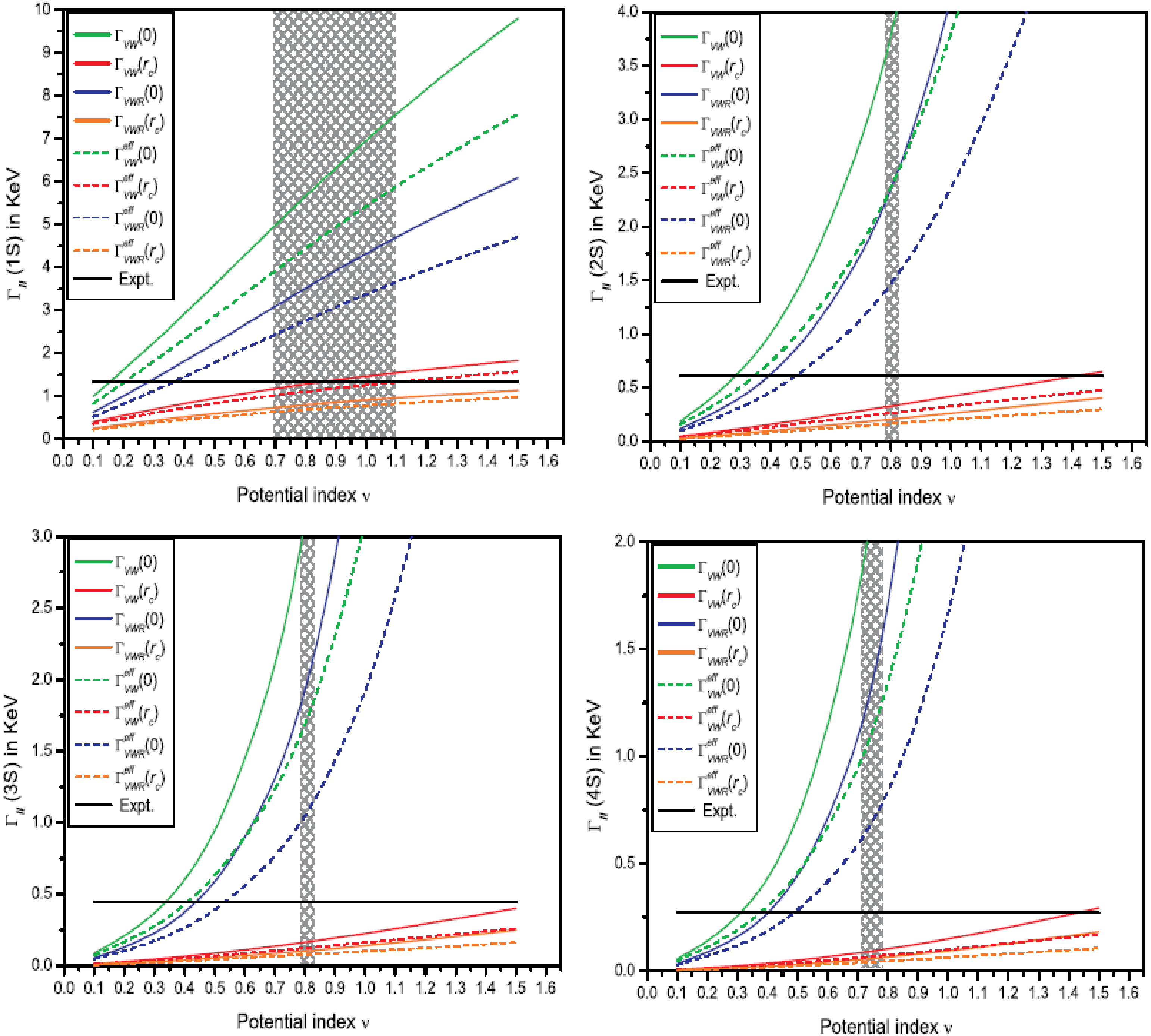
: The di-lepton decay widths of $c\bar{c}$ system with potential index ν in NRQCD formalism. The horizontal lines are the respective experimental values.

Di-lepton decay width $[\Gamma(1^{--} \to \ell^+ \ell^-)]$ (in keV) of $b\bar{b}$ ($1S-3S$ states) system in NRQCD formalism.

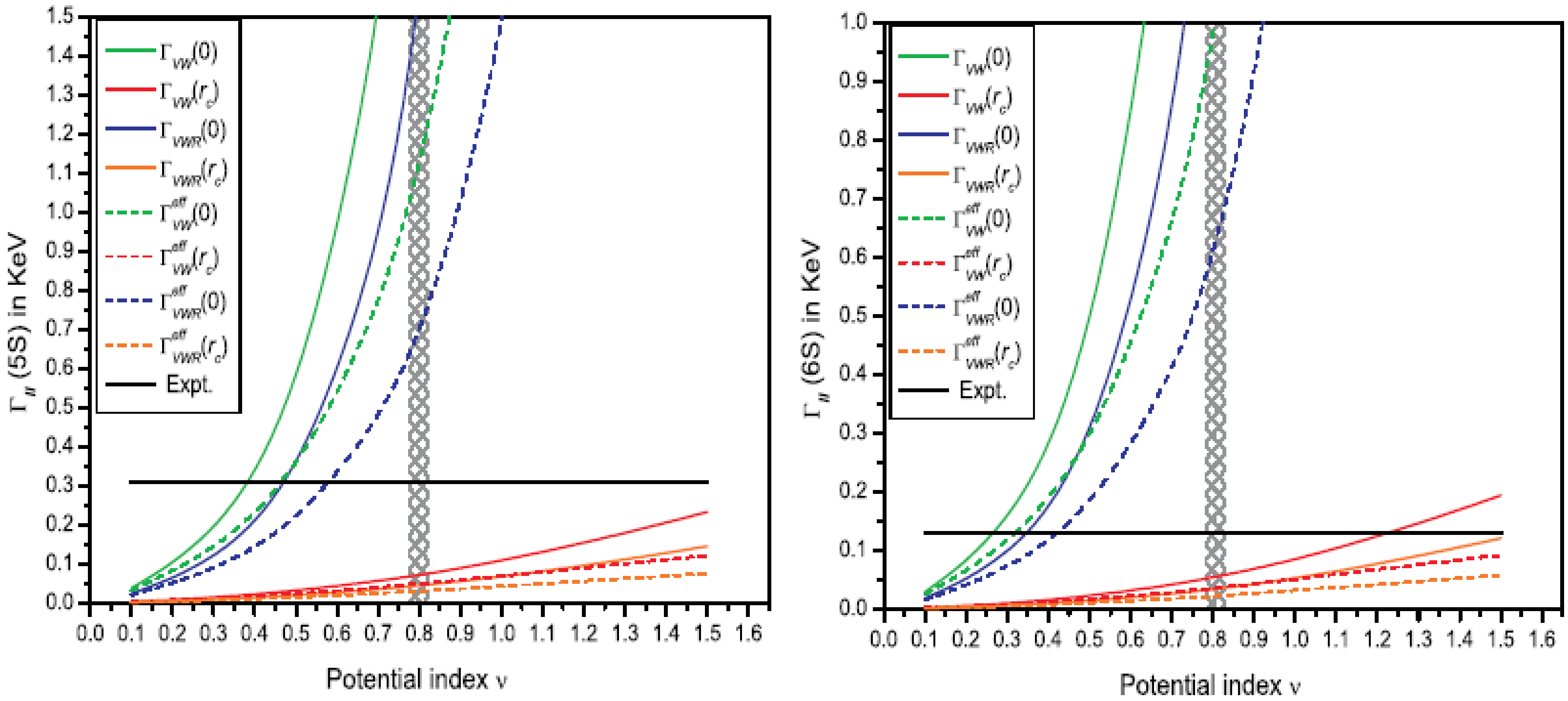
		With m_Q				With m_Q^{eff}				
State	PI	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	Others
$1S$	0.5	3.59	0.96	2.23	0.59	2.86	0.83	1.78	0.52	1.34±0.018 [7]
	0.7	4.98	1.18	3.10	0.73	3.93	1.02	2.44	0.63	1.4[132]
	1.0	6.96	1.46	4.33	0.91	5.43	1.26	3.38	0.78	1.320[55]
	1.1	7.58	1.54	4.71	0.96	5.90	1.33	3.67	0.82	0.98[129]
	1.3	8.74	1.69	5.43	1.05	6.78	1.45	4.21	0.90	
	1.5	9.80	1.82	6.09	1.13	7.57	1.56	4.70	0.97	
$2S$	0.5	1.37	0.20	0.85	0.12	0.99	0.16	0.61	0.10	0.612±0.011 [7]
	0.7	2.74	0.28	1.70	0.18	1.80	0.23	1.12	0.14	0.6[132]
	1.0	6.60	0.42	4.10	0.26	3.79	0.33	2.35	0.20	0.628[55]
	1.1	8.55	0.46	5.31	0.29	4.70	0.36	2.92	0.22	0.41[129]
	1.3	13.72	0.56	8.52	0.35	6.95	0.42	4.31	0.26	
	1.5	20.90	0.65	12.98	0.40	9.80	0.48	6.09	0.30	
$3S$	0.5	0.84	0.08	0.52	0.05	0.58	0.07	0.36	0.04	0.443±0.008 [7]
	0.7	2.01	0.13	1.25	0.08	1.20	0.10	0.74	0.06	0.263[55]
	1.0	6.60	0.23	4.10	0.14	3.07	0.16	1.91	0.10	0.27[129]
	1.1	9.48	0.26	5.89	0.16	4.07	0.18	2.53	0.11	
	1.3	18.62	0.33	11.57	0.20	6.81	0.22	4.23	0.14	
	1.5	34.30	0.40	21.31	0.25	10.77	0.26	6.69	0.16	

Di-lepton decay width [$\Gamma(1^{--} \rightarrow \ell^+ \ell^-)$] (in keV) of $b\bar{b}$ ($4S - 6S$ states) system in NRQCD formalism.

		With m_Q				With m_Q^{eff}				
State	PI	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	$\Gamma_{VW}(0)$	$\Gamma_{VW}(r_c)$	$\Gamma_{VWR}(0)$	$\Gamma_{VWR}(r_c)$	Others
$4S$	0.5	0.61	0.05	0.38	0.03	0.42	0.04	0.26	0.02	0.272±0.029 [7]
	0.7	1.64	0.08	1.02	0.05	0.91	0.06	0.57	0.04	0.104[55]
	1.0	6.72	0.15	4.18	0.09	2.65	0.10	1.64	0.06	0.20[129]
	1.1	10.44	0.17	6.49	0.11	3.66	0.11	2.27	0.07	
	1.3	23.85	0.23	14.81	0.14	6.63	0.14	4.12	0.09	
	1.5	50.61	0.29	31.44	0.18	11.27	0.17	7.00	0.11	
$5S$	0.5	0.48	0.03	0.30	0.02	0.32	0.02	0.20	0.01	0.31±0.071 [7]
	0.7	1.41	0.06	0.88	0.03	0.74	0.04	0.46	0.02	0.0404[55]
	1.0	6.90	0.11	4.29	0.07	2.35	0.07	1.46	0.04	0.16[129]
	1.1	11.41	0.13	7.09	0.08	3.35	0.08	2.08	0.05	
	1.3	29.45	0.18	18.29	0.11	6.45	0.10	4.00	0.06	
	1.5	70.07	0.23	43.53	0.14	11.50	0.12	7.15	0.08	
$6S$	0.5	0.40	0.02	0.25	0.01	0.27	0.02	0.17	0.01	0.13±0.03 [7]
	0.7	1.25	0.04	0.78	0.03	0.63	0.03	0.39	0.02	0.12[129]
	1.0	7.11	0.08	4.42	0.05	2.14	0.05	1.33	0.03	
	1.1	12.39	0.10	7.70	0.06	3.12	0.06	1.94	0.04	
	1.3	35.45	0.15	22.02	0.09	6.26	0.08	3.89	0.05	
	1.5	92.90	0.19	57.71	0.12	11.58	0.09	7.20	0.06	



$\Gamma_{VW}(0)$
$\Gamma_{VW}(r_c)$
$\Gamma_{VWR}(0)$
$\Gamma_{VWR}(r_c)$
$\Gamma^{eff}_{VW}(0)$
$\Gamma^{eff}_{VW}(r_c)$
$\Gamma^{eff}_{VWR}(0)$
$\Gamma^{eff}_{VWR}(r_c)$
Expt.
Γ_{ll} (1S) in KeV
Γ_{ll} (2S) in KeV
Γ_{ll} (3S) in KeV
Γ_{ll} (4S) in KeV
Potential index ν



The di-lepton decay widths of $b\bar{b}$ system with potential index ν in NRQCD formalism. The horizontal lines are the respective experimental values.

Quarkonia decay into Light Hadrons

The annihilation rate of the heavy quarkonium state (η_c, η_b, J/ψ and Υ) into light hadrons (G. T. Bodwin et al 1995)

$$\Gamma(^1S_0 \to LH) = \frac{N_c}{\pi}\frac{Im\ f_1(^1S_0)}{m_Q^2}\left|\overline{R_{\eta_c}}\right|^2 - \frac{N_c}{\pi}\frac{Im\ g_1(^1S_0)}{m^4}Re\left(\overline{R_s}\overline{\nabla^2 R_s}\right) + O(v^4\Gamma)$$

$$\Gamma(^3S_1 \to LH) = \frac{N_c}{\pi}\frac{Im\ f_1(^3S_1)}{m_Q^2}\left|\overline{R_{\psi}}\right|^2 - \frac{N_c}{\pi}\frac{Im\ g_1(^3S_1)}{m^4}Re\left(\overline{R_s}\overline{\nabla^2 R_s}\right) + O(v^4\Gamma)$$

The short distance coefficients

$$Im\ f_1(^1S_0) = \frac{\pi C_F}{2N_c}\alpha_s^2$$

$$Im\ g_1(^1S_0) = -\frac{2\pi C_F}{3N_c}\alpha_s^2$$

$$Im\ f_1(^3S_1) = \frac{(\pi^2-9)(Nc^2-4)C_F}{54Nc)}\alpha_s^2\left[1+(-9.46\ C_F+4.13\ C_A-1.161\ n_f)\alpha_s/\pi\right] + \pi Q^2\left(\sum_i Q_i^2\right)\alpha^2\left(1-\frac{13}{4}C_F\frac{\alpha_s}{\pi}\right)$$

The imaginary part of the $Im\ g_1(^3S_1)$ vanish at order α_s

The quarkonia decay into light hadrons (relative order v^4)

$$\begin{aligned}
\Gamma({}^1S_0 \to LH) &= \frac{F_1({}^1S_0)}{m_Q^2}\left\langle {}^1S_0|O_1({}^1S_0)|{}^1S_0\right\rangle + \frac{G_1({}^1S_0)}{m_Q^4}\left\langle {}^1S_0|P_1({}^1S_0)|{}^1S_0\right\rangle \\
&+\frac{F_8({}^3S_1)}{m_Q^2}\left\langle {}^1S_0|O_8({}^3S_1)|{}^1S_0\right\rangle + \frac{F_8({}^1S_0)}{m_Q^2}\left\langle {}^1S_0|O_8({}^1S_0)|{}^1S_0\right\rangle \\
&+\frac{F_8({}^1P_1)}{m_Q^4}\left\langle {}^1S_0|O_8({}^1P_1)|{}^1S_0\right\rangle + \frac{H_1^1({}^1S_0)}{m_Q^6}\left\langle {}^1S_0|Q_1^1({}^1S_0)|{}^1S_0\right\rangle \\
&+\frac{H_1^2({}^1S_0)}{m_Q^6}\left\langle {}^1S_0|Q_1^2({}^1S_0)|{}^1S_0\right\rangle
\end{aligned}$$

$$\begin{aligned}
\Gamma({}^3S_1 \to LH) &= \frac{F_1({}^3S_1)}{m_Q^2}\left\langle {}^3S_1|O_1({}^3S_1)|{}^3S_1\right\rangle + \frac{G_1({}^3S_1)}{m_Q^4}\left\langle {}^3S_1|P_1({}^3S_1)|{}^3S_1\right\rangle \\
&+\frac{F_8({}^1S_0)}{m_Q^2}\left\langle {}^1S_0|O_8({}^1S_0)|{}^1S_0\right\rangle + \frac{F_8({}^3S_1)}{m_Q^2}\left\langle {}^1S_0|O_8({}^3S_1)|{}^1S_0\right\rangle \\
&+\sum_{j=0,1,2}\frac{F_8({}^3P_J)}{m_Q^4}\left\langle {}^3S_1|O_8({}^3P_J)|{}^3S_1\right\rangle + \frac{H_1^1({}^3S_1)}{m_Q^6}\left\langle {}^3S_1|Q_1^1({}^3S_1)|{}^3S_1\right\rangle \\
&+\frac{H_1^2({}^3S_1)}{m_Q^6}\left\langle {}^3S_1|Q_1^2({}^3S_1)|{}^3S_1\right\rangle
\end{aligned}$$

G.T. Bodwin et al PRD, 66 (2002)

The short distance coefficients

$$F_1(^1S_0) \;=\; \frac{\pi\ C_F}{Nc}\alpha_s^2\left[1+(\frac{\pi^2}{4}-5)C_F+(\frac{199}{18}-\frac{13\pi^2}{24})\ C_A-\frac{8}{9}\ n_f)\frac{\alpha_s}{\pi}\right]$$

Where $Nc = 3$ is the no. of colour, $C_F = 4/3$

$$G_1(^1S_0) = -\frac{4\pi C_F}{3N_c}\alpha_s^2$$

$$\begin{aligned} F_8(^3S_1) \;&=\; \frac{\pi n_f}{3}\alpha_s^2\left[1+\frac{\alpha_s}{\pi}\left(-\frac{13}{4}\right)C_F+\left(\frac{133}{18}-\frac{2}{3}log2-\frac{\pi^2}{4}\right)\ C_A-\frac{10}{9}\ n_fT_F+2b_oLog\frac{\mu}{2m_Q}\right] \\ &+5\alpha_s^3\left(-\frac{73}{4}+\frac{67}{36}\pi^2)\right) \end{aligned}$$

$$F_8(^1S_0) \;=\; 2\pi B_F\alpha_s^2\left[1+\frac{\alpha_s}{\pi}\left(-5+\frac{\pi^2}{4}\right)C_F+\left(\frac{122}{9}-\frac{17}{24}\pi^2\right)\ C_A-\frac{16}{9}\ n_fT_F+2b_oLog\frac{\mu}{2m_Q}\right]$$

$$F_8(^1P_1) = \frac{\pi Nc}{6}\alpha_s^2$$

$$H_1^1(^1S_0) + H_1^2(^1S_0) = \frac{68\pi C_F}{45N_c}\alpha_s^2$$

$$H_1^1(^3S_1) + H_1^2(^3S_1) = \frac{(N_c^2-1)(N_c^2-4)}{N_c^2}\left(-\frac{833}{972} + \frac{(1609\pi^2)}{12960} + \frac{7}{81}log\frac{2m}{\mu_\Lambda}\right)\alpha_s^3$$

$\eta_c \rightarrow Light\ Hadrons$(in MeV)

	Expansion up to v^2		*Expansion up to* v^4		
ν	Γ_{NRQCD}	Γ_{wcr}	Γ_{NRQCD}	Γ_{wcr}	Γ_{Others}
0.5	11.34	5.82	77.01	41.93	
0.7	15.61	6.71	99.60	45.65	14.38
0.9	19.91	9.15	120.48	58.20	±
1.0	21.96	9.89	129.78	61.33	1.07
1.1	23.95	10.06	138.94	64.46	±
1.3	27.79	11.91	155.90	69.77	1.43
1.5	31.40	13.09	171.33	74.43	N. Faboano
1.7	34.76	14.07	185.22	78.48	(2002)
1.9	37.97	15.16	198.16	82.09	
2.0	39.47	15.61	204.31	83.77	

$\eta_b \rightarrow Light\ Hadrons$(in MeV)

	Expansion up to v^2		*Expansion up to* v^4		
ν	Γ_{NRQCD}	Γ_{wcr}	Γ_{NRQCD}	Γ_{wcr}	Γ_{Others}
0.5	6.30	4.13	21.57	14.13	
0.7	7.72	4.91	26.42	16.79	
0.9	8.67	5.62	29.62	19.22	
1.0	9.71	5.95	33.16	20.33	
1.1	10.34	6.27	35.32	21.42	
1.3	11.52	6.73	39.32	22.98	
1.5	12.58	7.38	42.90	25.16	
1.7	13.57	7.85	46.27	26.77	
1.9	12.21	7.18	45.31	26.65	
2.0	12.65	7.39	46.86	27.36	

$J/\psi \rightarrow Light\ Hadrons$(in MeV)

ν	$Expansion\ \ up\ \ to\ v^2$		$Expansion\ \ up\ \ to\ v^4$		
	Γ_{NRQCD}	Γ_{wcr}	Γ_{NRQCD}	Γ_{wcr}	Γ_{Others}
0.5	0.61	0.48	21.66	18.70	
0.7	0.82	0.54	26.57	20.60	
0.9	1.01	0.76	31.32	26.02	
1.0	1.11	0.82	33.39	27.52	
1.1	1.20	0.88	35.58	29.12	
1.3	1.37	0.99	39.63	32.00	
1.5	1.54	1.09	43.40	34.64	
1.7	1.69	1.18	46.83	37.01	
1.9	1.83	1.26	50.50	39.49	
2.0	1.90	1.31	52.21	40.65	

$\Upsilon \rightarrow Light\ Hadrons$(in MeV)

ν	$Expansion\ \ up\ \ to\ v^2$		$Expansion\ \ up\ \ to\ v^4$		
	Γ_{NRQCD}	Γ_{wcr}	Γ_{NRQCD}	Γ_{wcr}	Γ_{Others}
0.5	0.12	0.12	1.02	0.67	
0.7	0.23	0.14	1.25	0.79	
0.9	0.27	0.16	1.46	0.91	
1.0	0.28	0.17	1.56	0.96	
1.1	0.30	0.18	1.66	1.02	
1.3	0.34	0.19	1.85	1.09	
1.5	0.37	0.22	2.01	1.19	
1.7	0.39	0.23	2.16	1.26	
1.9	0.42	0.24	2.31	1.33	
2.0	0.44	0.25	2.38	1.36	

Summary

- We study the heavy flavour hadrons containing one or more heavy flavour quarks with minimum number of free parameters such as, the potential strength A of the mesonic system for each choices of ν and the quark mass parameters. The masses ($1S-6S$, low lying $P-$waves, $D-$waves and $F-$waves of $c\bar{c}$, $b\bar{b}$ and $b\bar{c}$ systems).
- The relative mean square velocities of bound quarks, the mean square radii of the meson states and the decay constants $f_{P/V}$
- The di-lepton, di-gamma and di-gluon widths of different states and E1 and M1 transitions rates.
- In the heavy-light flavour sector, the present study consists of the masses of few low-lying states of $Q\bar{q}$ systems (D, D_s, B, B_s), their relative mean square velocities of the bound quarks, mean square radii, the decay constants $f_{P/V}$, the inclusive semi-leptonic and leptonic branching ratios.
- The neutral flavour oscillations of $B^0-\bar{B}^0$ and $B_s^0-\bar{B}_s^0$ mesons.
- We also study in the heavy flavour baryon sector consisting the ground state masses, mean square radii, hyperfine mass splitting and the magnetic moments of single heavy flavour (Qqq), double heavy flavour (QQq) and triple heavy flavour (QQQ) systems based on hypercentral model using the variational approach.

List of Publications

1. Bhavin Patel, Ajay Kumar Rai and P C Vinodkumar: Masses and Magnetic Moments of Charmed Baryons Using Hypercental Model, Frascati Physics Series Vol XLVI(2007), 1431-1438; [arXiv:hep-ph/0803.0221].

2. P C Vinodkumar, Ajay Kumar Rai, Bhavin Patel and Jignesh Pandya : Open Flavour Charmed Mesons, Frascati Physics Series Vol XLVI(2007), 929-936.

3. Bhavin Patel, Ajay K. Rai and P C Vinodkumar: Masses and Magnetic Moments of Heavy Flavour Baryons using Hypercentral Model, J Phys. G : Nucl. Part. Phys. **35** (2008) 065001.

4. Bhavin Patel, Ajay K. Rai and P C Vinodkumar: Masses and Magnetic Moments of Charmed Baryons using Hypercentral Model, J Phys. G : Conference Series. **110**, 122010 (2008);[arXiv:hep-ph/0710.3828].

5. Ajay Majethiya, Bhavin Patel and P C Vinodkumar: Single Heavy Flavour Baryons using Coulomb plus Power law interquark Potential, Eur. Phys. J. **A38**, 307 (2008); [arXiv:hep-ph/0805.3439].

6. Ajay Kumar Rai, Bhavin Patel and P C Vinodkumar: Properties of $Q\bar{Q}$ mesons in NRQCD formalism, Phy. Rev. **C78**, 055202 (2008); [arXiv:hep-ph/0810.1832].

7. Ajay Kumar Rai, Bhavin Patel, J N Pandya and P C Vinodkumar: Decay Properties of $Q\bar{Q}$ Mesons in Potential Models and Effective Field Theories, Proceeding of Science (Accepted for Publication) (2008).

8. Bhavin Patel and P C Vinodkumar: Properties of $Q\bar{Q}$ ($Q \epsilon b, c$) mesons in Coulomb plus Power potential, J Phys. G : Nucl. Part. Phys. **36**, 035003 (2009); [arXiv:hep-ph/0808.2888].

9. Bhavin Patel, Ajay K. Rai and P C Vinodkumar: Heavy Flavour Baryons using Hypercentral Model, Pramana - J. Phys. **70** (2008) 797.

10. Bhavin Patel, Ajay Majethiya and P C Vinodkumar: Masses and Magnetic Moments of Triply Heavy Flavour Baryons using Hypercentral Model, Pramana - J. Phys. **72**, 679 (2009); [arXiv:hep-ph/0808.2880].

11. Ajay Majethiya, Bhavin Patel and P C Vinodkumar: Binding energy and masses of QQq baryons in analogy with H_2^{++} molecule Prajna Jnl. Pure and Appl. Scie. **14**(2008)121.

12. Bhavin Patel and P C Vinodkumar: Mixing and lifetime of B-mesons in Coulomb plus Power potential, (Communicated for publication).