# Magic Baseline and Magic Energy in Neutrino Oscillation with non-standard interactions

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# INTRODUCTION

- **Three types of neutrino:** $\nu_e, \nu_\mu, \nu_\tau$
- **Relation between flavour and mass eigenstates of neutrinos:**

$$|\nu_\alpha>= \sum_i U_{\alpha i}|\nu_i>$$

$$U = R_{23}R_{13}(\delta)R_{12}$$

CP violating phase

- **The Present experimental values:**

$$\Delta m^2_{21} = 7.10^{-5}eV^2$$
$$\Delta m^2_{31} = 2.5.10^{-3}eV^2$$

- **The neutrino oscillation probability :**

$$P(\nu_l \to \nu_m) = \delta_{lm} - 4\sum_{i>j} Re[J^{lm}_{ij}]\sin^2\Delta^{'}_{ij} + 2\sum_{i>j} Im[J^{lm}_{ij}]\sin 2\Delta^{'}_{ij}$$

where

$$J^{lm}_{ij} = U^{'}_{li}U^{'*}_{lj}U^{'*}_{mi}U^{'}_{mj}$$

$$\Delta^{'}_{ij} = \Delta^{'} m^2_{ij}L/(4E)$$

$$\Delta^{'} m^2_{ij} = m^{'2}_i - m^{'2}_j$$

$U^{'}$ **is the modified mixing matrix in presence of matter**

## Neutrino Interactions with matter in SM:

- **Interactions involving muon neutrinos and tau neutrinos is done through z exchange.**
- **Interactions involving electron neutrinos is done through both z and w exchange.**
- **Different forward scattering amplitudes.**
- **Leads to matter induced oscillation effect.**

**To find the matter effect we solve the evolution equation:**

$$i\frac{d}{dx}\nu = (\frac{1}{2E}UM_1^2U^\dagger + V)\nu$$

**where**

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$M_1^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

$$V = \sqrt{2}G_F n_e$$

$G_F$ **is the Fermi constant**

$n_e$ **is the electron number density**

# Neutrino Interaction with R parity violating Supersymmetry

- **R parity violating supersymmetry is an example of non standard interactions.**

- **R parity is defined as:**

$$R = (-1)^{3B+L+2S}$$

.

Where
**B** is the baryon number
**L** is the lepton number
**S** is the spin

- **The quark neutrino interaction Lagrangian:**

$$L_{\lambda'} = \lambda'_{ijk}\left[\tilde{d}^i_L \tilde{d}^k_R \nu^i_L + (\tilde{d}^k_R)^*(\bar{\nu}^i_L)^c(d^i_L)\right] + \text{h.c.}$$



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- **Such non-standard interactions have been considered for neutrinos interacting with matter in the context of neutrino oscillation in long baseline. (Adhikari, Agarwalla, Raychaudhuri, Physics Letter B 642(2006)111-118)**

- **The element of a matrix involving R parity violating supersymmetry:**

$$R_{ij}(\lambda') = \sum_m \left( \frac{\lambda'_{im1}\lambda'_{jm1} n_d}{4m^2(\tilde{d}_m)} + \frac{\lambda'_{i1m}\lambda'_{j1m} n_d}{4m^2(\tilde{d}_m)} \right)$$

- **This model has flavour changing and flavour diagonal neutral currents**

- **Thus effecting the probability of oscillation.**

- **Solve the evolution equation considering such models.**

# Conditions on baseline and on neutrino energy

**Let** $x = \Delta'_{31}$, $y = \Delta'_{32}$, $z = \Delta'_{12}$

**Trigonometric Identities:**

$$-\sin^2 x + \sin^2 y - \sin^2 z = 2\sin x \ \cos y \ \sin z$$

$$-\sin 2x + \sin 2y - \sin 2z = -4\sin x \ \sin y \ \sin z$$

x=y-z,
y=x+z,
z=y-x

$$\sin z = 0$$

$$P_{\nu_e \to \nu_\mu} = -4\left(Re\left[U'_{13}U'^{*}_{11}U'^{*}_{23}U'_{21}\right] + Re\left[U'_{13}U'^{*}_{12}U'^{*}_{23}U'_{22}\right]\right)\sin^2\Delta'_{31}$$
$$-2\left(Im\left[U'_{13}U'^{*}_{11}U'^{*}_{23}U'_{21}\right] + Im\left[U'_{13}U'^{*}_{12}U'^{*}_{23}U'_{22}\right]\right)\sin 2\Delta'_{31}$$

$$|\Delta'_{21}| = \pm n\pi$$

Baseline

$$\sin x = 0$$

$$P_{\nu_e \to \nu_\mu} = -4\left(Re\left[U'_{12}U'^{*}_{11}U'^{*}_{22}U'_{21}\right] + Re\left[U'_{13}U'^{*}_{12}U'^{*}_{23}U'_{22}\right]\right)\sin^2\Delta'_{12}$$
$$-2\left(Im\left[U'_{12}U'^{*}_{11}U'^{*}_{22}U'_{21}\right] - Im\left[U'_{13}U'^{*}_{12}U'^{*}_{23}U'_{22}\right]\right)\sin 2\Delta'_{12}$$

$$|\Delta'_{31}| = \pm n\pi$$

Energy

# Oscillation Probability considering $\theta_{13}$ to be small

- **To find the neutrino oscillation probability we need to solve the schrodinger equation:**

$$i\frac{d}{dt}|\nu(t)>= H|\nu(t)>$$

$10^{-2}$

where

$$\alpha = \Delta m^2_{21}/\Delta m^2_{31}$$

$$H \approx \frac{\Delta m^2_{31}}{2E} R_{23} M R^\dagger_{23}$$

Here

$$M = R_{13}R_{12}\frac{\Delta m^2_{31} diag(0,\alpha,1)}{\Delta m^2_{31}} R^\dagger_{12}R^\dagger_{13} + diag(A,0,0) + R^\dagger_{23}\begin{pmatrix} 0 & X & Y \\ X & B & C \\ Y & C & D \end{pmatrix} R_{23}$$

$$X = \frac{2E\epsilon_{12}}{\Delta m^2_{31}}, Y = \frac{2E\epsilon_{13}}{\Delta m^2_{31}}, B = \frac{2E\epsilon_{22}}{\Delta m^2_{31}}, C = \frac{2E\epsilon_{23}}{\Delta m^2_{31}}, D = \frac{2E\epsilon_{33}}{\Delta m^2_{31}}, A = \frac{2E\sqrt{2}G_F n_e}{\Delta m^2_{31}}$$

$\epsilon_{12}$ , $\epsilon_{13}$, $\epsilon_{22}$, $\epsilon_{23}$ and $\epsilon_{33}$ are the non standard interaction terms.

- **M can be written as** $M^{(0)} + M^{(1)} + M^{(2)}$

$M^{(0)}$**,** $M^{(1)}$ **and** $M^{(2)}$ **consists of terms which are zero order first order and second order in the expansion parameter** $\alpha$

- **Using pertubative way of calculation eigenvalues and eigenvectors are calculated upto second order in α.**

- **Mixing matrix** $U^{'} = R_{23}.W$ **; W is the normalized eigenvectors.**

- **The Probability of oscillation** $P(\nu_e \to \nu_\mu)$ **considering NSI terms:**

$$P = \frac{4c_{23}^2}{A^2}|b|^2 \sin^2(\frac{\Delta m_{31}^2 AL}{4E}) + \frac{4s_{23}^2}{(1-A)^2}|a|^2 \sin^2(\frac{\Delta m_{31}^2 (1-A)L}{4E})$$
$$+\frac{8c_{23}s_{23}}{A(1-A)} Re\,[a^* b] \sin(\frac{\Delta m_{31}^2 AL}{4E}) \sin(\frac{\Delta m_{31}(1-A)L}{4E}) \cos(\frac{\Delta m_{31} L}{4E})$$
$$+\frac{8c_{23}s_{23}}{A(1-A)} Im\,[a^* b] \sin(\frac{\Delta m_{31}^2 AL}{4E}) \sin(\frac{\Delta m_{31}(1-A)L}{4E}) \sin(\frac{\Delta m_{31} L}{4E})$$

(T.Kikuchi,H.Minakata,S.Uchinami,JHEP 0903:114,2009)

where

$$a = c_{23}Y + e^{-i\delta}c_{13}s_{13} + Xs_{23}$$
$$b = c_{23}X + c_{12}c_{13}\alpha s_{12} - Ys_{23}$$

- **Putting NSI terms equal to zero reduces the probability expression to that for SM**

(P.Huber,W.Winter,Phys.Rev.D68:037301,2003)

## Magic baseline and magic energy for small $\theta_{13}$

- **The magic baseline condition :**

$$L = \frac{2\pi}{\sqrt{2}G_F n_e}$$

(P.Huber,W.Winter,Phys.Rev.D68:037301,2003)

- **The Probability expression after using the baseline condition:**

$$P(\nu_e \to \nu_\mu) \approx 4\frac{s_{23}^2}{(1-A)^2}|a|^2 \sin^2 \frac{\Delta m_{31}^2 (1-A)L}{4E}$$

- **Magic energy condition:**

$$E = \Delta m_{31}^2 / \left( \pm 4n\pi/L + 2\sqrt{2}G_F n_e \right)$$

- **Here energy stands for average energy for neutrinos.**
- **The probability expression after the energy condition:**

$$P(\nu_e \to \nu_\mu) \approx \frac{4c_{23}^2}{A^2}|c_{23}X + c_{12}c_{13}\alpha s_{12} - Y s_{23}|^2 \sin^2 \left( \frac{\Delta m_{31}^2 AL}{4E} \right)$$

# Perturbative results for large $\theta_{13}$

$$M^{(0)} = \frac{\Delta m_{31}^2}{2E}\begin{pmatrix} A + s_{13}^2 & 0 & e^{-i\delta}s_{13}c_{13} \\ 0 & 0 & 0 \\ e^{i\delta}s_{13}c_{13} & 0 & c_{13}^2 \end{pmatrix}$$

$$M^{(1)} = \frac{\Delta m_{31}^2}{2E}\begin{pmatrix} \alpha s_{12}^2 & b & a_1 \\ b & \alpha c_{12}^2 & -e^{-i\delta}\alpha c_{12}s_{12}s_{13} \\ a_1^* & -e^{i\delta}\alpha c_{12}s_{12}s_{13} & 0 \end{pmatrix}$$

$$M^{(2)} = \frac{\Delta m_{31}^2}{2E}\begin{pmatrix} \alpha s_{13}^2 s_{12}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha s_{13}^2 s_{12}^2 \end{pmatrix}$$

where

$$a_1 = c_{23}Y - e^{-i\delta}c_{13}s_{13}\alpha s_{12}^2 + X s_{23}$$

- Using perturbative way of calculation eigenvalues and eigenvectors are calculated

- **Total eigenvalues:**

$$\frac{m_1'^{\,2}}{2E} \approx \frac{\Delta m_{31}^2}{2E}\left[\frac{1}{2}(A+1-x)+\alpha s_{12}^2-\alpha s_{13}^2 s_{12}^2+\frac{2|b|^2}{A+1-x}-\frac{|a|^2}{x}\right]$$

$$\frac{m_2'^{\,2}}{2E} \approx \frac{\Delta m_{31}^2}{2E}\left[\alpha c_{12}^2-\frac{2|b|^2}{A+1-x}-\frac{2(\alpha c_{12}s_{12}s_{13})^2}{A+1+x}\right]$$

$$\frac{m_3'^{\,2}}{2E} \approx \frac{\Delta m_{31}^2}{2E}\left[\frac{1}{2}(A+1+x)+\alpha s_{13}^2 s_{12}^2+\frac{|a^*|^2}{x}+\frac{2(\alpha c_{12}s_{12}s_{13})^2}{A+1+x}\right]$$

where

$$x=(1+A^2-2A\cos 2\theta_{13})^{1/2}$$

- **Modified magic baseline for large $\theta_{13}$ :**

$$L = \frac{8En\pi}{\Delta m^2_{31}(A+1-(1+A^2-2A\cos 2\theta_{13})^{1/2})}$$

**•The probability expression :**

$$P(\nu_e \to \nu_\mu) \approx -4Re[Z]\sin^2\frac{\Delta m^2_{31}Lx}{8E} - 2Im[Z]\sin 2\frac{\Delta m^2_{31}Lx}{8E}$$

**Probability is not independent of $\delta$**

where

$$\begin{aligned} Z &= \frac{s^2_{23}}{(1-\xi^2)}\left[-\xi^2k^2 - \frac{a_1\xi^2k^2}{x} + \frac{a_1^*\xi^3k^2}{x} - \frac{|c_{23}Y + s_{23}X|^2\xi^4k^2}{x^2} + \frac{|c_{23}Y + s_{23}X|^2\xi^2}{x^2}\right] \\ &+ \frac{c_{23}s_{23}}{(1-\xi^2)}\left[-\frac{4(c_{23}\beta^* - s_{23}\gamma^*)\xi k}{(A+1+x)}\right] \end{aligned}$$

$$\beta = C\, c_{23} + B\, s_{23}$$

$$\gamma = D\, c_{23} + C\, s_{23}$$

$$k = 1/[1 + (-A + \cos 2\theta_{13} + x)^2\, \mathrm{cosec}^2 2\theta_{13}]^{1/2}$$

$$\xi = (-A + \cos 2\theta_{13} + x)\mathrm{cosec}2\theta_{13}$$

- **Magic energy for large $\theta_{13}$ :**

$$E = \Delta m_{31}^2 S \cos 2\theta_{13} L^2 \left(-1 \pm \sqrt{1 + Q/R}\right) / (2Q)$$

where

$$Q = (2n\pi)^2 - S^2 L^2, R = S^2 \cos^2 2\theta_{13} L^4, S = \sqrt{2} G_F n_e$$

- **The probability expression after using the energy condition:**

$$P(\nu_e \rightarrow \nu_\mu) \approx \frac{16|b|^2 \xi^2 c_{23}^2}{(1+\xi^2)(A+1-x)^2} \sin^2\left(\frac{\Delta m_{31}^2 L(A+1-x)}{8E}\right)$$

- **Energy conditions gives $\delta$ independent probability**

- **Magic energy can be used to measure NSI coupling, $\theta_{13}$ and to resolve heirarchy problem**

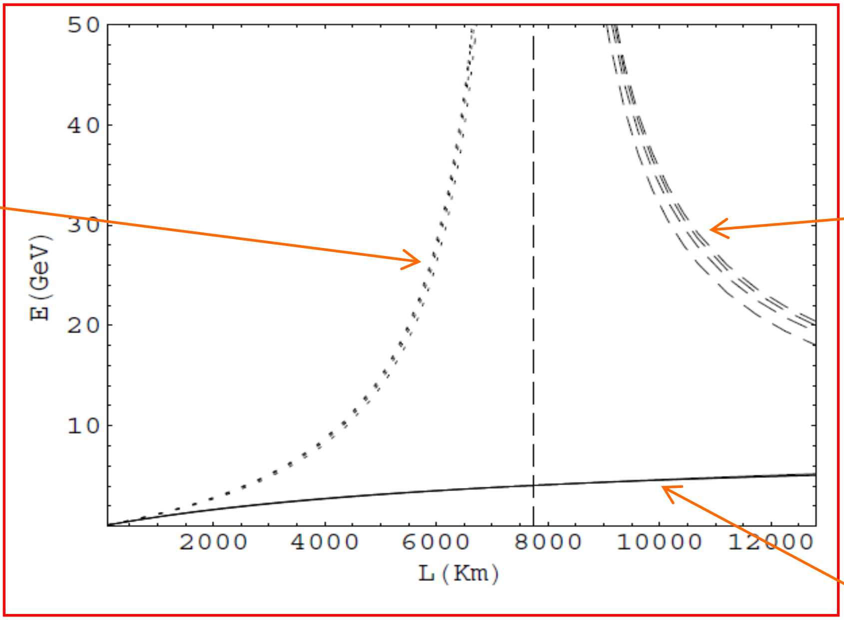


**Fig1: Neutrino energy vs length of baseline.**

- $\theta_{13} = 1, 5, 8, 12$
- **For normal heirarchy and –ve sign,** $L > \frac{\sqrt{2}n\pi}{G_F n_e}$
- **For normal and +ve sign, no bound in L**
- **For inverted and –ve sign,** $L < \frac{\sqrt{2}n\pi}{G_F n_e}$
- **Condition is not much sensitive to variation with** $\theta_{13}$

# Statistical Analysis

- **Sensitivity of unknown oscillation parameters like $\theta_{13}$ $and$ $\Delta m^2_{31}$**
- **Source: monoenergetic neutrino beam**
- **Baselines: (a) 650Km from CERN to megaton water Cerenkov detector in Canfranc**
  **(b) 7152 Km from CERN to 50Kton iron detector at INO in India**

  (S. K. Agarwalla,S. Choubey and A. Raychaudhuri, Nucl.Phys.B771,1-27(2007), S. Choubey et al,JHEP 0912:020(2009))
- **Flux = $10^{18}$ neutrinos per year**

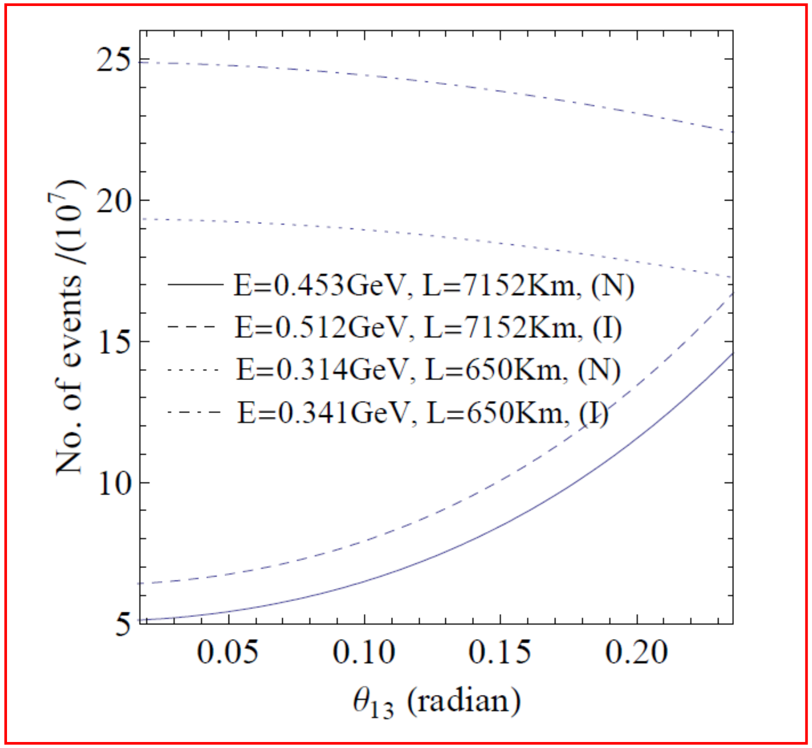


**Fig 2: Number of muon events vs $\theta_{13}$**

- **Number of events is large as neutrino energy is monochromatic instead of Gaussian distribution**

# PRECISION MEASUREMENT:

- **Precision in the measurement of** $cos^2\theta_{13}$

$$\chi^2_{total} = \left(\frac{N^{expt} - N^{th}}{\Delta N}\right)^2 + \left(\frac{|\Delta m^2_{31}| - |\Delta m^2_{31}(true)|}{\sigma\left(\Delta m^2_{31}\right)}\right)^2 + \left(\frac{\sin^2 2\theta_{23} - \sin^2 2\theta_{23}(true)}{\sigma\left(\sin^2 2\theta_{23}\right)}\right)^2$$

$$\Delta N = \Delta N_{pert} + \Delta N_{\alpha^2}$$

Gaussian Distribution

$$\Delta m^2_{31}(true) = 2.5.10^{-3} eV^2$$

$$\theta_{23}(true) = 45^\circ$$

$$\sigma(\Delta m^2_{31}) = 1.5\%$$

$$\sigma(\sin^2 2\theta_{23}) = 1.0\%$$

$$Precision = \frac{\cos^2\theta_{13}(min) - \cos^2\theta_{13}(max)}{\cos^2\theta_{13}(min) + \cos^2\theta_{13}(max)} 100\%$$

(S. K. Agarwalla,S. Choubey and A. Raychaudhuri, Nucl.Phys.B771,1-27(2007)

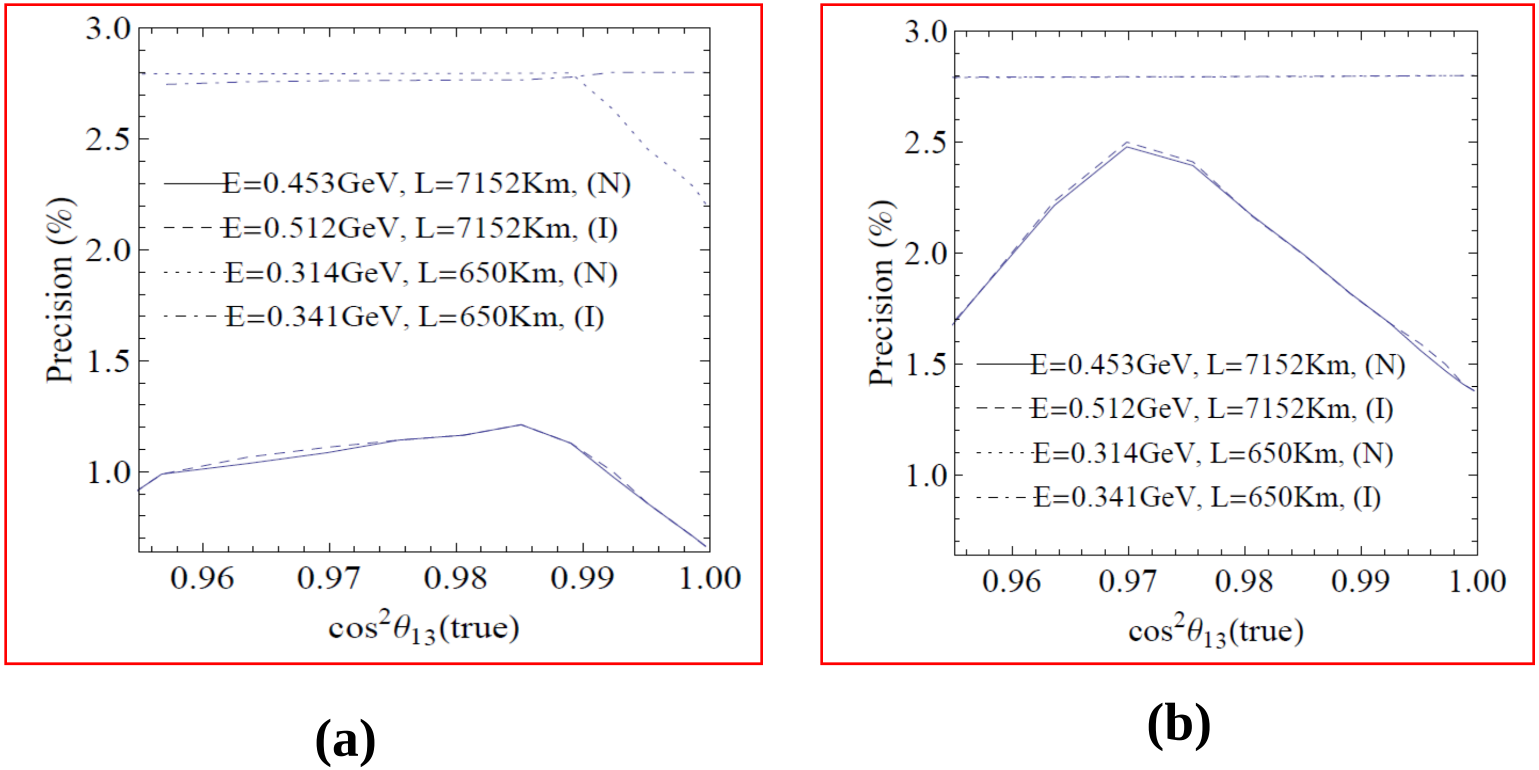


**Fig 3: Precision in the measurements of** $(a) cos^2\theta_{13}(1\sigma)$ **and** $(b) cos^2\theta_{13}(3\sigma)$

- **Precision do not vary much with heirarchy for both baselines**
- **For 650 km precision becomes better for higher values of** $cos^2\theta_{13} at\ 1(\sigma)$
- **For 7152 Km initially precision worsens and then becomes better with the increase of** $cos^2\theta_{13}$

## Conclusions

- **Magic baseline condition exists only for small $\theta_{13}$ and for NSI in 23 block**
- **Using magic energy the probability expression becomes independent of δ.**
- **Magic energy can be used to measure the NSI couplings and $\theta_{13}$**
- **To fix a particular energy seems to be easier than fixing a particular baseline.**
- **For any baseline we can find out the magic energy and find the oscillation probability where there is no δ dependence.**
- **The number of muon events are larger for neutrino beams with monochromatic energy in comparison to the energy with Gaussian distribution.**
- **Precision for $\theta_{13}$ is better in case of magic energy than that for magic baseline.**

**Thank You**