# An Origin of Quark-Lepton Complementarity in SO(10) Model with Flavour Symmetry

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**Reference: Phys. Lett. B (2010), [arXiv:1008.5061]**

# INTRODUCTION

**Quark-Lepton Complementarity**

- Present knowledge of flavor mixing parameters

**Flavor Mixing Patterns of Quark and Lepton sector**

| Quark Mixing Angles | Lepton Mixing Angles |
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| $\theta^q_{12} \equiv \theta_{us} \approx 13^o$ | $\theta^l_{12} = 34.5^o \binom{+3.2}{-2.8}$ |
| $\theta^q_{23} \equiv \theta_{cb} \approx 2.4^o$ | $\theta^l_{23} = 42.3^o \binom{+11.4}{-7.1}$ |
| $\theta^q_{13} \equiv \theta_{ub} \approx 0.2^o$ | $\theta^l_{13} \leq 13^o$ |

- There exists an interesting empirical relation...

$$\theta^l_{12} + \theta_{us} \sim \frac{\pi}{4}$$

- Similar relation can be written for 23 mixing angle

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**QLC from Quark-Lepton Unification: A Qualitative Picture**

[Minakata & Smirnov (2004); Raidal (2004)]

- Assume that the structure of fermion mass matrices at high scale are such that

$$V_{CKM} \equiv U_u^\dagger U_d = I \qquad \& \qquad V_{PMNS} \equiv U_e^\dagger U_\nu = U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Then both the mixing matrices get corrected by $\mathcal{O}(\theta_C)$ terms coming from the next leading order where the down quark and charged lepton mass matrices are equal.

$$M_d \approx M_e$$

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- Construction of realistic GUT model in which all fermion masses and mixing angles are correctly reproduced along with QLC is highly non-trivial.
- The original proposal was based on $SU(5)$ relation $M_e = M_d^T$ but detailed explanation of the fermionic spectrum was not developed.

[M.Raidal, Phys. Rev. Lett. 93 (2004)]

- Some models proposed to explain QLC are based on a smaller gauge group, namely Pati-Salam $SU(4)_c \times SU(2)_L \times SU(2)_R$ group.

[Frampton & Mohapatra (2005); Antusch, King & Mohapatra (2005); King (2005); Toorop, Bazzocchi, & Merlo (2010)]

- A complete and realistic model based on $SO(10)$ GUT has not been proposed so far.

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**SO(10) Models for Fermion Masses**

- $SO(10)$ unifies quarks and leptons at high scale and hence provides common and attractive platform to study the dissimilarities between quarks and leptons.
- provide a natural framework for understanding neutrino masses because of the seesaw mechanisms inherent in them.
- Neutrino masses arise in these models from two separate sources either from the vev of left-handed triplet (type-II) or from the right-handed triplet (type-I) Higgs.
- Renormalizable supersymmetric models based on the $SO(10)$ group are quite powerful in constraining the fermion mass structure.
- There also exist a class of models where appropriate flavor symmetry is integrated with SO(10) to construct a predictive theory which can simultaneously explain hierarchical nature of quark masses and mixing angles and large lepton mixing angles.

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# SO(10) model of Fermion Masses

## Fermion Masses in $SO(10)$

- Consider three families of **16**-dimensional fermions obtaining their masses through Yukawa interactions with four Higgs multiplets $\Phi, \Phi', \Phi''$ (**10**) and $\overline{\Sigma}$ ($\overline{\mathbf{126}}$)

$$W_Y = Y_{10}\psi\psi\Phi + Y_{\overline{126}}\psi\psi\overline{\Sigma} + Y_{10'}\psi\psi\Phi' + Y_{10''}\psi\psi\Phi''$$

- Decomposition under Pati-Salam ($SU(4)_{PS} \times SU(2)_L \times SU(2)_R$)

$$\begin{aligned} 16 &= (4,2,1) + (\overline{4},1,2) \\ 10 &= (1,2,2) + (6,1,1) \\ \overline{126} &= (10,1,3) + (\overline{10},3,1) + (15,2,2) + (6,1,1) \end{aligned}$$

- Starting from $SO(10)$, an effective MSSM is obtained by assuming that only two appropriate linear combinations of these Higgs doublets (red colored) remain light and are responsible for Fermion masses.
- VEV of $SU(2)_R(SU(2)_L)$ triplet (blue colored) generates light neutrino masses by type-I(type-II) seesaw mechanism.

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- After EWSB, the mass Lagrangian of the model is

$$-\mathcal{L}_{mass} = \overline{f}_L M_f f_R + \overline{\nu}_L M_D \nu_R + \frac{1}{2}\overline{\nu}_L M_L \nu_L^c + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + h.c.$$

- The final fermion mass relations can suitably written as,

$$\begin{aligned} M_d &= H + \ F + tH' + H'' \quad , \quad M_u = r(H + \ sF + H' + pH''), \\ M_l &= H - 3F + tH' + H'' \quad , \quad M_D = r(H - 3sF + H' + pH''), \\ M_L &= r_L \ F \quad , \quad M_R = r_R^{-1} \ F \end{aligned}$$

The light neutrino mass matrix is given by,

$$\mathcal{M}_\nu = r_L F - r_R M_D F^{-1} M_D^T \equiv \mathcal{M}_\nu^{II} + \mathcal{M}_\nu^{I}$$

- It is possible to have symmetry breaking pattern where **type-II term dominates** over the type-I contributions.

# Ansatz & its Phenomenology

$$M_d = H + F + tH' + H'' \quad , \quad M_u = r(H + sF + H' + pH''),$$
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$$H = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h \end{pmatrix}; \quad F = \begin{pmatrix} b+c & a & a \\ a & b & c \\ a & c & b \end{pmatrix}; \quad H' = \begin{pmatrix} 0 & -a' & a' \\ -a' & 0 & 0 \\ a' & 0 & 0 \end{pmatrix}; \quad H'' = x\ I$$

- In the limit of dominant contribution from $H$ (10-plet Higgs),

$$m_b = m_\tau = \frac{1}{r} m_t; \quad V_{CKM} = I; \quad V_{PMNS} = U_{BM}$$

  Correct $b - \tau$ unification & Bi-maximal lepton mixings are obtained with no mixing between quarks.
- Further, the contributions from other Higgs coupling matrices $F, H'$ and $H''$ make the model realistic.

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## Quark Sector

$$M_d = \begin{pmatrix} b+c+x & a-ta' & a+ta' \\ a-ta' & b+x & c \\ a+ta' & c & h+b+x \end{pmatrix}; \quad M_u = \begin{pmatrix} s(b+c)+x' & sa-a' & sa+a' \\ sa-a' & sb+x' & sc \\ sa+a' & sc & rh+sb+x' \end{pmatrix}$$

strongly hierarchical $h \gg b, c \gg a, a' \gg x, x'$

- Masses of 6 quarks fix 6 real parameters $h, b, x, r, s, x'$.
- It turns out that $\theta^u_{23} \ll \theta^d_{23}$ and $\theta^u_{13} \ll \theta^d_{13}$. Further, one has to assume that $a' \approx sa$ to keep $\theta^u_{12} \ll \theta^d_{12}$. In this limit, the quark mixing matrix takes the form

$$V_{CKM} = U_u^\dagger U_d \approx U_d \approx R_{23}(\theta^d_{23}) R_{13}(\theta^d_{13}) R_{12}(\theta^d_{12})$$

- The CKM elements fix the remaining parameters,

$$c \sim -V_{cb}\ h; \quad a - ta' \sim -V_{us}\ b; \quad a + ta' \sim -V_{ub}\ h$$

- Interesting relationship between $V_{us}$ and $V_{ub}$ is found in the limit $t \sim 0$.

$$V_{ub} \approx V_{us} \frac{m_s}{m_b} + \mathcal{O}\left(\frac{m_s^2}{m_b^2}\right)$$

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## Lepton Sector

$$M_e = \begin{pmatrix} -3(b+c)+x & -3a - ta' & -3a + ta' \\ -3a - ta' & -3b + x & -3c \\ -3a + ta' & -3c & h - 3b + x \end{pmatrix}; \quad M_\nu = \begin{pmatrix} b+c & a & a \\ a & b & c \\ a & c & b \end{pmatrix}$$

**Since all the parameters are fixed from Quark sector, the entire Lepton sector emerges as the prediction**

- It predicts $m_\tau \approx m_b$ and $m_\mu \approx -3m_s$.
- For $b = -c$, $m_e \approx m_d$ which is viable with observed values of $m_e$ and $m_d$ extrapolated at the GUT scale within $3\sigma$ deviations. However for $b \neq -c$, any desired value of $m_d/m_e$ can be obtained.
- For $t \sim 0$, $\theta^e_{12} \approx \theta_C$, $\theta^e_{23} \approx -3\theta_{cb}$ and $\theta^e_{13} \approx -3\theta_{ub}$.
- The ratio of the solar to atmospheric squared mass difference is

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \sqrt{2} V_{us} \left( \frac{m_s/m_b}{V_{cb}} \left(1 + \frac{m_s}{m_b}\right) - 1 \right)$$

$m_s/m_b \sim 1.08\ V_{cb}$ can generate the observed value $\Delta m^2_{sol}/\Delta m^2_{atm} (\sim 0.031)$

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- The leptonic mixing matrix will be

$$V_{PMNS} \equiv U_e^{\dagger} U_{\nu} = U_e^{T} U_{BM}$$

  where $U_e = R_{23}(-3\theta_{cb}) R_{13}(-3\theta_{ub}) R_{12}(\theta_C)$.

- The correction of $\mathcal{O}(\theta_C)$ from charged lepton generates

$$\theta_{12}^{l} \approx \frac{\pi}{4} - \frac{\theta_C}{\sqrt{2}}; \quad \theta_{23}^{l} \approx \frac{\pi}{4} + 3\theta_{cb}; \quad \theta_{13}^{l} \approx \frac{\theta_C}{\sqrt{2}}$$

- In this setup, since the neutrino mass spectrum follows normal hierarchy ($m_1 < m_2 \ll m_3$), the effect of RGE corrections are known to be negligible ($\sim 10^{-5}(1 + \tan^2\beta)$). So the above relations hold at low scale also.

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## An Example Set of Parameters & Fermion Spectrum

### Input Parameters

$h = 1.7$ GeV; $b = 0.0243$ GeV; $c = -0.022113$ GeV; $a = -0.0052$ GeV;
$x = 0.00325$ GeV; $r = 55.88$; $s = -8.64198$; $t = 0$;
$a' = (0.0344247 - 0.028885i)$ GeV; $x' = (0.0233596 - 0.00293374i)$ GeV

### Obtained Fermion Spectrum

$m_t = 94.8$ GeV; $m_c = 0.19$ GeV; $m_u = 0.65$ MeV;
$m_b = 1.73$ GeV; $m_s = 28.5$ MeV; $m_d = 4.21$ MeV;
$m_\tau = 1.63$ GeV; $m_\mu = 75.4$ MeV; $m_e = 0.35$ MeV.

$\sin\theta_{us} = 0.222$; $\sin\theta_{cb} = 0.015$; $\sin\theta_{ub} = 0.005$; $\delta_{CKM} = 60.9^\circ$;
$\sin^2\theta^l_{12} = 0.368$; $\sin^2\theta^l_{23} = 0.527$; $\sin^2\theta^l_{13} = 0.024$; $\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} = 0.030$.

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## A Model from Discrete Flavor Symmetry $S_4 \times Z_n$

| | $\psi$ | $\Phi$ | $\Phi'$ | $\Phi''$ | $\overline{\Sigma}$ | $\chi$ | $\phi$ | $\eta$ | $\sigma$ | $\sigma'$ | $\Psi_{V1}$ | $\overline{\Psi}_{V1}$ | $\Psi_{V2}$ | $\overline{\Psi}_{V2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SO(10) | **16** | **10** | **10** | **10** | $\overline{\mathbf{126}}$ | **1** | **1** | **1** | **1** | **1** | **16** | $\overline{\mathbf{16}}$ | **16** | $\overline{\mathbf{16}}$ |
| $S_4$ | $\mathbf{3_2}$ | $\mathbf{1_1}$ | $\mathbf{1_2}$ | $\mathbf{1_1}$ | $\mathbf{1_1}$ | $\mathbf{3_1}$ | $\mathbf{3_2}$ | $\mathbf{3_1}$ | $\mathbf{1_1}$ | $\mathbf{1_2}$ | $\mathbf{1_1}$ | $\mathbf{1_1}$ | $\mathbf{1_2}$ | $\mathbf{1_2}$ |
| $Z_n$ | 1 | $\omega^{-2m}$ | $\omega^{-(p+q)}$ | $\omega^{-2q}$ | $\omega^{-2k}$ | $\omega^{k}$ | $\omega^{m}$ | $\omega^{p}$ | $\omega^{k}$ | $\omega^{q}$ | $\omega^{m}$ | $\omega^{-m}$ | $\omega^{k}$ | $\omega^{-k}$ |

- The Yukawa superpotential invariant under $SO(10) \times S_4 \times Z_n$ is

$$\begin{aligned} W &= (\phi\psi)\overline{\Psi}_{V1} + \lambda \Psi_{V1}\Psi_{V1}\Phi + M_1 \Psi_{V1}\overline{\Psi}_{V1} \\ &+ (\chi\psi)\overline{\Psi}_{V2} + \lambda' \Psi_{V2}\Psi_{V2}\overline{\Sigma} + M_2 \Psi_{V2}\overline{\Psi}_{V2} \\ &+ \sum_i \frac{\alpha_i}{\Lambda^2}(\chi^2\psi\psi)_i\overline{\Sigma} + \frac{\beta}{\Lambda^2}\sigma(\chi\psi\psi)\overline{\Sigma} + \frac{\gamma}{\Lambda^2}\sigma^2(\psi\psi)\overline{\Sigma} \\ &+ \frac{\alpha'}{\Lambda^2}\sigma'(\eta\psi\psi)\Phi' + \frac{\alpha''}{\Lambda^2}\sigma'^2(\psi\psi)\Phi'' \end{aligned}$$

- The effective superpotential after integrating out heavy vector-like fields is

$$W_{eff} = \frac{\lambda}{M_1^2}(\phi\psi)(\phi\psi)\Phi + \frac{\lambda'}{M_2^2}(\chi\psi)(\chi\psi)\overline{\Sigma} + \sum_i \frac{\alpha_i}{\Lambda^2}(\chi^2\psi\psi)_i\overline{\Sigma} + \ldots\ldots$$

## A Model from Discrete Flavor Symmetry $S_4 \times Z_n$

| | $\psi$ | $\Phi$ | $\Phi'$ | $\Phi''$ | $\overline{\Sigma}$ | $\chi$ | $\phi$ | $\eta$ | $\sigma$ | $\sigma'$ | $\Psi_{V1}$ | $\overline{\Psi}_{V1}$ | $\Psi_{V2}$ | $\overline{\Psi}_{V2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SO(10) | **16** | **10** | **10** | **10** | $\overline{\mathbf{126}}$ | **1** | **1** | **1** | **1** | **1** | **16** | $\overline{\mathbf{16}}$ | **16** | $\overline{\mathbf{16}}$ |
| $S_4$ | $\mathbf{3_2}$ | $\mathbf{1_1}$ | $\mathbf{1_2}$ | $\mathbf{1_1}$ | $\mathbf{1_1}$ | $\mathbf{3_1}$ | $\mathbf{3_2}$ | $\mathbf{3_1}$ | $\mathbf{1_1}$ | $\mathbf{1_2}$ | $\mathbf{1_1}$ | $\mathbf{1_1}$ | $\mathbf{1_2}$ | $\mathbf{1_2}$ |
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# The Model for Ansatz

## A Model from Discrete Flavor Symmetry $S_4 \times Z_n$

- The $SO(10) \times S_4 \times Z_n$ is broken to $SO(10)$ by vevs of the flavon fields.
- The proposed ansatz is correctly obtained by choosing a particular set of vacuum alignments of flavon fields.

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \upsilon_\phi; \quad \langle\chi\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \upsilon_\chi; \quad \langle\eta\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \upsilon_\eta$$

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# SUMMARY

- It is indeed possible to obtain Quark-Lepton Complementarity in realistic $SO(10)$ model with 10 and $\overline{126}$-dimensional Higgs fields that give mass to fermions and dominant type-II seesaw mechanism.
- QLC relation is obtained by proposing specific ansatz that leads to the bimaximal lepton mixing at leading order, which is then corrected by $\mathcal{O}(\theta_C)$ corrections related to quark mixing.
- The ansatz is capable of explaining the entire fermionic spectrum and not only QLC relation.
- It is shown that the proposed ansatz can be derived in SO(10) model with $S_4 \times Z_n$ discrete symmetry, $S_4$ non-singlet flavon fields and two pairs of 16-dimensional vector-like fermions.
- A generic prediction of this approach is $\theta^l_{13} \approx \theta_C/\sqrt{2}$ which can be confirmed or excluded by the current generation of neutrino oscillation experiments.

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