

Fermion Masses and Mixing in Non-supersymmetric SO(10)

Ketan Patel
(with Anjan S. Joshipura)

Physical Research Laboratory,
Ahmedabad - 380 009

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SO(10) Models of Fermion Masses and Mixing

- ▶ $SO(10)$ GUT which unify strong and electroweak interactions also provide constrained and unified description of the fermion masses and mixing angles.
- ▶ All the known SM fermions plus three right-handed neutrinos fit into three copies of 16-dimensional spinorial representation of $SO(10)$.
- ▶ It predicts the existence of right-handed neutrinos and offers an appealing explanation for the smallness of neutrino masses through seesaw mechanisms.
- ▶ In renormalizable theories based on $SO(10)$ group, only three Yukawa coupling matrices Y_{10} , $Y_{\overline{126}}$, Y_{120} and relative strengths between them determine the entire spectrum of fermion masses and mixing angles.
- ▶ Because of the success of SUSY unification, most of the attention in recent years has been focused on supersymmetric $SO(10)$.

Unification with Supersymmetry

- ▶ The minimal version of SUSY $SO(10)$ constructed with $10 + 126 + \overline{126} + 210$ Higgs fields was found to be very predictive and studied in great details. [Aulakh, Bajc, Melfo, Senjanovic, Vissani (2004)]
- ▶ It has been shown recently that there is an inherent tension between
 - ▶ the need for relatively large atmospheric mass-squared difference in the neutrino sector
(indicating $M_{B-L} \sim 10^{13} \text{ GeV}$ quite below $M_{GUT} \sim 10^{16} \text{ GeV}$) and
 - ▶ the requirements of the gauge coupling convergence
(preferring M_{B-L} close to M_{GUT})
- ▶ This issue is absent in the Non-Supersymmetric case because the separation between the GUT scale and intermediate scales is necessary for successful gauge coupling unification.

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INTRODUCTION

Non-Supersymmetric $SO(10)$

- ▶ The recent detailed studies on non-supersymmetric $SO(10)$ show that
[Bertolini, Luzio, Malinsky (2009, 2010)]
 - ▶ Two or three steps breaking of $SO(10)$ to the Standard Model can be achieved through vev of 45_H and $\overline{126}_H$ (or $\overline{16}_H$). The presence of intermediate scale allows the gauge coupling unification.
 - ▶ The required pattern of the Higgs vevs for the gauge symmetry breaking can emerge from the minimization of the 1-loop corrected Higgs potential.
- ▶ This observation opens the option of considering the minimal non-susy $SO(10)$ model as a reference framework for model building.
- ▶ A complete and consistent model of this kind must also reproduce the viable and realistic fermion mass spectrum. Thus it is worth looking at detail fits to fermion masses in non-susy theory.
- ▶ A preliminary simplified two generation analytical study has been done in this direction but **Complete and systematic three generation analysis is not carried out so far.**

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Fermion Masses in SO(10)

The masses of 16_F fermions arise from the renormalizable couplings with Higgs fields belonging to $10_H, \overline{126}_H$ and 120_H , since

$$16 \times 16 = 10 + 126 + 120$$

Identifying the Minimal Higgs fields for Realistic Fermion Spectrum

- ▶ Decomposition under Pati-Salam ($SU(4)_{PS} \times SU(2)_L \times SU(2)_R$)

$$16 = (4, 2, 1) + (\bar{4}, 1, 2)$$

$$10 = (1, 2, 2) + (6, 1, 1)$$

$$120 = (1, 2, 2) + (6, 3, 1) + (6, 1, 3) + (15, 2, 2) + (10, 1, 1) + (\overline{10}, 1, 1)$$

$$\overline{126} = (10, 1, 3) + (\overline{10}, 3, 1) + (15, 2, 2) + (6, 1, 1)$$

- ▶ VEV of $SU(2)_R(SU(2)_L)$ triplet $\in \overline{126}$ generates light neutrino masses by type-I(type-II) seesaw mechanism.

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The Minimal Higgs Sector

① $10_H + \overline{126}_H$

② $120_H + \overline{126}_H$

The Simplest Version: real $10_H/120_H$

- ▶ The most economical possibility would be to assume a real $10_H/120_H$.
- ▶ It is argued that a real $10_H/120_H$ cannot fit even two generation case. (as they predict $m_t/m_b \approx \mathcal{O}(1)$) [Bajc, Melfo, Senjanovic, Vissani (2006)]
- ▶ Thus both the cases require complexifying the Higgs fields $10_H/120_H$.

Complex $10_H/120_H$: Call for $U(1)_{PQ}$ symmetry

- ▶ This introduces an extra set of Yukawa couplings since both the real and imaginary parts of $10_H/120_H$ can independently couple to fermions.
- ▶ This can be avoided by assigning a Peccei-Quinn (PQ) charge

$$16_F \rightarrow e^{i\alpha} 16_F \quad \text{and} \quad (10_H, \overline{126}_H, 120_H) \rightarrow e^{-2i\alpha} (10_H, \overline{126}_H, 120_H)$$

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Minimal Model: $10_H + \overline{126}_H$ Higgs

In this case, the most general Yukawa interaction is

$$\mathcal{L}_Y = \mathbf{16}_F [Y_{10} \mathbf{10}_H + Y_{126} \overline{\mathbf{126}}_H] \mathbf{16}_F + h.c.$$

After EWSB, the final fermion mass relations can suitably written as

$$\begin{aligned} M_d &= H + F \quad , \quad M_u = r(H + s F), \\ M_l &= H - 3F \quad , \quad M_D = r(H - 3s F), \\ M_L &= r_L F \quad , \quad M_R = r_R^{-1} F, \end{aligned}$$

The light neutrino mass matrix is given by,

$$\mathcal{M}_\nu = r_L F - r_R M_D F^{-1} M_D^T$$

- ▶ The model has 18 real parameters (compared to 56 in SM + RH neutrinos) which have to explain 20 observables of fermion masses and mixing angles. (16 measured at low energy + 4 predictions)
- ▶ Similar model was studied in great detail in context of SUSY $SO(10)$ and found to be consistent with low energy fermion mass spectrum.

Minimal Model: Numerical Analysis

Numerical Study of Model: χ^2 Analysis

- ▶ We do the χ^2 fitting to check the viability of the model.
- ▶ We construct

$$\chi^2(\alpha_j) = \sum_i \left(\frac{X_i(\alpha_j) - O_i}{\sigma_i} \right)^2$$

Where,

X_i are the fermion masses and mixing as complex nonlinear functions of parameters α_j calculated from the given model at GUT scale.

O_i (σ_i) are the input mean values (1σ errors) of respective masses and mixing angles evaluated at $M_{GUT} = 2 \times 10^{16}$ GeV.

- ▶ We assume normal hierarchy in neutrino masses and consider two separate cases corresponding to the individual dominance of type-I and type-II seesaw mechanism.
- ▶ Then the data are fitted by minimizing the χ^2 function with respect to parameters α_j using an algorithm based on the downhill simplex method.

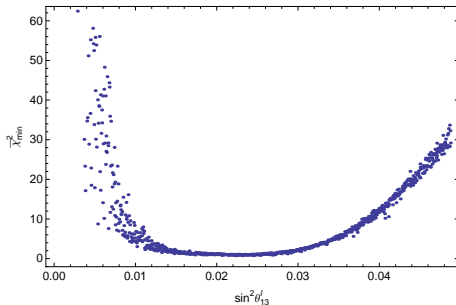
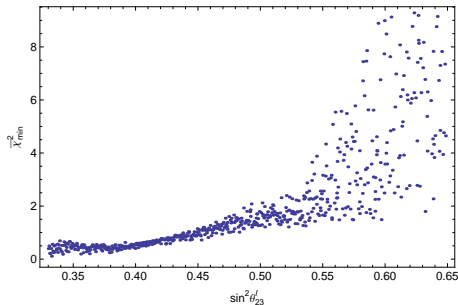
Minimal Model: Numerical Analysis

Results of χ^2 minimization

Observables	Type-I		Type-II	
	Fitted value	pull	Fitted value	pull
m_d	0.000810163	-0.687161	0.00101285	-0.264898
m_s	0.0208099	-0.198354	0.0225915	0.0844982
m_b	0.999667	-0.00831657	1.08201	2.05031
m_u	0.000495023	0.0751133	0.000507336	0.13668
m_c	0.237348	0.0670883	0.237096	0.0598882
m_t	73.9427	-0.0154941	74.3006	0.075144
m_e	0.000469652	-	0.000469652	-
m_μ	0.0991466	-	0.0991466	-
m_τ	1.68558	-	1.68558	-
$\left(\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}\right)$	0.030526	0.127968	0.0297114	-0.235285
$\sin \theta_{12}^q$	0.224651	0.0464044	0.224499	-0.0916848
$\sin \theta_{23}^q$	0.0420499	0.0392946	0.0421308	0.103004
$\sin \theta_{13}^q$	0.00349369	-0.0974312	0.00353053	0.0389979
$\sin^2 \theta_{12}^l$	0.323245	0.148134	0.3108	-0.610792
$\sin^2 \theta_{23}^l$	0.435096	-0.369178	0.113306	-7.02461
$\sin^2 \theta_{13}^l$	0.0244287	-	0.0176863	-
$\delta_{CKM} [^\circ]$	69.5262	-0.0314447	69.2051	-0.128759
$\delta_{MNS} [^\circ]$	318.465	-	14.5386	-
$\alpha_1 [^\circ]$	21.5053	-	345.645	-
$\alpha_2 [^\circ]$	215.128	-	141.905	-
χ_{min}^2		0.710777		54.1197

Minimal Model: Numerical Analysis

Results & Predictions



- ▶ There is no definite prediction for θ_{23} angle and very good fits are possible with values of θ_{23} in the whole range allowed by the data.
- ▶ There is a clear prediction for reactor angle $\sin^2 \theta_{13}$ in the range of (0.005 - 0.035).
- ▶ We also found that the model allows whole range for CP violating Dirac phase in lepton sector δ_{PMNS} .

Minimal Model: Numerical Analysis

Failure of Type-II seesaw: An Analytical Understanding

- ▶ In the case of type-II dominance, the neutrino mass matrix can be written as

$$\mathcal{M}_\nu = r_L F \propto M_d - M_l \simeq m_b \begin{pmatrix} \epsilon & \epsilon \\ \epsilon & \frac{m_b - m_\tau}{m_b} \end{pmatrix}$$

- ▶ It has been shown that the above relation offers an interesting connection between $b - \tau$ unification and the large atmospheric mixing angle.

[Bajc, Senjanovic, Vissani (2004)]

- ▶ In SUSY version, the extrapolated data at the GUT scale shows $m_b \simeq m_\tau$ which generates large atmospheric angle.
- ▶ In NON-SUSY version, $b - \tau$ unification fails badly ($m_\tau \simeq 1.6m_b$) at GUT scale and it generates small atmospheric mixing angle.

Non-unification of $b - \tau$ masses in non-susy spectrum disfavors type-II dominance in the minimal model.

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Fitted Structure of Mass Matrices

- ▶ The fitted fermion mass matrices can approximately be parametrized as

$$M_f \approx m_{f3} \begin{pmatrix} (1 + \epsilon_{11}^f)\lambda^4 & (1 + \epsilon_{12}^f)\lambda^3 & (1 + \epsilon_{13}^f)\lambda^2 \\ (1 + \epsilon_{12}^f)\lambda^3 & (1 + \epsilon_{22}^f)\lambda^2 & (1 + \epsilon_{23}^f)\lambda \\ (1 + \epsilon_{13}^f)\lambda^2 & (1 + \epsilon_{23}^f)\lambda & 1 \end{pmatrix} \quad \text{where } f = d, u, l, D, R$$

- ▶ Phenomenological consequences of such structure has been studied earlier by Dorsner & Smirnov (2004) and it was found that it generates correct hierarchy of the quark masses and mixings.
- ▶ After seesaw mechanism, the light neutrino mass matrix takes the form

$$\mathcal{M}_\nu \approx m_0 \begin{pmatrix} (1 + \epsilon_{11})\lambda^2 & (1 + \epsilon_{12})\lambda^2 & (1 + \epsilon_{13}) \\ (1 + \epsilon_{12})\lambda^2 & (1 + \epsilon_{22}) & (1 + \epsilon_{23}) \\ (1 + \epsilon_{13}) & (1 + \epsilon_{23}) & 1 \end{pmatrix}$$

- ▶ Such structure generally predicts hierarchical neutrinos, near maximal atmospheric mixing and large 13 mixing angle.
- ▶ Such structure provides hint into possible flavor structure of fermions and can be a starting point to uncover the underlying flavor symmetries.

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Minimal Model-II: $120_H + \overline{126}_H$ Higgs

In this case, the final fermion mass relations can suitably written as

$$\begin{aligned}M_d &= G + F \quad , \quad M_u = t_u G + s F, \\M_l &= t_l G - 3F \quad , \quad M_D = t_D G - 3s F,\end{aligned}$$

where G is an anti-symmetric matrix. The light neutrino mass matrix is given by,

$$\mathcal{M}_\nu = r_L F - r_R M_D F^{-1} M_D^T$$

- ▶ We studied the viability of above relations through detailed numerical analysis.
- ▶ This model is found to be highly inconsistent with all the data extrapolated at the GUT scale and show poor fits:
 - ▶ $\chi_{min}^2 \sim 60$ (in case of type-I seesaw)
 - ▶ $\chi_{min}^2 \sim 72$ (in case of type-II seesaw)

So the model with $120_H + \overline{126}_H$ Higgs fields is unable to reproduce the fermion spectrum and can be ruled out.

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So the model with $120_H + \overline{126}_H$ Higgs fields is unable to reproduce the fermion spectrum and can be ruled out.

An Extended Model: $10_H + \overline{126}_H + 120_H$

- ▶ The other model (studied extensively in SUSY context) is constructed with all three Higgs fields which has 28 real parameters in general.
- ▶ Considerable reduction in parameters can be achieved by restricting to the Hermitian mass matrices which can be achieved by assuming Spontaneous CP Violation (SCPV).
- ▶ In this case, the fermion mass relations can be written as

$$\begin{aligned}M_d &= H + F + iG \quad , \quad M_u = r(H + sF + it_u G), \\M_l &= H - 3F + it_l G \quad , \quad M_D = r(H - 3sF + it_D G)\end{aligned}$$

where H, F are real symmetric and G is real anti-symmetric matrices. r, s, t_f are real dimensionless parameters. The light neutrino mass matrix is given by,

$$\mathcal{M}_\nu = r_L F - r_R M_D F^{-1} M_D^T \equiv \mathcal{M}_\nu^{\prime\prime} + \mathcal{M}_\nu^{\prime}$$

- ▶ This model has 17 real parameters (one less compared to minimal model) and provide predictive framework for fermion spectrum.

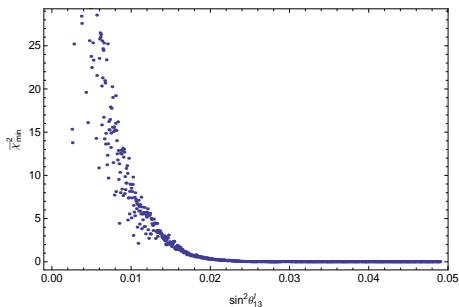
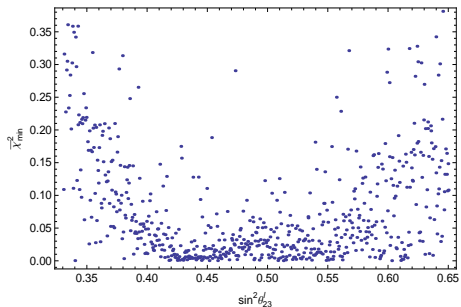
Extended Model: Numerical Analysis

Results of χ^2 minimization

Observables	Type-I		Type-II	
	Fitted value	pull	Fitted value	pull
m_d	0.00113968	-0.000676838	0.00108711	-0.110189
m_s	0.0219909	-0.00150966	0.0142689	-1.28852
m_b	1.	0.0000376219	1.19665	4.9162
m_u	0.000480133	0.000666686	0.000486627	0.0331338
m_c	0.235007	0.000211758	0.240819	0.166268
m_t	73.9997	-0.0000888053	77.4295	0.857367
m_e	0.000469652	0	0.000469652	0
m_μ	0.0991466	0	0.0991466	0
m_τ	1.68558	0	1.68558	0
$\left(\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}\right)$	0.0302402	0.000545016	0.0260106	-1.88556
$\sin \theta_{12}^q$	0.224601	0.00105776	0.224567	-0.0304356
$\sin \theta_{23}^q$	0.0420001	0.0000431604	0.0431393	0.897068
$\sin \theta_{13}^q$	0.00351992	-0.000308192	0.00338234	-0.509862
$\sin^2 \theta_{12}^l$	0.320821	0.000292661	0.278093	-2.6052
$\sin^2 \theta_{23}^l$	0.453034	0.000947066	0.343286	-2.26804
$\sin^2 \theta_{13}^l$	0.0306736	-	0.00538748	-
$\delta_{CKM} [^\circ]$	69.6278	-0.000660788	72.7155	0.935014
$\delta_{MNS} [^\circ]$	355.719	-	46.8148	-
$\alpha_1 [^\circ]$	60.079	-	60.6202	-
$\alpha_2 [^\circ]$	214.691	-	250.978	-
χ_{min}^2		$\sim 10^{-6}$		44.0801

Extended Model: Numerical Analysis

Results & Predictions



- ▶ Like minimal model, there is no definite prediction for θ_{23} angle and very good fits are possible with values of θ_{23} in the whole range allowed by the data.
- ▶ Model predicts relatively large values for reactor angle $\sin^2 \theta_{13} \gtrsim 0.01$.
- ▶ It also allows whole range for CP violating Dirac phase in lepton sector δ_{PMNS} .

SUMMARY

- ▶ We studied the fit of quark and lepton masses and mixing data within a class of non-supersymmetric $SO(10)$ models.
- ▶ Among the various possible options for constructing a predictive model, we found the minimal model with $10_H + \overline{126}_H$ Higgs fields and an additional $U(1)_{PQ}$ symmetry can reproduce the entire fermion spectrum if neutrinos get their masses through type-I seesaw mechanism.
- ▶ It predicts the value of atmospheric angle $\sin^2 \theta_{13} \sim (0.005-0.035)$ which can be tested by current generation of neutrino oscillation experiments.
- ▶ This minimal model together with 45_H Higgs may provide the platform for exploring the possibility of a realistic and predictive GUT, along the lines of the recent efforts in the SUSY context.
- ▶ We also found that an alternative model with extended Higgs sector and with the assumption of spontaneous CP violation can also provide a predictive and viable scenario for fermion spectrum in non-susy $SO(10)$.

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