2+1 Flavor Quark Meson Linear Sigma Model with Polyakov Loop Potential and Symmetry Restoration Effects

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- Theoretical considerations indicate that at very high temperature and baryon densities, there should be a phase transition from ordinary hadronic matter to plasma of quarks and gluons.
- The rich phase structure of Quantum Chromodynamics (QCD - the theory of strong interaction), is studied through the dynamics of Quark Hadron Phase transition.
- We know that the basic QCD Lagrangian has the global $SU_V(3)xSU_A(3)xU_A(1)$ symmetry.

- In the zero quark mass limit, chiral condensate works as an order parameter for the spontaneous breakdown of the chiral symmetry in the low energy hadronic vacuum of QCD.
- For the infinitely heavy Quarks in the pure gauge $SU_C(3)$ QCD, the $Z(3)$ (center symmetry of the QCD color gauge group) symmetry which is the symmetry of hadronic vacuum, gets spontaneously broken in the high temperature/density regime of QGP.
- The expectation value of the Wilson line (Polyakov Loop) is related to the free energy of the static color charge, hence it serves as the order parameter of the confinement - deconfinement phase transition.

- Lot of effective model building activity is centered around combining the features of spontaneous breakdown of both the chiral symmetry as well as the center $Z(3)$ symmetry of QCD in one single model.
- In these models, chiral condensate and Polyakov loop are simultaneously coupled to the Quark degrees of freedom. Several such studies have been done in PNJL models.
- Since chiral symmetry restoration is signaled by parity doubling, one looks for the patterns of emerging convergence in the masses of the chiral partners in pseudo scalar (π, η, η', K) and scalar mesons (σ, a_0, f_0, κ).

- Schaefer et al [PRD 79, 014018(2009)] in Quark Meson Sigma (QMS) Model studied the consequences of $SU(3)$ chiral symmetry restoration for scalar and pseudo-scalar meson masses and mixing angles in the 2+1 flavor symmetry breaking scenario.

- We are investigating how the inclusion of Polyakov Loop in the 2+1 flavor QMS Model, qualitatively and quantitatively affects the convergence in the masses of chiral partners.

- We are also studying the effect of polyakov loop potential on the temperature variation of octet - singlet mixing angle in the scalar and pseudoscalar meson channels. The interplay of $SU_A(3)$ chiral symmetry and $U_A(1)$ symmetry restoration is also being studied.

- The model Lagrangian is

$$\mathcal{L}_{PQMS} = \mathcal{L}_{QMS} - \mathcal{U}\big(\Phi, \Phi^*, T\big) \tag{1}$$

where the Lagrangian in Quark Meson Chiral Sigma model

$$\mathcal{L}_{QMS} = \bar{q}_f\big(i\gamma^\mu D_\mu - g\ T_a(\sigma_a + i\gamma_5\pi_a)\big)q_f + \mathcal{L}_m \tag{2}$$

- The coupling of quarks with uniform temporal background gauge field is effected by the replacement ∂_μ to $D_\mu = \partial_\mu - iA_\mu$ and $A_\mu = \delta_{\mu 0}A_0$ (Polyakov Gauge), where $A_\mu = g_s A^a_\mu \lambda^a/2$. g_s is the gauge coupling.
- The mesonic Lagrangian

$$\begin{aligned}\mathcal{L}_m &= \mathrm{Tr}\left(\partial_\mu M^\dagger \partial^\mu M\right) - m^2\mathrm{Tr}(M^\dagger M) - \lambda_1\left[\mathrm{Tr}(M^\dagger M)\right]^2 \\ &\quad -\lambda_2\mathrm{Tr}\left(M^\dagger M\right)^2 + c\big[det(M) + det(M^\dagger)\big] \\ &\quad +\mathrm{Tr}\left[H(M+M^\dagger)\right] \end{aligned} \tag{3}$$

- The chiral field M is a 3×3 complex matrix comprising of the nine scalars (σ_a) and nine pseudo scalars (π_a) mesons.
$$M = T_a \xi_a = T_a(\sigma_a + i\pi_a) \qquad T_a = \frac{\lambda_a}{2}$$
Here $T_a = \frac{\lambda_a}{2}$ represent 9 generators of $U(3)$ with $a = 0, 1 \ldots 8$. λ_a are standard Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{2}{3}}\,\mathbf{1}$.
- The generators follow $U(3)$ algebra $[T_a, T_b] = if_{abc} T_c$ and $\{T_a, T_b\} = d_{abc} T_c$
- f_{abc} and d_{abc} are standard antisymmetric and symmetric structure constants respectively.
- $f_{ab0} = 0$ and $d_{ab0} = \sqrt{\frac{2}{3}}\,\mathbf{1}\,\delta_{\mathbf{ab}}$ and matrices are normalized as $\mathrm{Tr}(T_a T_b) = \frac{\delta_{ab}}{2}$.

- Since the $\bar{\xi}$ must have the quantum numbers of vacuum, explicit chiral symmetry breaking is only possible with three nonzero parameters h_0, h_3 and h_8.
- The $SU_L(3) \times SU_R(3)$ chiral symmetry is explicitly broken by the term

$$H = T_a h_a$$

  Here H is a 3×3 matrix with nine external parameters h_a.
- spontaneous symmetry breaking is effected by non zero condensates σ_0 and σ_8
- In the explicit symmetry breaking consideration two flavors are treated on equal footing, thus $h_0,\ h_8 \neq 0 \rightarrow$ 2+1 flavor breaking scenario
- In absence of $U_A(1)$ anomaly: $\rightarrow c = 0$ (no det M contribution)

- The Polyakov Loop field $\Phi(\vec{x})$ is defined as the thermal expectation value of color trace of Wilson loop in temporal direction.

$$\Phi = \frac{1}{N_c}\mathrm{Tr}_c L, \qquad \Phi^* = \frac{1}{N_c}\mathrm{Tr}_c L^\dagger \tag{4}$$

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_0(\vec{x},\tau)\right] \tag{5}$$

Here $\mathcal{P}$ is the path ordering, A_0 is temporal Vector field, $\beta = \frac{1}{T}$

- We have taken two choices of the effective potential for the polyakov loop.

[C. Ratti et al PRD 73,014019(2006)]

- The first choice is based on the polynomial expansion in terms of Polyakov loop order parameter Φ and this effective potential is

$$\frac{\mathcal{U}_{\rm pol}\left(\Phi,\Phi^*,T\right)}{T^4} = -\frac{b_2}{4}\left(|\Phi|^2+|\Phi^*|^2\right)-\frac{b_3}{6}(\Phi^3+\Phi^{*3}) + \frac{b_4}{16}\left(|\Phi|^2+|\Phi^*|^2\right)^2 \qquad (6)$$

- Coefficient $b_2(T)$ governs the confinement-deconfinement phase transition where

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

- The other parameters have the following value

$$a_0 = 6.75\ , \quad a_1 = -1.95\ , \quad a_2 = 2.625\ ,$$
$$a_3 = -7.44, \quad b_3 = 0.75\ , \quad b_4 = 7.5$$

[S. Rößner et al PRD 75,034007(2007)]

- The other choice of effective potential has the logarithmic form

$$\frac{\mathcal{U}_{\log}(\Phi, \Phi^*, T)}{T^4} = -\frac{a(T)}{2}\Phi^*\Phi + b(T)$$
$$\ln[1 - 6\Phi^*\Phi + 4(\Phi^{*3} + \Phi^3) - 3(\Phi^*\Phi)^2] \qquad (7)$$

- The temperature dependent coefficients are as follow

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

- The parameters are

$$a_0 = 3.51\ , \qquad a_1 = -2.47\ ,$$
$$a_2 = 15.2\ , \qquad b_3 = -1.75$$

Grand Potential

- Mean-field grand potential in the presence of Polyakov loop

$$\Omega(T,\mu) = -\frac{T\ln Z}{V} = U(\sigma_x,\sigma_y) + \mathcal{U}(\Phi,\Phi^*,T) + \Omega_{\bar{q}q}(T,\mu) \quad (8)$$

- Mesonic potential in the nonstrange - strange basis

$$U(\sigma_x,\sigma_y) = \frac{m^2}{2}\left(\sigma_x^2+\sigma_y^2\right) - h_x\sigma_x - h_y\sigma_y - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}\left(2\lambda_1+\lambda_2\right)\sigma_x^4 + \frac{1}{8}\left(2\lambda_1+2\lambda_2\right)\sigma_y^4 , \quad (9)$$

Quark/antiquark contribution in the presence of Polyakov loop

$$\Omega_{\bar{q}q}(T,\mu) = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[\ln g_f^+ + \ln g_f^-\right] \tag{10}$$

where

$$g_f^+ = \left[1 + 3\Phi e^{-E_f^+/T} + 3\Phi^* e^{-2E_f^+/T} + e^{-3E_f^+/T}\right]$$

$$g_f^- = \left[1 + 3\Phi^* e^{-E_f^-/T} + 3\Phi e^{-2E_f^-/T} + e^{-3E_f^-/T}\right]$$

Here we use the notation $E_f^\pm = E_f \mp \mu$ and E_f is the flavor dependent single particle energy of quark/antiquark.

$$E_f = \sqrt{p^2 + m_f{}^2}, \qquad m_x = g\frac{\sigma_x}{2}, \qquad m_y = g\frac{\sigma_y}{\sqrt{2}}$$

Here m_x and m_y are masses of nonstrange and strange quarks respectively.

- The temperature dependence of σ_x, σ_y, $\langle\Phi\rangle$ and $\langle\Phi^*\rangle$ is obtained from the solution of the gap equations

$$\frac{\partial\Omega}{\partial\sigma_x} = \frac{\partial\Omega}{\partial\sigma_y} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\Phi^*}\bigg|_{\sigma_x=\bar{\sigma}_x,\sigma_y=\bar{\sigma}_y,\Phi=\bar{\Phi},\Phi^*=\bar{\Phi}^*} = 0\,. \qquad (11)$$

- The curvature of grand potential at the global minimum gives scalar and pseudoscalar meson masses and mixing angles.

$$m^2_{\alpha,ab} = \frac{\partial^2\Omega(T,\mu)}{\partial\xi_{\alpha,a}\partial\xi_{\alpha,b}}\bigg|_{min} \qquad \tan 2\theta_\alpha = \left(\frac{2m^2_{\alpha,08}}{m^2_{\alpha,00} - m^2_{\alpha,88}}\right) \qquad (12)$$

Here α stands for scalar and pseudoscalar field.

Model Parametrs

- The vacuum masses of pion and kaon m_π, m_K, the pion and kaon decay constant f_π, f_K, mass square of η, η' and m_σ are used to fix six unknown parameters.

	C[MeV]	m^2 $[MeV^2]$	λ_1	λ_2	h_x $[MeV^3]$	h_y $[MeV^3]$
W/$U_A(1)$	4807.84	$(342.52)^2$	1.40	46.48	$(120.73)^3$	$(336.41)^3$
W/o$U_A(1)$	0	$-(189.85)^2$	-17.01	82.47	$(120.73)^3$	$(336.41)^3$

- The value of Yukawa coupling g has been fixed from the nonstrange constituent quark mass $m_q = 300$ MeV and is equal to 6.5.

- The temperature dependence of $\langle \Phi \rangle$, σ_x and σ_y at zero chemical potential, is obtained from the solution of the gap equations
- The inflection points of these order parameters respectively give the characteristic temperature (pseudocritical temperature)

	QMS	PQMS:pol	PQMS:log
T_c^{χ} (MeV)	146	204	206
T_s^{χ} (MeV)	248	262	274
T_c^{Φ} (MeV)	—	204	206

- The inclusion of Polyakov loop in $2 + 1$ flavor linear sigma model leads to the sharper melting of the nonstrange condensate while strange condensate also shows stronger melting in comparison to QMS model results.

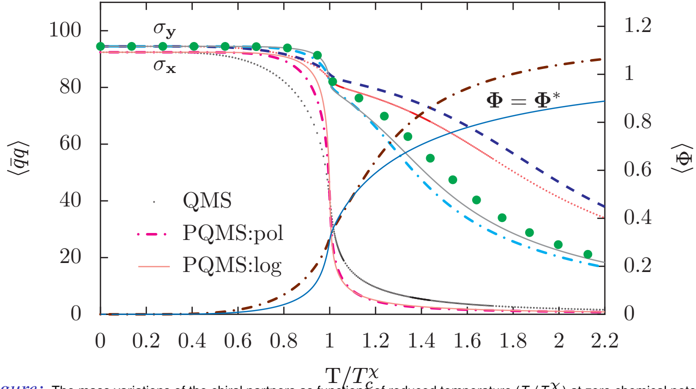

Figure: The mass variations of the chiral partners as functions of reduced temperature (T/T_c^χ) at zero chemical potential ($\mu = 0$), in the presence of axial $U_A(1)$ breaking term, are plotted for (σ, π) and (a_0, η') in left panel and the corresponding mass variations, in the absence of the $U_A(1)$ axial breaking term, are plotted in right panel.

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- Mass degeneration trend of the chiral partners (σ, π) and (a_0, η') becomes sharper in PQMS model.

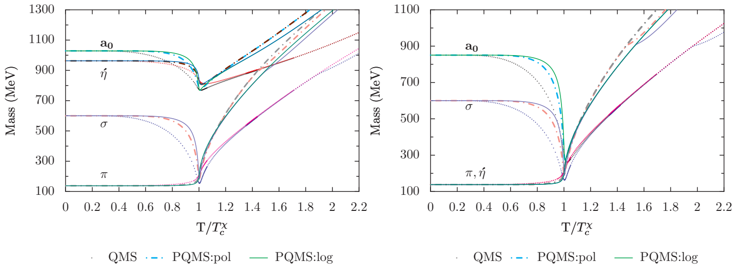

Figure: The mass variations of the chiral partners as functions of reduced temperature (T/T_c^χ) at zero chemical potential $(\mu = 0)$, in the presence of axial $U_A(1)$ breaking term, are plotted for (σ, π) and (a_0, η') in left panel and the corresponding mass variations, in the absence of the $U_A(1)$ axial breaking term, are plotted in right panel.

- Mass gap $(m_\pi = m_\sigma < m_{a_0} = m_{\eta'})$between two sets of chiral partners (σ, π) and (a_0, η') results due to the opposite sign of anomaly term $\sqrt{2}c\sigma_y$.
- This gap will be reduced due to the sharper and stronger melting of strange condensate σ_y in PQMS models. Hence $U_A(1)$ restoration trend sets up early in the PQMS models.

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- Mass degeneration of the chiral partners (K, κ) with the η occurs at $T/T_c^\chi \sim 1.3$ in two PQMS models while it occurs in QMS model at $T/T_c^\chi \sim 1.5$.

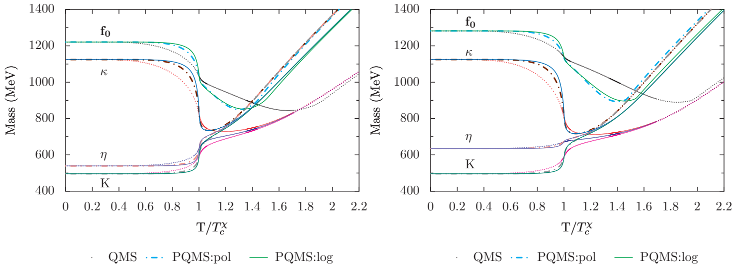

Figure: The mass variations of the chiral partners as functions of reduced temperature, in the presence of axial $U_A(1)$ breaking term, are plotted for (η, f_0) and (K, κ) in left panel, while the corresponding mass variations, in the absence of the $U_A(1)$ axial breaking term, are plotted in right panel.

- Intersection point of f_0 and η masses occurs at $T/T_c^\chi \sim 1.4$ in two PQMS models while in QMS model it is found at $T/T_c^\chi \sim 1.7$.
- All these mass degenerations occur due to sharper and stronger melting of strane condensate as a result of Polyakov loop inclusion in QMS model.

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- The smooth approach of the pseudoscalar mixing angle θ_P towards the ideal mixing in the QMS model, becomes sharper and faster in the PQMS models due to the influence of the Polyakov loop potential. Further the ideal mixing is achieved earlier in the PQMS models.

- Faster growth of θ_S towards ideal value is observed in PQMS model. θ_S drops to -51° at $T/T_c^\chi \sim 1.50$ in PQMS model and at $T/T_c^\chi \sim 1.85$ in QMS model.

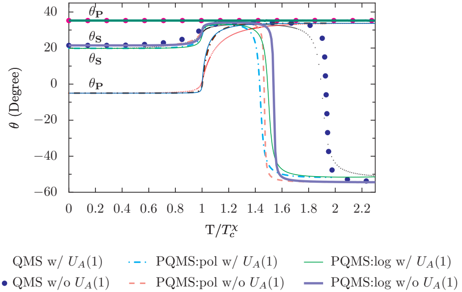

Figure: the temperature variation of scalar θ_S and pseudoscalar θ_P mixing angle at zero chemical potential are plotted in presence and absence of axial $U_A(1)$ symmetry breaking term.

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- The smooth mass convergence trend of the pure QMS model in $m_{\eta'} \rightarrow m_{\eta_{NS}}$ and $m_{\eta} \rightarrow m_{\eta_{S}}$ approach, becomes sharp around $T/T_c^{\chi} = 1$ in the PQMS models.

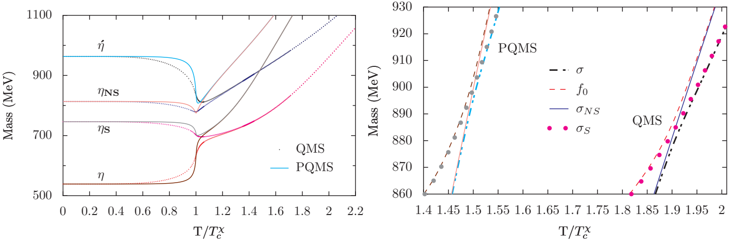

Figure: Left panel shows the mass variations for the physical η, η' and the nonstarnge-strange η_{NS}, η_S complex, on the reduced temperature scale and zero chemical potential.The masses of the physical σ and f_0 anticross and the nonstraange-strange $\sigma_{NS} - \sigma_S$ system masses cross in right panel.

- The masses of physical σ_0 and f_0 anticross and the nonstrange - strange (σ_{NS} - σ_S) system masses cross in vicinity of $T/T_c^{\chi} = 1.9$ in QMS model and $T/T_c^{\chi} = 1.5$ in PQMS model.

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Conclusions

- Inclusion of Polyakov loop potential drives the critical temperature of chiral phase transition to higher value.
- Significant melting of strange condensate is observed in PQMS model.
- Chiral symmetry restoration in non strange and strange sector becomes sharper and faster in PQMS model.
- Sharp mass degeneration trend is observed for scalar and pseudoscalar mesons in the PQMS model calculations and it sets up early on the reduced temperature scale.
- Earlier setting up of axial $U_A(1)$ symmetry restoration trend on reduced temperature scale in PQMS model.
- Sharper and faster approach towards ideal mixing is found for the pseudoscalar (θ_P) and scalar (θ_S) mixing angles in PQMS model.

THANK YOU !

	Scalar Meson Sector		Pseudoscalar Meson Sector
$m^2_{a_0}$	$m^2 + \lambda_1(x^2+y^2) + \frac{3\lambda_2}{2}x^2 + \frac{\sqrt{2}c}{2}y$	m^2_{π}	$m^2 + \lambda_1(x^2+y^2) + \frac{\lambda_2}{2}x^2 - \frac{\sqrt{2}c}{2}y$
m^2_{κ}	$m^2 + \lambda_1(x^2+y^2) + \frac{\lambda_2}{2}(x^2 + \sqrt{2}xy + 2y^2) + \frac{c}{2}x$	m^2_{K}	$m^2 + \lambda_1(x^2+y^2) + \frac{\lambda_2}{2}(x^2 - \sqrt{2}xy + 2y^2) - \frac{c}{2}x$
$m^2_{s,00}$	$m^2 + \frac{\lambda_1}{3}(7x^2 + 4\sqrt{2}xy + 5y^2) + \lambda_2(x^2+y^2) - \frac{\sqrt{2}c}{3}(\sqrt{2}x + y)$	$m^2_{p,00}$	$m^2 + \lambda_1(x^2+y^2) + \frac{\lambda_2}{3}(x^2+y^2) + \frac{c}{3}(2x + \sqrt{2}y)$
$m^2_{s,88}$	$m^2 + \frac{\lambda_1}{3}(5x^2 - 4\sqrt{2}xy + 7y^2) + \lambda_2(\frac{x^2}{2} + 2y^2) + \frac{\sqrt{2}c}{3}(\sqrt{2}x - \frac{y}{2})$	$m^2_{p,88}$	$m^2 + \lambda_1(x^2+y^2) + \frac{\lambda_2}{6}(x^2 + 4y^2) - \frac{c}{6}(4x - \sqrt{2}y)$
$m^2_{s,08}$	$\frac{2\lambda_1}{3}(\sqrt{2}x^2 - xy - \sqrt{2}y^2) + \sqrt{2}\lambda_2(\frac{x^2}{2} - y^2) + \frac{c}{3\sqrt{2}}(x - \sqrt{2}y)$	$m^2_{p,08}$	$\frac{\sqrt{2}\lambda_2}{6}(x^2 - 2y^2) - \frac{c}{6}(\sqrt{2}x - 2y)$
m^2_{σ}	$m^2_{s,00}\cos^2\theta_s + m^2_{s,88}\sin^2\theta_s + 2m^2_{s,08}\sin\theta_s\cos\theta_s$	$m^2_{\eta'}$	$m^2_{p,00}\cos^2\theta_p + m^2_{p,88}\sin^2\theta_p + 2m^2_{p,08}\sin\theta_p\cos\theta_p$
$m^2_{f_0}$	$m^2_{s,00}\sin^2\theta_s + m^2_{s,88}\cos^2\theta_s - 2m^2_{s,08}\sin\theta_s\cos\theta_s$	m^2_{η}	$m^2_{p,00}\sin^2\theta_p + m^2_{p,88}\cos^2\theta_p - 2m^2_{p,08}\sin\theta_p\cos\theta_p$
$m^2_{\sigma_{NS}}$	$\frac{1}{3}(2m^2_{s,00} + m^2_{s,88} + 2\sqrt{2}m^2_{s,08})$	$m^2_{\eta_{NS}}$	$\frac{1}{3}(2m^2_{p,00} + m^2_{p,88} + 2\sqrt{2}m^2_{p,08})$
$m^2_{\sigma_S}$	$\frac{1}{3}(m^2_{s,00} + 2m^2_{s,88} - 2\sqrt{2}m^2_{s,08})$	$m^2_{\eta_S}$	$\frac{1}{3}(m^2_{p,00} + 2m^2_{p,88} - 2\sqrt{2}m^2_{p,08})$

Table: The squared vacuum masses of scalar and pseudoscalar mesons as calculated in nonstrange-strange basis. In the table x denotes σ_x and y denotes σ_y. The masses of nonstrange σ_{NS}, strange σ_S, nonstrange η_{NS} and strange η_S mesons are given in the last two rows.

- The expression for the meson mass modification due to quark contribution at finite temperature in QMS model, has been evaluated by Schaefer et al PRD 79, 014018 (2009) and is given as

$$\delta m^2_{\alpha,ab} = \frac{\partial^2 \Omega_{\bar{q}q}(T,\mu)}{\partial \xi_{\alpha,a} \partial \xi_{\alpha,b}}\Big|_{min} = \nu_c \sum_{f=x,y} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_f} \Bigg[(a_f^+ + a_f^-) \left(m^2_{f,ab} - \frac{m^2_{f,a} m^2_{f,b}}{2E_f^2} \right) - (b_f^+ + b_f^-) \left(\frac{m^2_{f,a} m^2_{f,b}}{2E_f T} \right) \Bigg]$$

$m^2_{f,a} \equiv \partial m^2_f / \partial \xi_{\alpha,a}$ is the first derivative and $m^2_{f,ab} \equiv \partial m^2_{f,a} / \partial \xi_{\alpha,b}$ is the second derivative of squared quark mass with respect to meson fields $\xi_{\alpha,b}$. The number of internal quark degrees of freedom, $\nu_c = 2N_c = 6$. Here $a_f^{\pm}$ are quark/antiquark occupation numbers; given as

$$a_f^{\pm} = \frac{1}{1 + e^{E^{\pm}/T}}$$

the notations $b_f^{\pm} = a_f^{\pm} - (a_f^{\pm})^2$ stand for particle $(+)$ and antiparticle $(-)$ in QMS model.

- We are obtaining the following formula for the mass modification that results on account of quark contribution in the PQMS model

$$\delta m^2_{\alpha,ab} = \frac{\partial^2 \Omega_{\bar{q}q}(T,\mu)}{\partial \xi_{\alpha,a}\partial \xi_{\alpha,b}}\Big|_{min} = 3\sum_{f=x,y}\int \frac{d^3p}{(2\pi)^3}\frac{1}{E_f} \left[(A_f^+ + A_f^-)\left(m^2_{f,ab} - \frac{m^2_{f,a}m^2_{f,b}}{2E_f^2}\right) + (B_f^+ + B_f^-)\left(\frac{m^2_{f,a}m^2_{f,b}}{2E_f T}\right)\right]$$

The notations $A_f^{\pm}$ and $B_f^{\pm}$ have the following definitions

$$A_f^+ = \frac{\Phi e^{-E_f^+/T} + 2\Phi^* e^{-2E_f^+/T} + e^{-3E_f^+/T}}{g_f^+} \qquad A_f^- = \frac{\Phi^* e^{-E_f^-/T} + 2\Phi e^{-2E_f^-/T} + e^{-3E_f^-/T}}{g_f^-}$$

and $B_f^{\pm} = 3(A_f^{\pm})^2 - C_f^{\pm}$, where we again define

$$C_f^+ = \frac{\Phi e^{-E_f^+/T} + 4\Phi^* e^{-2E_f^+/T} + 3e^{-3E_f^+/T}}{g_f^+} \qquad C_f^- = \frac{\Phi^* e^{-E_f^-/T} + 4\Phi e^{-2E_f^-/T} + 3e^{-3E_f^-/T}}{g_f^-}$$

		$m^2_{x,a}m^2_{x,b}/g^4$	$m^2_{x,ab}/g^2$	$m^2_{y,a}m^2_{y,b}/g^4$	$m^2_{y,ab}/g^2$
σ_0	σ_0	$\frac{1}{3}\sigma_x^2$	$\frac{2}{3}$	$\frac{1}{3}\sigma_y^2$	$\frac{1}{3}$
σ_1	σ_1	$\frac{1}{2}\sigma_x^2$	1	0	0
σ_4	σ_4	0	$\sigma_x \frac{\sigma_x+\sqrt{2}\sigma_y}{\sigma_x^2-2\sigma_y^2}$	0	$\sigma_y \frac{\sqrt{2}\sigma_x+2\sigma_y}{2\sigma_y^2-\sigma_x^2}$
σ_8	σ_8	$\frac{1}{6}\sigma_x^2$	$\frac{1}{3}$	$\frac{2}{3}\sigma_y^2$	$\frac{2}{3}$
σ_0	σ_8	$\frac{\sqrt{2}}{6}\sigma_x^2$	$\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}\sigma_y^2$	$-\frac{\sqrt{2}}{3}$
π_0	π_0	0	$\frac{2}{3}$	0	$\frac{1}{3}$
π_1	π_1	0	1	0	0
π_4	π_4	0	$\sigma_x \frac{\sigma_x-\sqrt{2}\sigma_y}{\sigma_x^2-2\sigma_y^2}$	0	$\sigma_y \frac{\sqrt{2}\sigma_x-2\sigma_y}{\sigma_x^2-2\sigma_y^2}$
π_8	π_8	0	$\frac{1}{3}$	0	$\frac{2}{3}$
π_0	π_8	0	$\frac{\sqrt{2}}{3}$	0	$-\frac{\sqrt{2}}{3}$

Table: First and second derivative of squared quark mass in nonstrange-strange basis with respect to meson fields are evaluated at minimum. Sum over two light flavors, denoted by symbol x, are in third and fourth columns. The last two columns have only strange quark mass flavor denoted by the symbol y.