Scale Determination in LQCD with Dynamical Wilson Quarks

Sangita De Sarkar

Saha Institute of Nuclear Physics

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Collaborators: Asit K. De, A. Harindranath,

Jyotirmoy Maiti, Anwesa Sarkar, Santanu Mondal

Aim of the Talk

- Setting the cut-off in dynamical lattice regularized QCD using Sommer parameter.

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- Study the quark mass dependence of the cut-off.

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- Lattice spacing a act as inverse cut-off.

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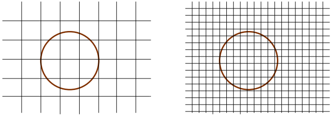

- Lattice QCD is a gauge invariant regularization of QCD.
- Lattice spacing a act as inverse cut-off.

At the end of the day, we should remove the cut-off i.e set $a \rightarrow 0$ (Continuum Limit).

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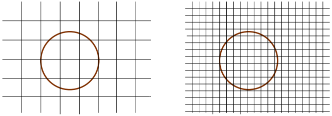

Tuning g, m accordingly, physics remains invariant.

$\hat{\xi}(g, m) \rightarrow \infty$

when $g \rightarrow g_c, m \rightarrow m_c$

RG equation

According to the definition of β function,

$$a(g) \quad = \quad \frac{1}{\Lambda_L}(\beta_0 g^2)^{\frac{-\beta_1}{2\beta_0^2}} e^{-\frac{1}{2\beta_0 g^2}} = \frac{1}{\Lambda_L} R(g)$$

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$\Rightarrow$ a decreases with decrease in g. One should study $g \to 0$ i.e. $\beta = \frac{6}{g^2} \to \infty$ limit to achieve CL.

Continuum Physics can be obtained not only in the CL but also in the scaling region.

$\Rightarrow$ Physical quantities are calculated in units of the cut-off a.

Thus lattice scale determination is necessary for comparing lattice observables with the continuum ones.

Wilson Fermion

We have done the analysis with Wilson fermions (2 degenerate flavors).

+ Removes doublers.

+ Conceptually easy.

– Explicitly breaks chiral symmetry.

- Exceptional Configuration.
  - But large-volume regime of lattice QCD with Wilson fermions is safe. (Luscher '06)

– $\mathcal{O}(a)$ cut-off effect.

– For non-zero lattice spacing, there is an unphysical phase in which flavor and parity are spontaneously broken. (S. Aoki '96)

- But it has been seen for $a \sim 0.2$fm & higher. (Sharpe- Singleton '98)

Wilson Fermion requires a detailed study....

- In the literature, there are also other fermion formulations like Overlap, Staggered, Domain-wall fermions.

- Each has its own advantages & disadvantages also.

- To start with, we chose to keep the things simple at the fundamental level. (Wilson's Formulation)

- In future, many improvements will be incorporated in our study.
  - $\mathcal{O}$(a) improvement- -
    - Clover fermion action.
    - Symanzik's improved gauge action.
  - Improved sources, fat links.
  - More realistic case with $2+1$ flavors.

Wilson Action

The Wilson action is

$$S = S_G + S_F + S_W .$$

where

Gauge action $S_G = \beta \sum_{Plaq.} \left(1 - \frac{1}{3} ReTrU_{Plaq.}\right)$

Fermion action $S_F + S_W = \sum_{x,y} \bar{\psi}_x M(U)_{xy} \psi_y$

with $M(U)_{xy} = \kappa \sum_{\mu} \delta_{x,y+\hat{\mu}} (r + \gamma_\mu) U_{x\mu} - \delta_{xy}$

- 2 input parameters $\beta = \frac{6}{g^2}$ and $\kappa = \frac{1}{2am_0 + 8r}$.

Simulation details

Unimproved Wilson gauge and dynamical fermions

2 flavors of degenerate sea quarks

- $\beta = 5.6$

16^3**x**32	24^3**x**48	32^3**x**64
κ=0.156		
0.1565		
0.15675		
0.157		
0.15725		
0.1575	0.1575	
0.15775	0.15775	
0.158	0.158	
	0.158125	
		0.15815
	0.15825	0.15825
		0.1583

- $\beta = 5.8$

24^3**x**48	32^3**x**64
κ=0.1535	
0.1538	
0.154	0.154
	0.15455
	0.15462
	0.15470
	0.15475

Simulation details......

- **Preliminary results.**
- Data for another gauge coupling $\beta = 5.7$ and also for higher volumes ($48^3 \times 64$) are ready to analyze.

Scale Determination using Sommer Parameter

- Need to fix one dimensionful quantity ⇒ Sommer Parameter r_0.

It is defined as

$$r^2 \frac{dV}{dr}\Big|_{r=r_0(c)} = c. \qquad \text{[Sommer '93]}$$

$V(r_0)$ is the potential between a heavy q and $\bar{q}$ at the intermediate distance r_0.

Comparing with the expt. data for the $b\bar{b}$ and $c\bar{c}$ spectra (E.Eitchen et al., '80), Sommer found that

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Now, $\quad V(r) = V_0 + \frac{\alpha}{r} + \sigma r \quad \Rightarrow \quad r_0(c = 1.65) = 0.49 fm.$

So we have used $r^2 \frac{dV}{dr}\Big|_{r=0.49} = 1.65$.

$$\Rightarrow \frac{a}{r_0} = \frac{a\sqrt{\sigma}}{\sqrt{1.65+\alpha}}$$

The Wilson loop is the observable for static $q\bar{q}$ potential $V(R)$.

$$\langle W_{\mathcal{L}}(R,T)\rangle = C(R)e^{-TV(R)} + \text{ higher state contribution. } \; T = at.$$

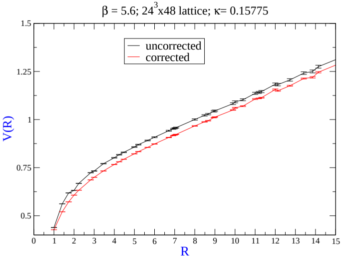
So we can calculate $V(R)$ from the large t behaviour of the Wilson loop.

- The breaking of rotational invariance on lattice is prominent at small R. So, there should be a correction for the finite lattice.

$$\left(\frac{1}{R}\right)_{lat} = \left[\frac{1}{R}\right] \quad = \quad \frac{4\pi}{a}\int_{-\pi}^{\pi}\frac{d^3k}{(2\pi)^3}\frac{e^{ik.R/a}}{4\sum_{i=1}^{3}\sin^2(k_i/2)} \neq \frac{1}{R}$$

(C. Michael '92)

- On lattice, the potential should be $V_{lat}(R) = V_0 + \alpha\left[\frac{1}{R}\right] + \sigma R$
- More generally, $V_{lat}(R) = \underbrace{V_0 + \alpha\frac{1}{R} + \sigma R} + \delta_{rot}\left(\left[\frac{1}{R}\right] - \frac{1}{R}\right)$.

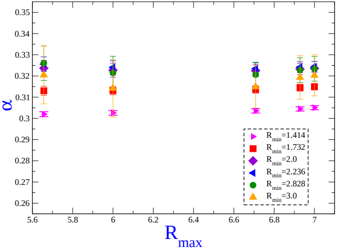


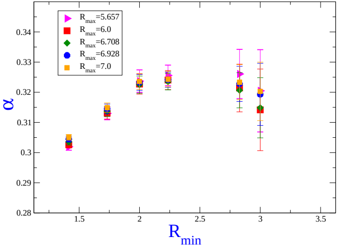
The kinks at small R region disappear after the correction.

Properties of α

α is familiar from the continuum. It resembles the strong coupling constant.

Focus to the reliable determination of α.





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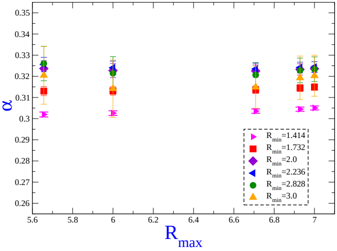
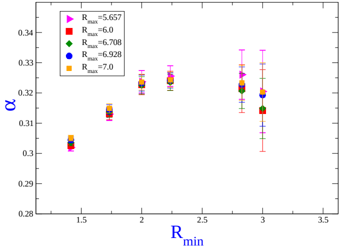


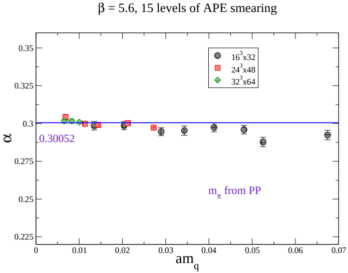


- α is indep. of R_{max} for $R_{min} = \sqrt{2}$, $\sqrt{3}$ & 2.
- α is growing larger as R_{min} increases as long as α is determined reliably, consistent with the expected running of α.

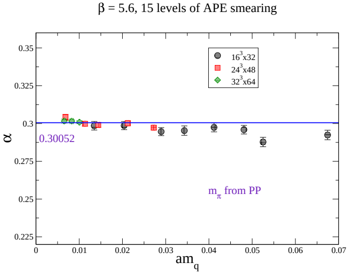
From the fit value of the parameters $\alpha, a^2\sigma, \delta_{rot}$ we have seen that

- The value of δ_{rot} is similar to that of α as expected.
- We have calculated the $\frac{a}{r_0}$ value for each κ.
- It shows some non-trivial am_q dependence.
- To study its am_q dependence, it is necessary to investigate the behaviour of α & σ.

α vs. am_q


β = 5.6, 15 levels of APE smearing
α
0.35
0.325
0.3
0.275
0.25
0.225
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
am_q
16^3x32
24^3x48
32^3x64
0.30052
m_π from PP

α vs. am_q



- α is indep. of am_q for $am_q \lesssim 0.035$.

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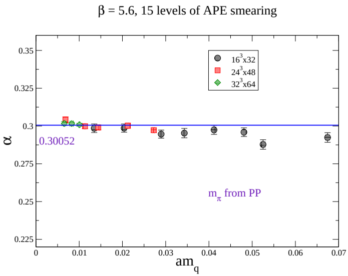
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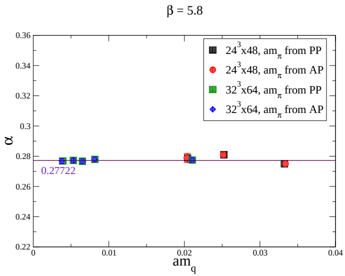
β = 5.8

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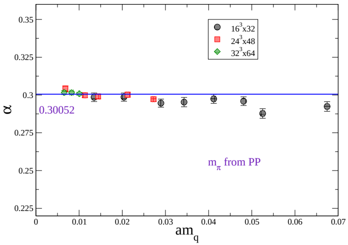
α vs. am_q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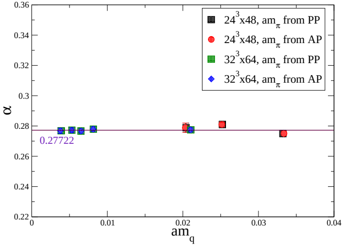
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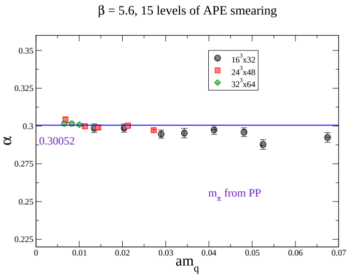
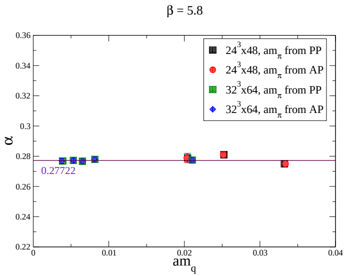
- α is indep. of am_q for $am_q \lesssim 0.035$.

- α is indep. of am_q for $am_q \lesssim 0.027$.

α vs. am_q

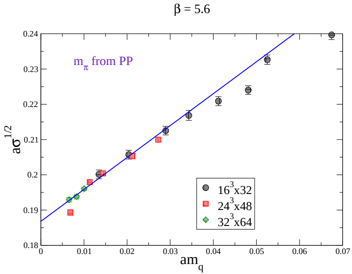




- α is indep. of am_q for $am_q \lesssim 0.035$.
- α is indep. of am_q for $am_q \lesssim 0.027$.

To conclude, am_q dependence of $\frac{a}{r_0}$ is entirely due to σ.

$a\sqrt{\sigma}$ vs. am_q



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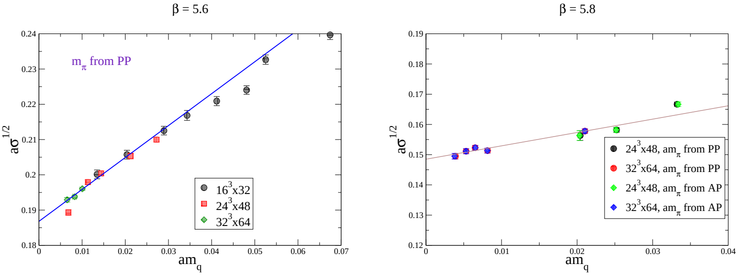

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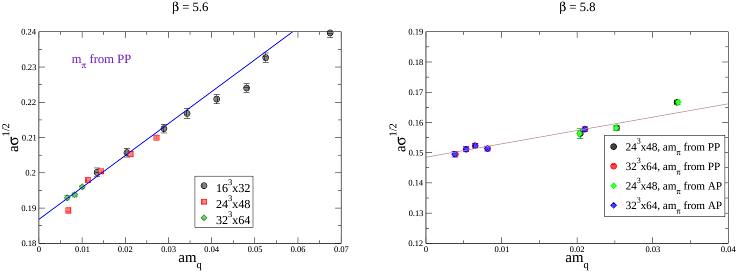
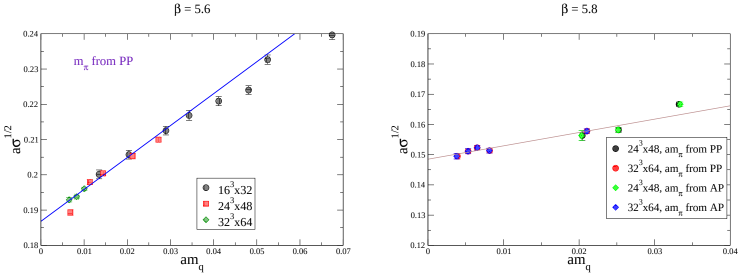


For both $\beta = 5.6$ and $\beta = 5.8$, the figures show linear dependence i.e.,

$$a\sqrt{\sigma} = C_1 + C_2 am_q$$

$a\sqrt{\sigma}$ vs. am_q



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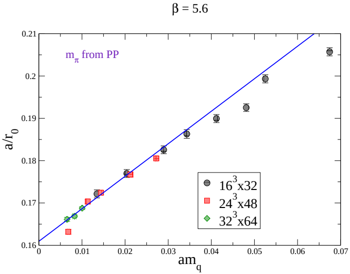
$$a\sqrt{\sigma} = C_1 + C_2 am_q$$

The fits have been done only for the scaling window.

$\frac{a}{r_0}$ vs. am_q



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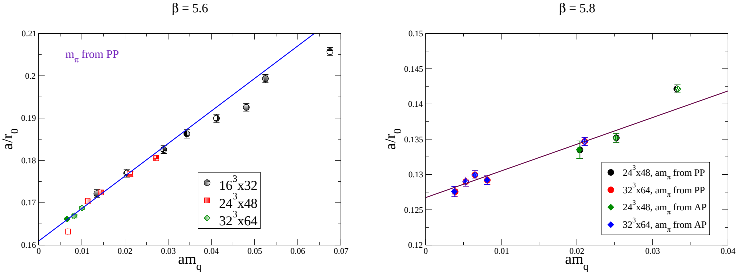

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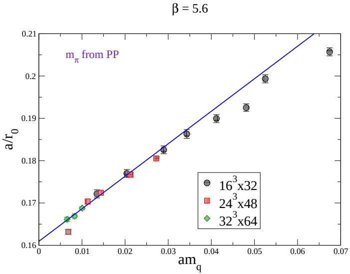
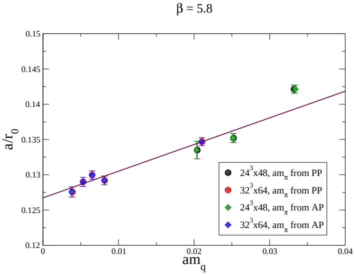




For both $\beta = 5.6$ and $\beta = 5.8$, the figures show linear dependence, similar to $a\sqrt{\sigma}$ i.e.,

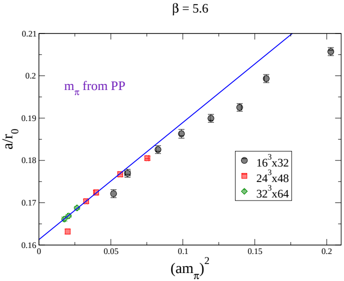
$$\frac{a}{r_0} = A_1 + A_2 am_q$$

Scaling Region

- Since α is a dimensionless parameter, we have used the am_q variation of α to get an idea about the scaling region.
- $\frac{m_1(a)}{m_2(a)} = \frac{m_1(0)}{m_2(0)} + C_{12}(am_1)^p$.
- We have seen that α is indep. of am_q for small enough am_q at both values of β.
- It is an indication about the distinction between the scaling region & the scale-violating region.
- Always remain in the scaling window for further observation.
- Mass indep. scheme: The scale a is constant for a fixed β but r_0 changes with am_q in this window.
- Within this scaling window, only change of β amounts to a change in the cut-off.

Chiral Extrapolation

For the extrapolation to physical point, we have used $(am_\pi)^2$ instead of am_q.



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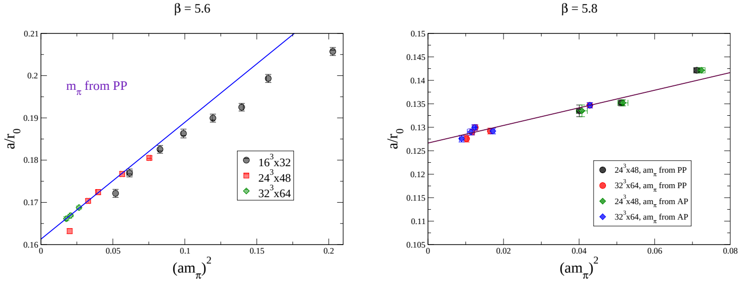

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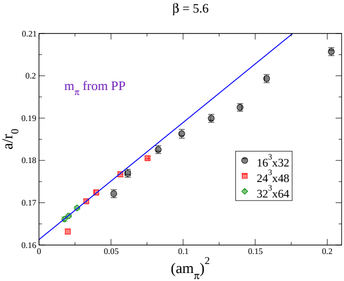

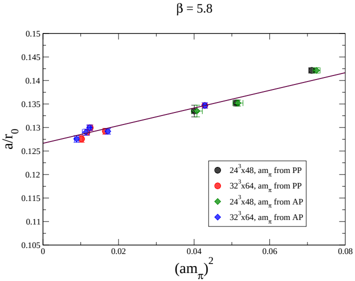

- Since $(am_\pi)^2$ is linear in am_q in the LO in χPT,

  $$\frac{a}{r_0^{phys}} = A_3 + A_4 (am_\pi^{phys})^2$$

- Using $m_\pi^{phys} = 139.6$MeV and solving the quadratic eq. we get

  $a = 0.079$fm i.e., $a^{-1} = 2.479$ GeV for $\beta = 5.6$

  $a = 0.062$fm i.e., $a^{-1} = 3.166$ GeV for $\beta = 5.8$

Summary: Parameters in physical unit

$\beta = 5.6; a = 0.0795 fm$

- Smallest quark mass ≈ 20.4 MeV for $(2.5fm)^3$.
- Pion mass ranging from ~ 320 MeV to 550 MeV.
- $M_\pi L = 4.25$ for smallest pion mass.
- $\frac{m_\rho}{\sqrt{\sigma}} = 1.58$ at the chiral limit.

$\beta = 5.8; a = 0.0622 fm$

- Smallest quark mass ≈ 15 MeV for $(2fm)^3$.
- Pion mass ranging from ~ 300 MeV to 850 MeV.
- $M_\pi L = 3.12$ for smallest pion mass.
- $\frac{m_\rho}{\sqrt{\sigma}} = 1.55$ at the chiral limit.

The approximate equality of $\frac{m_\rho}{\sqrt{\sigma}}$ for $\beta = 5.6$ & $\beta = 5.8$ is an indication of absence of scaling violation.

Numerical calculation are carried out on a Cray XDI, supported by Theory division, SINP under the DAE, Govt. of India.

This work is in part based on the MILC collaborations public lattice gauge theory code(http://physics.utah.edu/dtar/milc.html) and also on the DDHMC code developed by Luscher (http://luscher.web.cern.ch/luscher/DD-HMC/index.html).

Thank you.