FRAGMENTATION OF NONET VECTOR MESON USING (BROKEN) SU(3) MODEL

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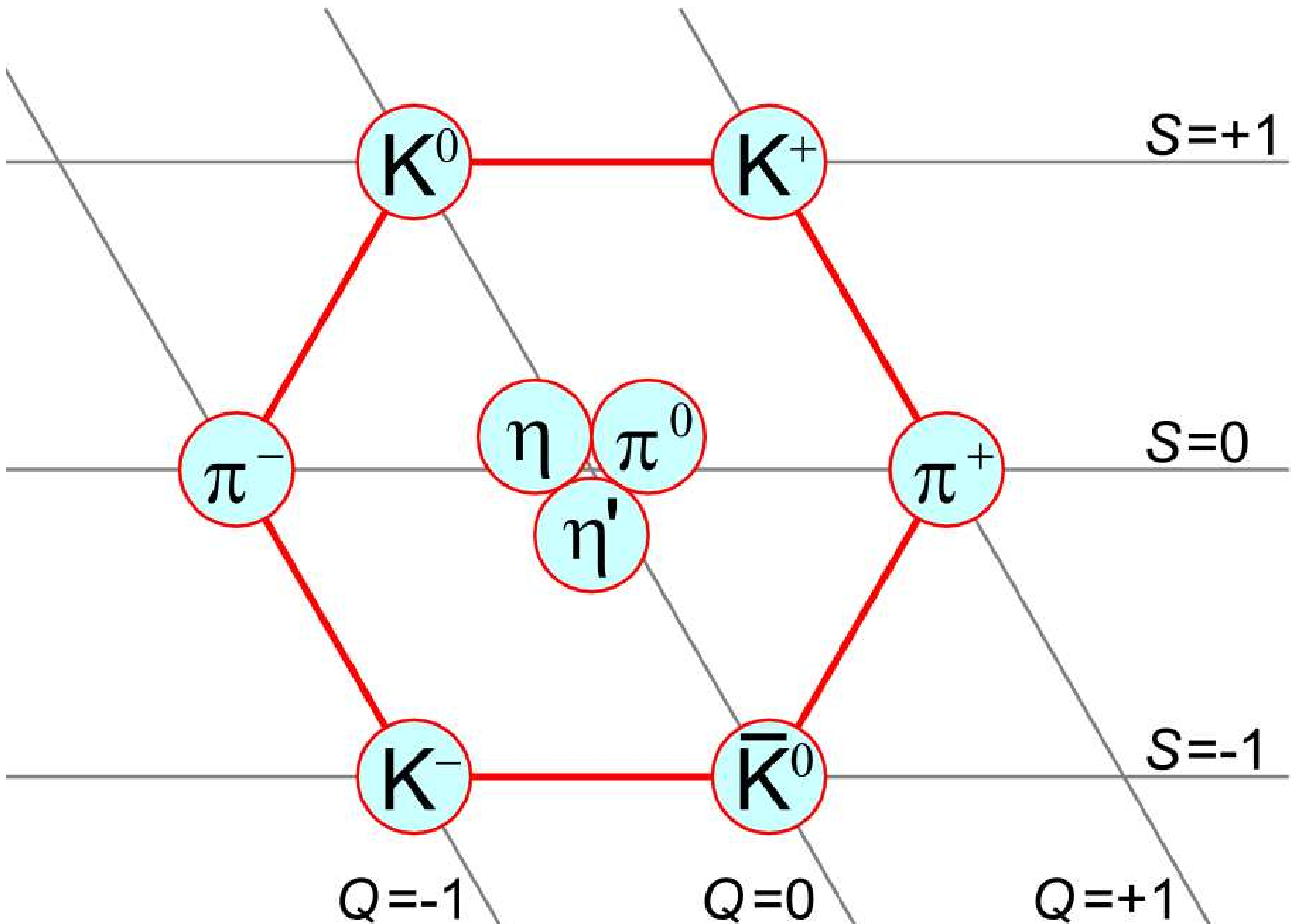
What is the matter made of?

- Matter consists of fermions including quarks and leptons. Interactions between these particles are mediated by *forces carriers* which are usually vector bosons.
- Leptons are thought to be point-like; no structure has been seen for these leptons, which include the familiar electron and muon, tau.
- Hadrons are not point-like particles but composite objects made of *quarks*.
- Baryons (such as protons, neutrons) and mesons (such as pions) are classified into *hadrons*.
- Baryons are made up of three quarks while mesons are a quark-antiquark pair.

Mesons

- On their spin and parity basis mesons are classified as follows:
- (i) Pseudoscalar Mesons: (π^+, π^-, π^0); (K^+, K^-), $(K^0, \overline{K}^0)$, η, which form the pseudoscalar octet.

  η mixes with the singlet to form the physical η and η' states.



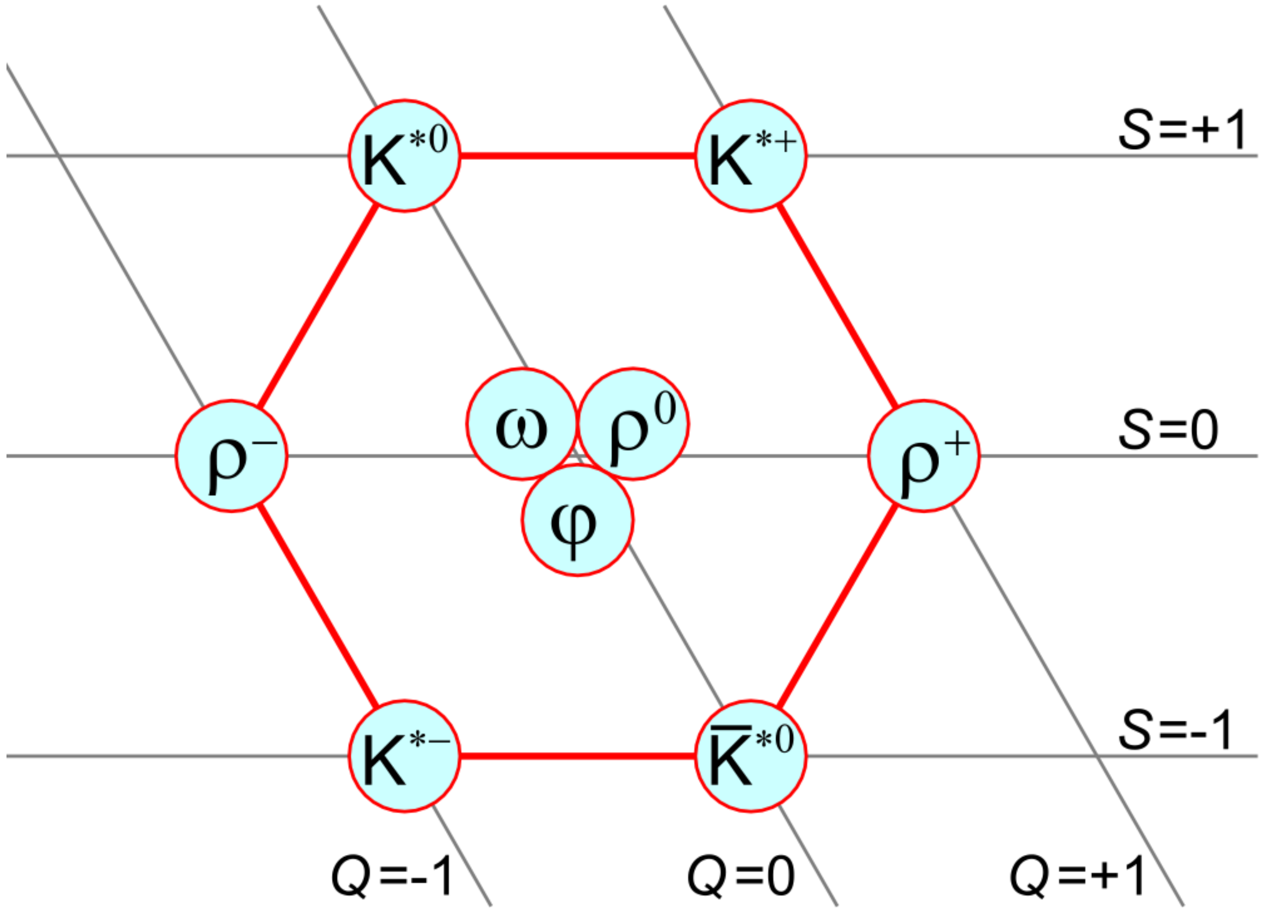
Quantum numbers of pseudoscalar mesons

- Mesons of bound states where the spins of the quark and anti quark point in the opposite directions with odd parity $J^P = 0^-$.

$particle$	$symbol$	$Mass(MeV)$	$Isospin, I$	I_z
π^+	u$\bar{d}$	140	1	1
π^-	d$\bar{u}$		1	-1
π^0	(u$\bar{u}$–d$\bar{d}$)$/\sqrt{2}$	135	1	0
K^+	u$\bar{s}$	494	1/2	1/2
K^-	s$\bar{u}$	494	1/2	$-1/2$
K^0	d$\bar{s}$	"	1/2	$-1/2$
$\overline{K^0}$	s$\bar{d}$	"	1/2	$1/2$
η	(u$\bar{u}$+d$\bar{d}$ $-$ 2s$\bar{s}$)$/\sqrt{6}$	548	0	0
η'	(u$\bar{u}$+d$\bar{d}$+s$\bar{s}$)$/\sqrt{3}$	958	0	0

Vector Mesons

- There exists an identical octet $+$ singlet, with spin 1. These are the vector mesons.



- Mesons of bound states where the spins of the quark and antiquark form a vector state with parity $J^P = 1^+$.

Quantum numbers of Vector mesons

$particle$	$symbol$	$Mass(MeV)$	$Isospin, I$	I_z
ρ^+	u$\overline{d}$	770	1	1
ρ^-	d$\overline{u}$	"	1	-1
ρ^0	$(u\overline{u} - d\overline{d})/\sqrt{2}$	"	1	0
K^{*+}	u$\overline{s}$	892	1/2	1/2
K^{*-}	s$\overline{u}$	"	1/2	$-1/2$
K^{*0}	d$\overline{s}$	"	1/2	$-1/2$
$\overline{K^{*0}}$	s$\overline{d}$	"	1/2	$1/2$
ω	$(u\overline{u} + d\overline{d} - 2s\overline{s})/\sqrt{6}$	782	0	0
ϕ	(u$\overline{u}$+d$\overline{d}$+s$\overline{s}$)$/\sqrt{3}$	1020	0	0

Table 1: Valence quark content of vector mesons.

Fragmentation Process

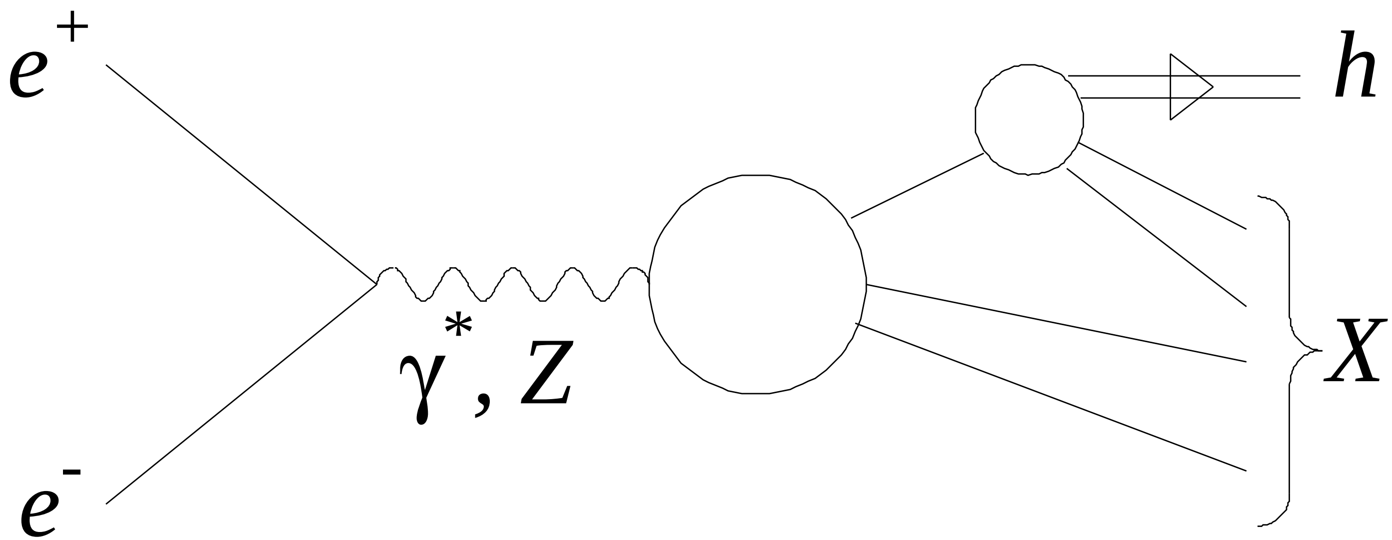
- Quarks and antiquarks cannot be seen individually due to their confinement property.
- The quark-antiquark pair obtained in the $e^+ e^-$ process are observed only through the hadrons they produce. This is known to be *hadronisation or fragmentation process*.
- These fragmentation processes are characterised by fragmentation functions. QCD which explains this process at the quark level cannot explain the origin of these fragmentation functions. However, QCD can explain their scale (Q^2) dependence.
- A comparison with experimental data allows the fragmentation functions to be determined at any given scale.
- A model is needed to determine these fragmentation functions from the observed event rates! Our model uses (a broken) SU(3) symmetry.

Why SU(3) Symmetry?

- Each meson has seven fragmentation functions associated with its production: $D_q(x,Q^2)$, $q=u,d,s$, and $D_g(x,Q^2)$.
- Since we are considering both quarks and antiquarks in the process we have to predict 48 (8×6) unknown quark fragmentation functions. It is quite hard. To make the case simpler we are looking into SU(3) symmetry.
- SU(3) symmetry is a good description of the octet of known vector mesons: the ρ(ρ^+, ρ^-,ρ^0), the K^*(K^{*+}, K^{*-}, K^{*0}, $\overline{K}^{*0}$), and the ω mesons.
- A simple model for a light quark (u,d,s) to fragment into a vector meson is proposed using SU(3) symmetry of octet hadrons.Charge conjugation and isospin invariance reduce the functions further.
- All fragmentation functions are described in terms of three SU(3) symmetric functions $\alpha(x,Q^2)$, $\beta(x,Q^2)$, $\gamma(x,Q^2)$. These correspond to fragmentation of a quark into an octet hadron and $3, \overline{6}, 15$-plet respectively.
- Subsequently, an SU(3) breaking parameter (λ) (suppression factor) is also introduced. All these are to be determined by comparison with data.

Hadron Production

For the extraction of the fragmentation functions from data, the particle production in $e^+ e^-$ annihilation is the choice process.



A schematic of the fragmentation processes in e^+e^- scattering, $e^+ + e^- \to (\gamma^*, Z) \to h + X$. From J. Binnewies, ArXiv/hep-ph/9707269. We focus on real Z-mediated interaction. x, the energy fraction, is $x \equiv x_E = E_h / \frac{\sqrt{s}}{2}$.

Cross Section

- To leading order in the coupling constant, the cross section for producing a hadron h can be expressed in terms of the unknown fragmentation functions , $D_q^h(x_E, Q)$, as

(1) $$\frac{1}{\sigma_{tot}} \frac{\mathrm{d}\sigma^h}{\mathrm{d}x_E} = \frac{\sum_q c_q D_q^h(x_E, Q)}{\sum_q c_q} .$$

- Here c_q are the electroweak charge factors associated with a quark q_i of flavour i. We can also re-express the cross section in terms of the non-singlet and singlet fragmentation function combinations, as

(2) $$\frac{1}{\sigma_{tot}} \frac{\mathrm{d}\sigma^h}{\mathrm{d}x_E} = \frac{a_0 D_0(x_E, Q) + a_3 D_3 + a_8 D_8 + a_{15} D_{15} + \cdots}{\sum_q c_q} ,$$

Input fragmentation functions

- We use the starting scale $Q_0^2 = 2$ GeV^2, where there are 3 light flavours.
- Then the singlet D_0 and the non-singlet D_3 and D_8 are the only relevant combinations:

(3)
$$
\begin{aligned}
D_0 &= D_u + D_d + D_s + \text{anti-quarks} \\
D_3 &= D_u - D_d + \text{ anti-quarks} \\
D_8 &= D_u + D_d - 2D_s + \text{ anti-quarks}
\end{aligned}
$$

Here $a_0 = (c_u + c_d + c_s)/3$; $a_3 = (c_u - c_d)/2$ and $a_8 = (c_u + c_d - 2c_s)/6$.

- The charm and bottom fragmentation functions are zero at the starting scale
- As the fragmentation functions evolve across the various thresholds, the contributions of the charm, bottom, etc., are included apropriately in the singlet and non-singlet combinations.
- This gives the model a great deal of predictability with small number of inputs.

The octet fragmentation functions

fragmenting quark		K^{*+}	fragmenting quark		K^{*0}
u	:	$\alpha+\beta+\frac{3}{4}\gamma$	u	:	$2\beta+\gamma$
d	:	$2\beta+\gamma$	d	:	$\alpha+\beta+\frac{3}{4}\gamma$
s	:	2γ	s	:	2γ
fragmenting quark		ω	**fragmenting quark**		ρ^0
u	:	$\frac{1}{6}\alpha+\frac{9}{6}\beta+\frac{9}{8}\gamma$	u	:	$\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{11}{8}\gamma$
d	:	$\frac{1}{6}\alpha+\frac{9}{6}\beta+\frac{9}{8}\gamma$	d	:	$\frac{1}{2}\alpha+\frac{1}{2}\beta+\frac{11}{8}\gamma$
s	:	$\frac{4}{6}\alpha+\frac{9}{6}\gamma$	s	:	$2\beta+\gamma$

Application of SU(3) invariance leads to $\beta=\gamma/2$. Hence there are two independent combinations: $V(x,Q^2)\equiv\alpha+\beta-5/4\gamma$ and $\gamma(x,Q^2)$, apart from the gluon fragmentation function.

Computations And Fits

- The ρ and K^* fragmentation functions can be written in terms of *sea and valence parts at Q_0^2.*

$$
\begin{aligned}
D_0^{\rho+} &= 2V + 12\gamma \ , \\
D_0^{K^{*0}} &= V(1+\lambda) + 12\lambda\gamma \ .
\end{aligned}
$$

- The functional form of V, γ, D_g at Q_0^2 needed to fit the data is

$$
F_i(x) = a_i x^{b_i} (1-x)^{c_i} (1 + d_i x + e_i x^2) \ ;
$$

where a, b, c, d and e are the parameters to be determined from the fit.

• The function should go to zero as $x \to 1$; this is governed by c_i. The small-x behaviour is governed by b_i.

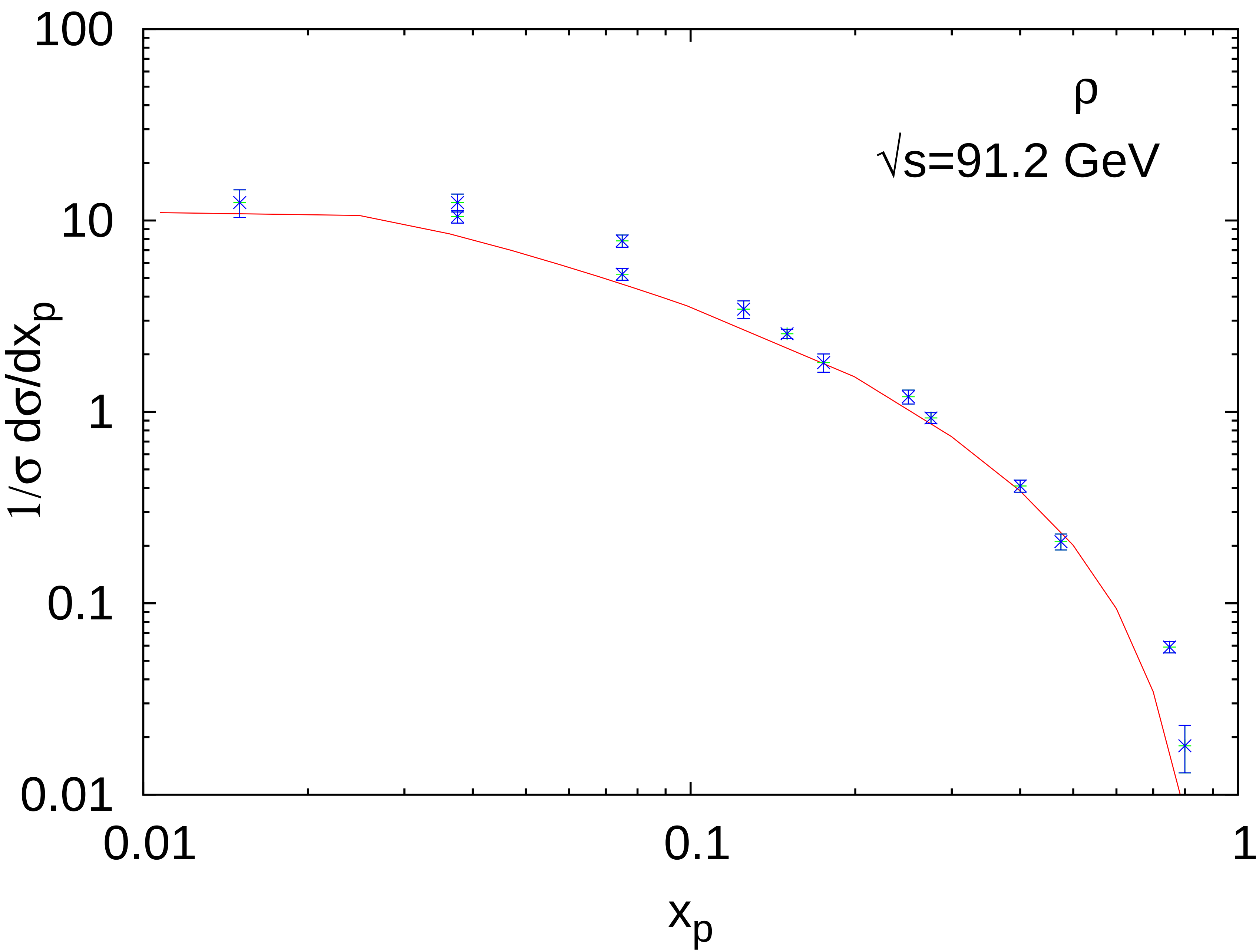
Results from ρ and K^*

- The fragmentation functions are evolved according to *LO* evolution to the Z-pole including heavier quarks which are set to zero at starting scale of Q_0^2.

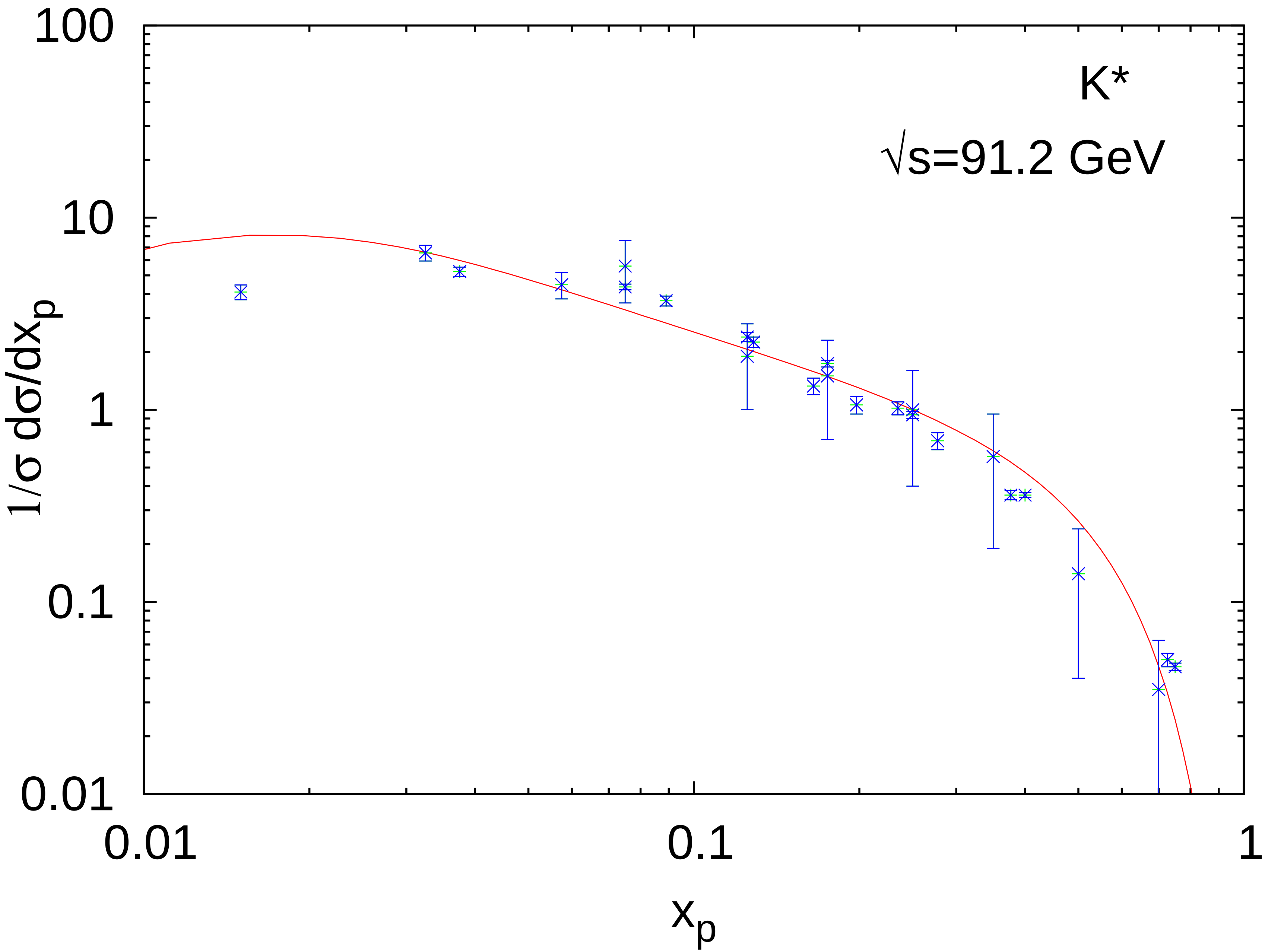
	V	γ	$gluon$
a	5.7	1.2	3.6
b	0.9	-0.2	1.5
c	2.8	10.8	7.2
d	5.4	1.9	2.0
e	-5.1	61.6	0.0
λ	0.2		

- V and γ are determined from the large- and small-x dependence of the ρ cross-section data.
- The strangeness-suppression parameter λ is fixed from the relative size of the ρ and K^* data.
- The gluon is not well-known at LO since it does not directly appear in the cross-section expressions.

Fits to ρ (LEP) data



Fit to K^* (LEP) data



Fits for ω and ϕ Mesons

- We have to assume some more parameters for ω and ϕ since they are mixtures of singlet and octet.
- Using the functional forms of valence and sea part of ρ and K^* mesons we can fit the octet part of ω and ϕ.

$$\begin{aligned} |\omega\rangle &= \sin\theta|\omega_8\rangle + \cos\theta|\omega_1\rangle \,, \\ &= \sin\theta \left| \frac{u\overline{u} + d\overline{d} - 2s\overline{s}}{\sqrt{6}} \right\rangle + \cos\theta \left| \frac{u\overline{u} + d\overline{d} + s\overline{s}}{\sqrt{3}} \right\rangle \,. \end{aligned}$$

Now

$$\begin{aligned} D_u^{\omega_8} &= \frac{1}{6}\alpha + \frac{9}{6}\beta + \frac{9}{8}\gamma = \frac{V}{6} + 2f_8^{\omega}\gamma \,, \\ D_s^{\omega_8} &= \frac{4}{6}\alpha + \frac{9}{6}\gamma = \frac{2}{3}\lambda V + 2f_8^{\omega}\gamma \,. \end{aligned}$$

- where f_8^{ω} is the sea quark suppression factor.

ω singlet part

- There is a *single* fragmentation function for the singlet meson ($q \to M^1 + X$)
- Ansatz: We assume the singlet part of ω to be some constant factor(f_d^ω) times the octet part.
- Then the u ($= d$) and s fragmentation functions can be written as follows:

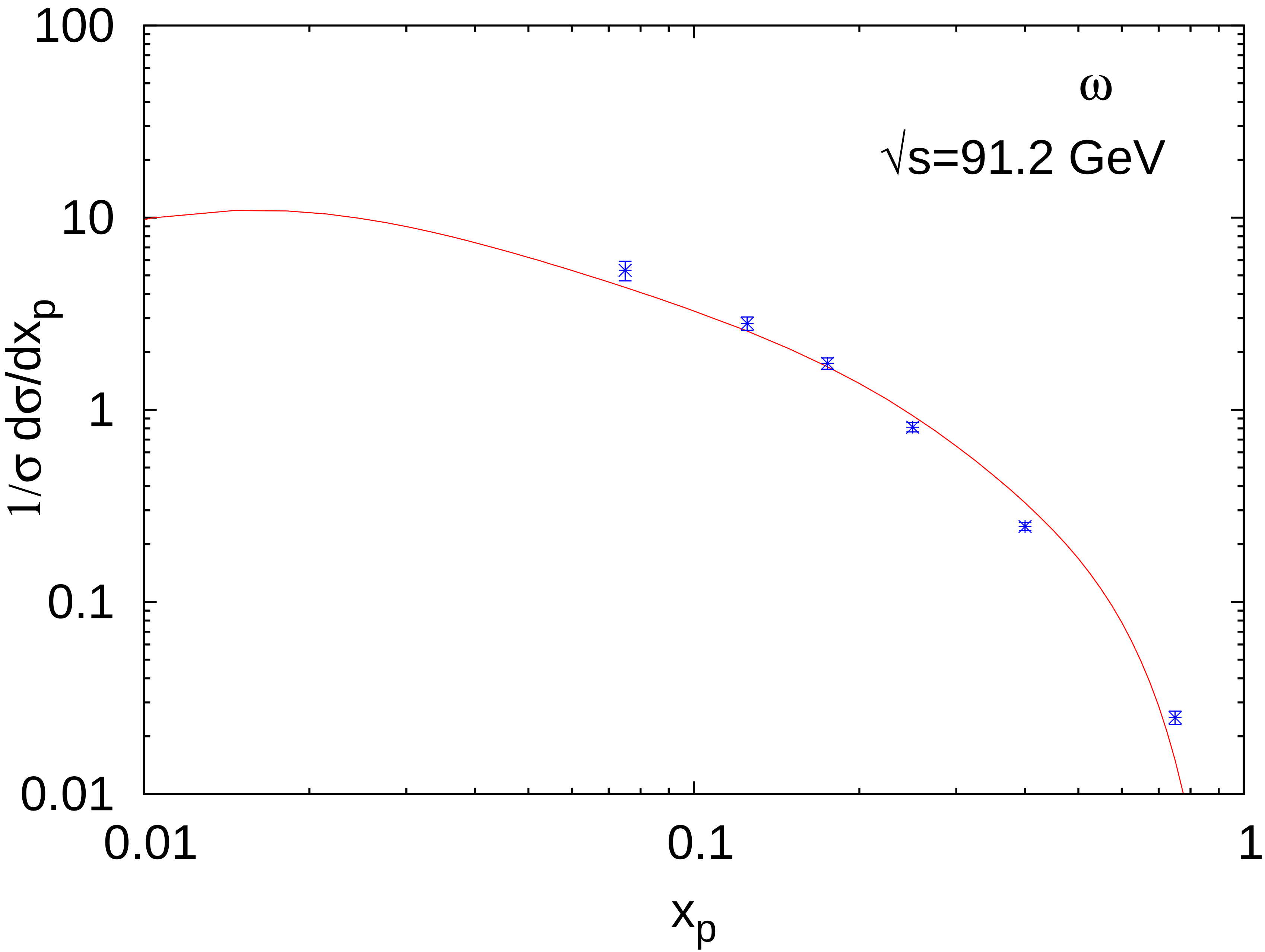
$$D_0^{\omega_1} = f_d^\omega D_0^{\omega_8} \; ; \qquad D_8^{\omega_1} = f_d^\omega D_8^{\omega_8} \; ,$$

  Using the definitions (at the starting scale)

$$D_0^{\omega_8} = D_8^{\omega_8} = V + 12 f_8^\omega \gamma \; ,$$

- we solve to get the fragmentation functions for u and s quarks in the singlet sector.
- We use the known mixing angle $\theta = 35^\circ$; the best fit to the data yield $f_8^\omega = 0.6$ and $f_d^\omega = 1.0$.

Fit to ω (LEP) data



The ϕ meson fits

- The same process has to be followed for ϕ meson which is the state orthogonal to ω. However, its relatively higher mass means that it is not really a "good" member of the nonet.

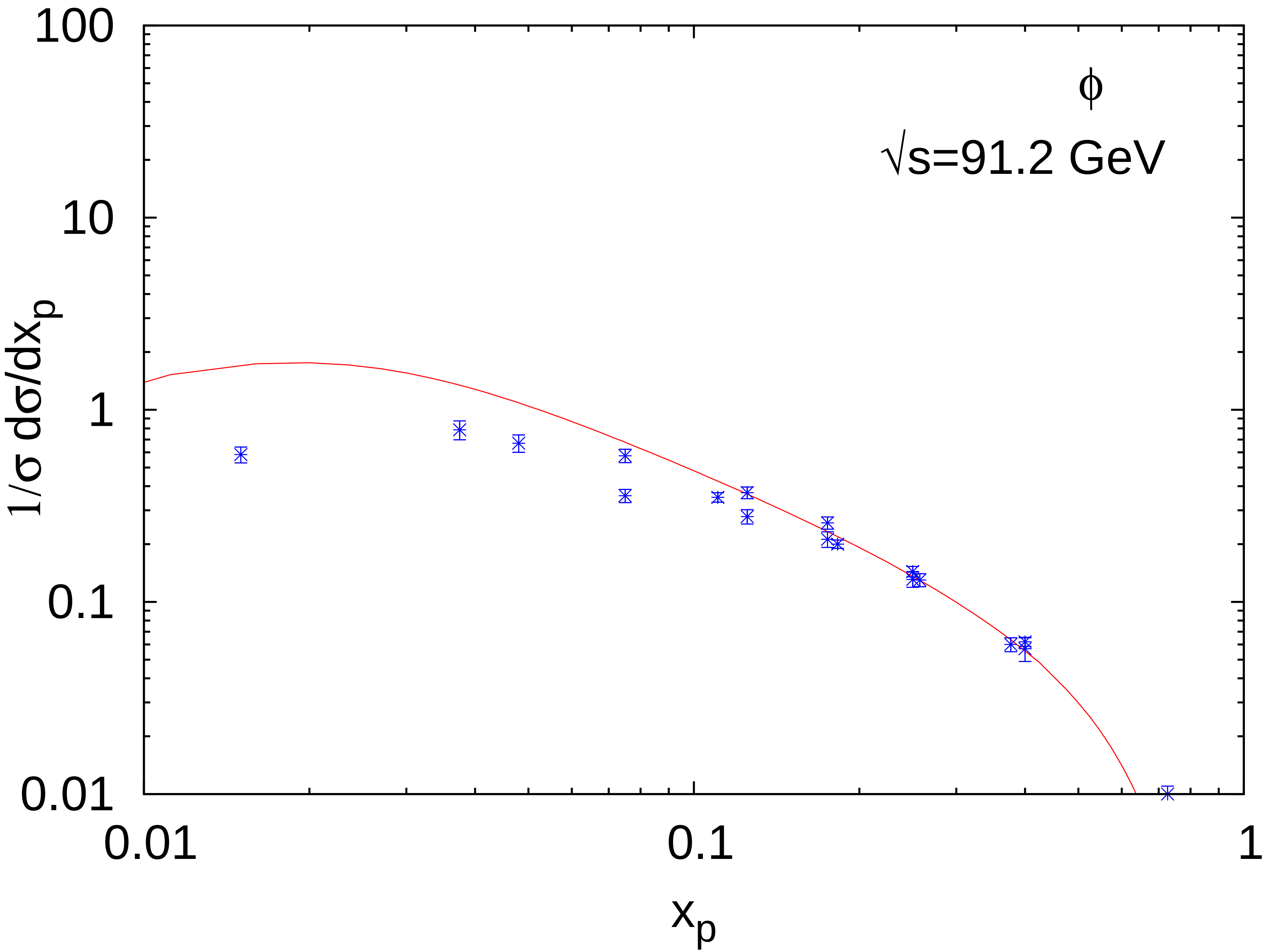
$$
\begin{aligned}
|\phi\rangle &= \cos\theta|\omega_8\rangle - \sin\theta|\omega_1\rangle \; , \\
&= \cos\theta \left| \frac{u\overline{u} + d\overline{d} - 2s\overline{s}}{\sqrt{6}} \right\rangle - \sin\theta \left| \frac{u\overline{u} + d\overline{d} + s\overline{s}}{\sqrt{3}} \right\rangle \; .
\end{aligned}
$$

Again, we relate

$$
\begin{aligned}
D_0^{\phi_1} &= f_d^\phi D_0^{\phi_8} \; , & D_8^{\phi_1} &= f_d^\phi D_8^{\phi_8} \; , \\
\text{where } \; D_0^{\phi_8} &= \lambda V + 3 f_8^\phi \gamma \; , & D_8^{\phi_8} &= -2\lambda V - 6 f_8^\phi \gamma \; .
\end{aligned}
$$

- Solving the above two equations we can get fragmentation functions for u and s quark of singlet sector.
- Using $\theta = 35^0$ as before, we get a best fit of $f_8^\phi = 0.2$ and $f_d^\phi = 0.0$.
- Note that the octet suppression factors for both ω and ϕ are close to half their mass-dependent suppressions: $m_\rho^2/m_{\omega,\phi}^2$.

Fit to ϕ (LEP) data



Result And Discussion

- Using a simple SU(3) model and LO QCD evolution, we fitted the quark fragmentation functions for vector mesons ρ and K^*. The model has high predictive power because fragmentation functions of 7 mesons (ρ^0, ρ^+, ρ^-, K^{*+}, K^{*-}, K^{*0}, $\overline{K^{*0}}$) are parametrised in terms of just two quark fragmentation functions, a gluon fragmentation function, and a single parameter, λ.

- The model was then applied to the ω-ϕ sector. With few parameters describing singlet-octet mixing and no additional fragmentation functions, we got reasonable fits with the data.

- The model is simple and explains the data of the entire vector nonet with very few parameters. Since the results are encouraging, we plan to extend the fits to other data (including $e\,p$ scattering, $p\,p$ scattering, etc).

- Finally, we plan to extend the results using NLO QCD evolution.

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