Four Texture Zero Fermion Mass Matrices in SO (10) GUT

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Four Texture Zero Mass Matrices

$$M_f = \begin{pmatrix} 0 & a_f e^{i\phi_f} & 0 \\ a_f e^{-i\phi_f} & b_f & c_f e^{i\psi_f} \\ 0 & c_f e^{-i\psi_f} & d_f \end{pmatrix} \qquad M_\nu = \begin{pmatrix} 0 & a_\nu & 0 \\ a_\nu & b_\nu & c_\nu \\ 0 & c_\nu & d_\nu \end{pmatrix}$$

- We consider a four texture zero Ansatz in which the charged fermion mass matrix M_f and the neutrino mass matrix M_ν have the same texture zero structure with zero entries at (1,1) and (1,3) places.

  G. C. Branco, D. Emmanuel-Costa, R. Gonza´lezFelipe, H. Serodio, *Phys. Lett. B* 670, 340(2009).

  S. Dev, Sanjeev Kumar, Surender verma and Shivani Gupta, *Mod. Phys. Lett. A* 24, 2251(2009).
- We assume hermitian mass matrices for charged fermions.
- Here $f = e$, u and d for charged leptons, up type and down type quarks respectively.

Four Texture Zero Mass Matrices

- The unitary matrices V_f diagonalize the mass matrices M_f as: $M_f = V_f M_f^d V_f^\dagger$
- Here $M_f^d = (-m_{f1}, m_{f2}, m_{f3})$

  and $V_f = P_f O_f$
- where $P_f = diag(e^{i\phi_f}, 1, e^{i\psi_f})$
- O_f is the diagonalizing matrix for the real symmetric mass matrix $M_f^{(r)} = P_f^\dagger M_f P_f$

Four Texture Zero Mass Matrices

- The orthogonal matrix O_f is:

$$\left.\begin{aligned}
O_{f11} &= \sqrt{\frac{m_{f2}m_{f3}(d_f+m_{f1})}{d_f(-m_{f1}-m_{f2})(-m_{f1}-m_{f3})}} \\
O_{f12} &= \sqrt{\frac{-m_{f1}m_{f3}(d_f-m_{f2})}{d_f(m_{f2}-m_{f3})(m_{f2}+m_{f1})}} \\
O_{f13} &= \sqrt{\frac{-m_{f1}m_{f2}(d_f-m_{f3})}{d_f(m_{f3}-m_{f2})(m_{f3}+m_{f1})}} \\
O_{f21} &= -\sqrt{\frac{m_{f1}(d_f+m_{f1})}{(-m_{f1}-m_{f2})(-m_{f1}-m_{f3})}} \\
O_{f22} &= \sqrt{-\frac{m_{f2}(d_f-m_{f2})}{(m_{f2}-m_{f3})(m_{f2}+m_{f1})}} \\
O_{f23} &= \sqrt{-\frac{m_{f3}(d_f-m_{f3})}{(m_{f3}-m_{f2})(m_{f3}+m_{f1})}} \\
O_{f31} &= \sqrt{\frac{-m_{f1}(d_f-m_{f2})(d_f-m_{f3})}{d_f(-m_{f1}-m_{f2})(-m_{f1}-m_{f3})}} \\
O_{f32} &= -\sqrt{\frac{m_{f2}(d_f-m_{f3})(d_f+m_{f1})}{d_f(m_{f2}-m_{f3})(m_{f2}+m_{f1})}} \\
O_{f33} &= \sqrt{\frac{m_{f3}(d_f+m_{f1})(d_f-m_{f2})}{d_f(m_{f3}-m_{f2})(m_{f3}+m_{f1})}}
\end{aligned}\right\}$$

Four Texture Zero Mass Matrices

- Using the invariants Tr(M_f^r), Tr$(M_f^r)^2$, and Det(M_f^r) we have

$$b_f = -m_{f1} + m_{f2} + m_{f3} - d_f,$$
$$a_f = \left(\frac{m_{f1}m_{f2}m_{f3}}{d_f}\right)^{\frac{1}{2}},$$
$$c_f = \left[-\frac{(d_f + m_{f1})(d_f - m_{f2})(d_f - m_{f3})}{d_f}\right]^{\frac{1}{2}}.$$

- Here the free parameter d_f should be in the range $m_{f2} < d_f < m_{f3}$ for a_f and c_f to be real.

Four Texture Zero Mass Matrices

- The neutrino mass matrix is complex symmetric for which all four elements are in general complex and is diagonalized by a unitary matrix V_ν as $M_\nu = V_\nu M_\nu^{diag} V_\nu^T$.
- The lepton mixing matrix is given by:

$$U_{PMNS} = V_e^\dagger V_\nu$$

- We parameterize $U_{PMNS} = U.P$ in terms of three mixing angles and three CP violating phases.

Four Texture Zero Mass Matrices

- *U* is given as:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

and *P* is given as:

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}$$

The neutrino mass matrix can be written as: $M_\nu = P_e O_e U_{PMNS} M_\nu^{diag} U_{PMNS}^T O_e^T P_e.$

Four Texture Zero Mass Matrices

- For vanishing (1,1) and (1,3) entries in the neutrino mass matrix we get two complex equations: $m_1 a^2 + \tilde{m}_2 b^2 + \tilde{m}_3 c^2 = 0,$

$$m_1 ad + \tilde{m}_2 bg + \tilde{m}_3 ch = 0$$

where $\tilde{m}_2 = m_2 e^{2i\alpha}, \; \tilde{m}_3 = m_3 e^{2i(\beta+\delta)}$

and the complex coefficients *a, b, c, d*, *g* and *h* are given by:

$$\left.\begin{aligned}
a &= O_{e11}U_{e1} + O_{e12}U_{m1} + O_{e13}U_{t1},\\
b &= O_{e11}U_{e2} + O_{e12}U_{m2} + O_{e13}U_{t2},\\
c &= O_{e11}U_{e3} + O_{e12}U_{m3} + O_{e13}U_{t3},\\
d &= O_{e31}U_{e1} + O_{e32}U_{m1} + O_{e33}U_{t1},\\
g &= O_{e31}U_{e2} + O_{e32}U_{m2} + O_{e33}U_{t2},\\
h &= O_{e31}U_{e3} + O_{e32}U_{m3} + O_{e33}U_{t3}.
\end{aligned}\right\}$$

Four Texture Zero Mass Matrices

- Solving the two texture zero equations for two mass ratios, we obtain:

$$\frac{m_1}{m_2} e^{-2i\alpha} = \frac{b(cg-bh)}{a(ah-cd)},$$

$$\frac{m_1}{m_3} e^{-2i\beta} = \frac{c(bh-cg)}{a(ag-bd)} e^{2i\delta}$$

- These equations have parameters from both the charged lepton sector and the neutrino sector.

Four Texture Zero Mass Matrices

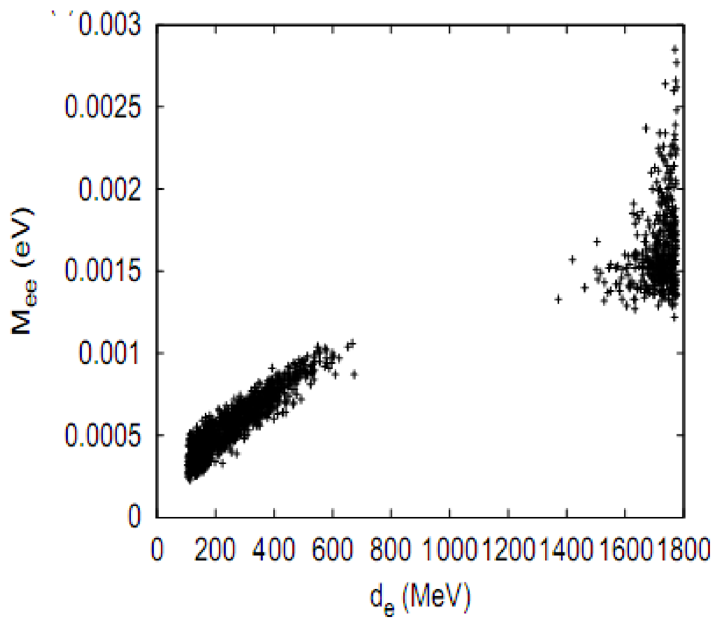
- For the simultaneous existence of texture zeros at (1,1) and (1,3) positions, the two values of m_1 given by:

$$m_1 = \left|\frac{m_1}{m_3}\right| \sqrt{\frac{\Delta m_{12}^2 + \Delta m_{23}^2}{1 - \left|\frac{m_1}{m_3}\right|^2}}, \quad m_1 = \left|\frac{m_1}{m_2}\right| \sqrt{\frac{\Delta m_{12}^2}{1 - \left|\frac{m_1}{m_2}\right|^2}}$$

calculated using the mass ratios $\left(\frac{m_1}{m_2}, \frac{m_1}{m_3}\right)$ must be equal to within the errors of oscillation data.

Four Texture Zero Mass Matrices

- Using these equations and the experimental inputs of charged lepton masses, two mass squared differences from neutrino sector and the lepton mixing angles we get two allowed regions for d_e from its full range.
- From the charged lepton sector full range of d_e was allowed but the input of texture zeros in the neutrino sector restricted this range to two small regions



SO(10) GUT

- A desirable feature of a complete model of quark and lepton masses and mixings is that it should be consistent with an underlying Grand Unified Theory (GUT)
- In present work we use Supersymmetric SO(10) GUT.
- Fermion masses arise in renormalizable SO(10) models through their couplings to Higgs fields transforming as 10, $\overline{126}$ and 120 representations.

Four Texture Zero and SO(10) GUT

- In the next step we combine the four zero texture Ansatze with the constraints obtained from SO(10) GUT.
- Such an analysis will require renormalization group (RG) running from weak scale to the GUT scale.
- However, it is known that the effects of RG running are negligible for mass matrices with normal hierarchy.
- Having obtained the texture zero constraints on the free parameter d_e, we now further limit it by imposing SO(10) relations.

Four Texture Zero and SO(10) GUT

- Six mass matrices included in SO(10) have the form:

$$M_u = S + \delta'' A + \epsilon S' \equiv S_u + A_u,$$
$$M_d = \eta S + \delta' A + S' \equiv S_d + A_d,$$
$$M_e = \eta S + A - 3S' \equiv S_e + A_e,$$
$$M_D = S + \delta''' A - 3\epsilon S' \equiv S_D + A_D,$$
$$M_L = \rho S' \equiv S_L,$$
$$M_R = \gamma S' \equiv S_R.$$

- Where *S*, *S'* are the symmetric parts coming from the 10 and $\overline{126}$ Higgs fields, *A* is the antisymmetric part coming from 120 Higgs field.
- Here $\eta,\ \epsilon,\ \rho,\ \gamma,\ \delta',\ \delta''$ and δ''' represent the relative coefficients of the vacuum expectation values (vevs)

Four Texture Zero and SO(10) GUT

- There are additional constraints among the parameters in the four texture zero model, when it is embedded in SO(10).
- We first discuss the constraints in the quarks and the charged lepton sector.
- S_f and A_f ($f = u, d, e$) now satisfy:

$$4\eta S_u = (3 + \eta\epsilon)S_d + (1 - \eta\epsilon)S_e,$$

$$\delta' A_u = \delta'' A_d = \delta'\delta'' A_e.$$

Four Texture Zero and SO(10) GUT

- In the component form these equations can be written as:

$$4\eta a_u \cos(\Delta\phi + \phi_d) = (3 + \eta\epsilon)a_d \cos\phi_d + (1 - \eta\epsilon)a_e \cos\phi_e,$$
$$4\eta c_u \cos(\Delta\psi + \psi_d) = (3 + \eta\epsilon)c_d \cos\psi_d + (1 - \eta\epsilon)c_e \cos\psi_e,$$
$$4\eta b_u = (3 + \eta\epsilon)b_d + (1 - \eta\epsilon)b_e,$$
$$4\eta d_u = (3 + \eta\epsilon)d_d + (1 - \eta\epsilon)d_e,$$
$$\delta' a_u \sin(\Delta\phi + \phi_d) = \delta'' a_d \sin\phi_d = \delta'\delta'' a_e \sin\phi_e,$$
$$\delta' c_u \sin(\Delta\psi + \psi_d) = \delta'' c_d \sin\psi_d = \delta'\delta'' c_e \sin\psi_e.$$

- $\Delta\phi = \phi_u - \phi_d$ and $\Delta\psi = \psi_u - \psi_d$
- We have eight independent equations.

Four Texture Zero and SO(10) GUT

- We have 12 parameters coming from S_f ($f = u, d, e$), 6 from A_f, 4 are the parameters δ', η, ϵ and δ'' totaling 22
- Whereas we have the experimental input of 9 charged fermion masses, 3 CKM mixing angles and the Dirac phase, totaling 13.
- Using above 8 equations we have in total 21 inputs.
- We are left with only one unknown parameter.

Four Texture Zero and SO(10) GUT

- The parameters d_u, d_d, $\Delta\Phi$ and $\Delta\Psi$ can be constrained by the observed CKM mixing matrix, the best fit values realized for these parameters are given by :

Koichi Matsuda, Hiroyuki Nishiura *Phys. Rev. D* 74, 033014 (2006)

$$\Delta\phi = \frac{\pi}{2},$$

$$\Delta\psi = -0.12,$$

$$d_u = 0.95 m_t,$$

$$d_d = 0.94 m_b.$$

Four Texture Zero and SO(10) GUT

- Using the eight equations we have

$$F^2\left(4\alpha a_u \cos\left(\Delta\phi+\phi_d\right)-(3+\kappa)a_d\cos\phi_d\right)^2-$$
$$\left(4\alpha c_u \cos\left(\Delta\psi+\psi_d\right)-(3+\kappa)c_d\cos\psi_d\right)^2=(1-\kappa)^2\left(a_e^2F^2-c_e^2\right)$$

- Where $F \equiv \frac{c_d sin\psi_d}{a_d sin\phi_d} = \frac{c_e sin\psi_e}{a_e sin\phi_e}.$

$$\cos\phi_e \equiv \frac{4\eta a_u \cos\left(\Delta\phi+\phi_d\right)-(3+\kappa)a_d\cos\phi_d}{(1-\kappa)a_e},$$

$$\cos\psi_e \equiv \frac{4\eta c_u \cos\left(\Delta\psi+\psi_d\right)-(3+\kappa)c_d\cos\psi_d}{(1-\kappa)c_e}.$$

Four Texture Zero and SO(10) GUT

$$\tan\phi_d = \frac{a_u \sin\Delta\phi}{r a_d - a_u \cos\Delta\phi},$$

$$\tan\psi_d = \frac{c_u \sin\Delta\psi}{r c_d - c_u \cos\Delta\psi}.$$

- Where $\kappa = \alpha\eta$ and $r = \frac{\delta''}{\delta'}$

$$\eta = \frac{(m_d + m_s + m_b)d_e - (m_e + m_\mu + m_\tau)d_d}{(m_d + m_s + m_b - m_e - m_\mu - m_\tau)d_u - (m_u + m_c + m_t)(d_d - d_e)},$$

$$\kappa = \frac{(m_u + m_c + m_t)(3d_d + d_e) - (3(m_d + m_s + m_b) + (m_e + m_\mu + m_\tau))d_u}{(m_d + m_s + m_b - m_e - m_\mu - m_\tau)d_u - (m_u + m_c + m_t)(d_d - d_e)}.$$

Four Texture Zero and SO(10) GUT

- In our numerical analysis we use the best fit values for quarks and charged leptons estimated at the unification scale given in:

  Takeshi Fukuyama, Koichi Matsuda and Hiroyuki Nishiura, *Int. J. Mod. Phys. A 22, 5325-5343 (2007).*

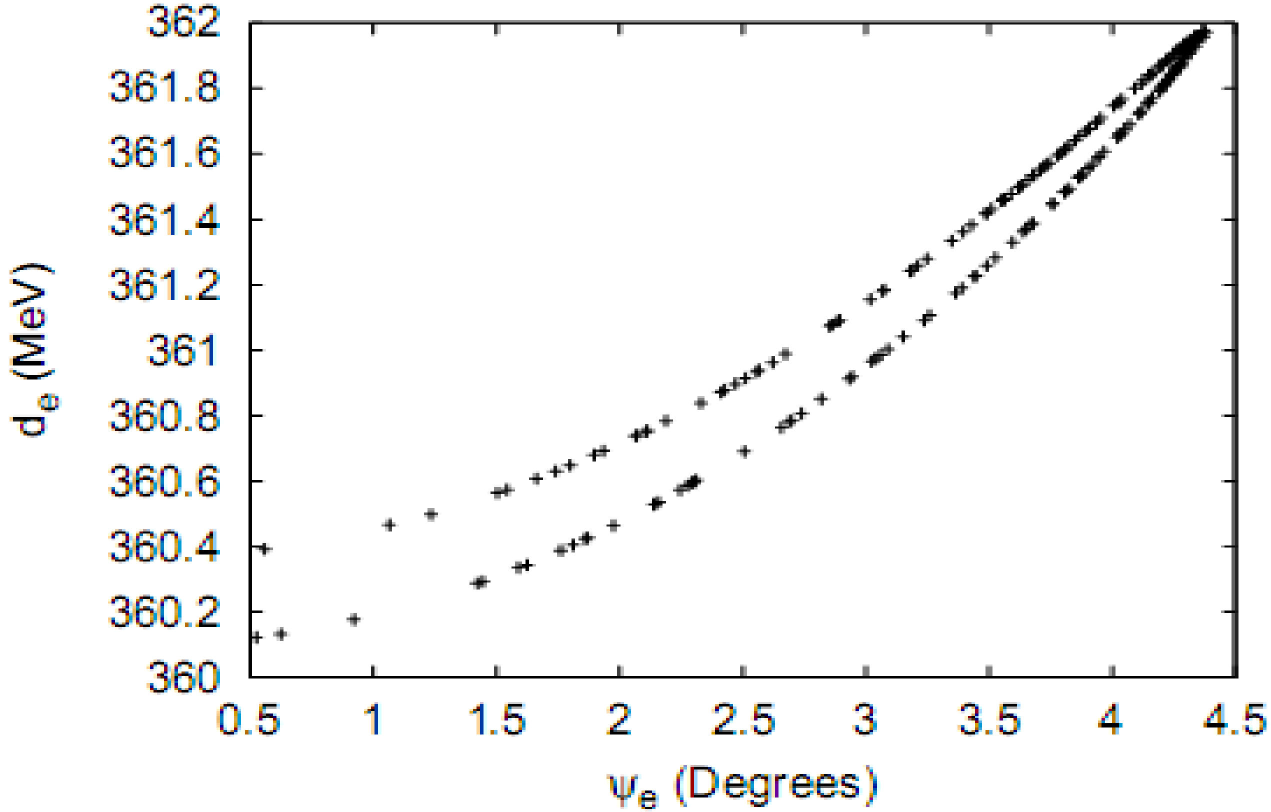
- We vary the parameter *r* freely
- Using the phenomenologically allowed regions of the free parameter d_e, along with the SO(10) relations, we get a constrained d_e- *r* plane.

Four Texture Zero and SO(10) GUT

- Now only one of the two regions of d_e is allowed while in earlier analysis done by
  Takeshi Fukuyama, Koichi Matsuda and Hiroyuki Nishiura, *Int. J. Mod. Phys. A 22, 5325-5343 (2007).*
  two regions were allowed after the input of SO(10) as the information of texture zeros in the neutrino mass matrix was not utilized. They used full range of the parameter d_e i.e. $m_{e2} < d_e < m_{e3}$

Four Texture Zero and SO(10) GUT

- A small range of d_e is now permissible



Four Texture Zero and SO(10) GUT

- So far we have succeeded in fitting the unknown parameters of the charged fermions sector.
- In the neutrino sector we have three more unknown parameters ρ, γ and δ'''.
- The light neutrino mass matrix from type-**I** + type-**II** seesaw mechanism is given by:

$$M_\nu = M_L - M_D M_R^{-1} M_D^T$$

Four Texture Zero and SO(10) GUT

- The lepton mixing matrix is given by:

$$U_{PMNS} = V_e^\dagger V_\nu$$

- Diagonalization of charged lepton mass matrix is the same as discussed earlier.
- The neutrino mass matrix M_ν after seesaw is complex symmetric.
- According to Takagi Factorization M_ν can be diagonalized as: $M_\nu = V_\nu M_\nu^{diag} V_\nu^T$

Takagi, *Japan J. Math.* 1, 83 (1923); R. A. Horn and C. R. Johnson, *Matrix Analysis* (Cambridge University Press)

Four Texture Zero and SO(10) GUT

- The lepton mixing angles can be obtained from U_{PMNS} as:

$$tan^2\theta_{12} = \frac{|(U_{PMNS})_{12}|^2}{|(U_{PMNS})_{11}|^2},$$

$$sin^2\theta_{23} = 4|(U_{PMNS})_{23}|^2|(U_{PMNS})_{33}|^2,$$

and

$$sin\theta_{13} = |(U_{PMNS})_{13}|$$

- To obtain consistent values of the lepton mixing angles we have to fine tune the three unknown parameters ρ, γ and δ'''.

Four Texture Zero and SO(10) GUT

- For the value of d_e = 360.545 *MeV* we obtain the following mixing angles:

$$\theta_{12} = 34.02^o$$

$$\theta_{23} = 42.97^o$$

$$\theta_{13} = 2.39^o$$

- The unknown parameters are fine tuned to be $\rho\gamma = 3.4533 \times 10^5$ and $\delta''' = 1.533$

Four Texture Zero and SO(10) GUT

- However we obtain a small value for

$$R_\nu = \frac{\Delta m^2_{12}}{|\Delta m^2_{23}|} = 2.4998 \times 10^{-7}$$

- Where as the experimental range of this parameter is :

$$2.540 \times 10^{-3} < R_\nu < 3.975 \times 10^{-3}$$

Thank You