Neutrino masses and different mixing scenario with charged lepton correction

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Objective

- Different mixings viz. Tri-bi-maximal, bi-maximal and Hexagonal can predict the neutrino oscillation parameters with correction.
- Corrected mixing matrix also fulfill the property $U=U^T$ and the Quark Lepton Complementary.

Neutrino Mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left(U_{PMNS} \right) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

↑ Flavor Eigenstate ↑ Mass Eigenstate

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \Rightarrow \theta_{13}, \theta_{12}, \theta_{23}$$

Different Mixings

- Bi-maximal :

$$U_{BM}=\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\sin^2\theta_{12}=\frac{1}{2},\quad \sin^2\theta_{23}=\frac{1}{2}$$

and $\sin\theta_{13}=0$

- Tri-Bi-maximal :

$$U_{TBM}=\begin{pmatrix}\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0\\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}\\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}\end{pmatrix}$$

$$\sin^2\theta_{12}=\frac{1}{3},\quad \sin^2\theta_{23}=\frac{1}{2}$$

and $\sin\theta_{13}=0$

- Hexagonal :

$$U_{HM}=\begin{pmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2} & 0\\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\sin^2\theta_{12}=\frac{1}{4},\quad \sin^2\theta_{23}=\frac{1}{2}$$

and $\sin\theta_{13}=0$

(arXiv:1004.2798)

Tri-bi-maximal

- We use three different kinds of neutrino mass models for our study:

1. Degenerate: $(m_1 \approx m_2 \approx m_3)$

(i)

$$m_{LL} = \begin{pmatrix} \delta_1 - 2\delta_2 & -\delta_1 & -\delta_1 \\ -\delta_1 & \frac{1}{2} - \delta_2 & -\frac{1}{2}\delta_2 \\ -\delta_1 & -\frac{1}{2} - \delta_2 & \frac{1}{2} - \delta_2 \end{pmatrix} m_0.$$

(ii)

$$m_{LL} = \begin{pmatrix} 1 - \delta_1 - 2\delta_2 & \delta_1 & \delta_1 \\ \delta_1 & 1 - \delta_2 & -\delta_2 \\ \delta_1 & -\delta_2 & 1 - \delta_2 \end{pmatrix} m_0.$$

(iii)

$$m_{LL} = \begin{pmatrix} 1 - \delta_1 - 2\delta_2 & \delta_1 & \delta_1 \\ \delta_1 & -\delta_2 & 1 - \delta_2 \\ \delta_1 & 1 - \delta_2 & -\delta_2 \end{pmatrix} m_0.$$

Cont..

2. Inverted hierarchical

(m_1, m_2>> m_3)

$$m_{LL} = \begin{pmatrix} 1+2\delta_1 & \delta_1 & \delta_1 \\ \delta_1 & \frac{1}{2} & \frac{1}{2}+\delta_2 \\ \delta_1 & \frac{1}{2}+\delta_2 & \frac{1}{2} \end{pmatrix} m_0.$$

$$m_{LL} = \begin{pmatrix} \alpha_1 & 1 & 1 \\ 1 & \alpha_2 & \alpha_3 \\ 1 & \alpha_3 & \alpha_2 \end{pmatrix} m_0'.$$

3. Normal hierarchical

(m_1, m_2<< m_3)

$$m_{LL} = \begin{pmatrix} -\delta_2 & -\delta_1 & -\delta_1 \\ -\delta_1 & 1-\delta_1 & -1 \\ -\delta_1 & -1+\delta_2 & 1-\delta_1 \end{pmatrix} m_0.$$

- After diagonalising these mass matrices we have the following data:

Type	$\nabla^2 m^2_{21}[10^{-5} eV^2]$	$\nabla^2 m^2_{23}[10^{-3} eV^2]$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$
I(A)	7.84	2.60	0.33	0.5	0
I(B)	7.94	2.47	0.33	0.5	0
I(C)	7.94	2.47	0.33	0.5	0
II(A)	8.36	2.61	0.33	0.5	0
II(B)	8.33	2.40	0.33	0.5	0
III	8.36	2.88	0.33	0.5	0

- Using the following parameters

Type	δ_1	δ_2	m_o
I(A)	0.6611	0.1653	0.4
I(B)	8.31 10^{-5}	0.0039	0.4
I(C)	8.31 10^{-5}	0.00395	0.4
II(A)	0.0055	0.5165	0.4
II(B)	0.6610	0.6610	0.5
III	0.176	0.0	0.03

Satisfy TBM conditions

• **To get deviation from TBM we adopted the method known as "charged lepton correction":**

In this method one can write the *U_{PMNS}* matrix as

$$U_{PMNS} = U_l^{+} U_\nu$$

(Ref. :Phys.Lett.B654:177-188,2007)

where,

$$U_\nu = \begin{pmatrix} c_{13}^\nu c_{12}^\nu & c_{13}^\nu s_{12}^\nu & s_{13}^\nu \\ -c_{23}^\nu s_{12}^\nu - c_{12}^\nu s_{13}^\nu s_{23}^\nu & c_{12}^\nu c_{23}^\nu - s_{12}^\nu s_{13}^\nu s_{23}^\nu & c_{13}^\nu s_{23}^\nu \\ s_{12}^\nu s_{23}^\nu - c_{12}^\nu s_{13}^\nu c_{23}^\nu & -c_{12}^\nu s_{23}^\nu - c_{23}^\nu s_{13}^\nu s_{12}^\nu & c_{13}^\nu c_{23}^\nu \end{pmatrix}$$

← Neutrino part

$$U_l = \begin{pmatrix} c_{13}^l c_{12}^l & c_{13}^l s_{12}^l & s_{13}^l \\ -c_{23}^l s_{12}^l - c_{12}^l s_{13}^l s_{23}^l & c_{12}^l c_{23}^l - s_{12}^l s_{13}^l s_{23}^l & c_{13}^l s_{23}^l \\ s_{12}^l s_{23}^l - c_{12}^l s_{13}^l c_{23}^l & -c_{12}^l s_{23}^l - c_{23}^l s_{13}^l s_{12}^l & c_{13}^l c_{23}^l \end{pmatrix}$$

← Charged lepton part

- **Calculation of Neutrino part** (U_ν)**:** **(Ref. JHEP 09 (2002) 011)**

1. Calculation of θ_{23}^{ν} **:**

$$\tan 2\theta_{23}^{\nu} = \frac{2\left|m_{23}^{\nu}\right|}{\left|m_{33}^{\nu}\right| - \left|m_{22}^{\nu}\right|}$$

2. Calculation of θ_{13}^{ν} **:**

$$\theta_{13}^{\nu} = \frac{\tilde{m}_{13}}{\tilde{m}'_3}$$

where

$$\tilde{m}'_3 = \left(s_{23}^{\nu}\right)^2\left|m_{22}^{\nu}\right| + 2 s_{23}^{\nu} c_{23}^{\nu}\left|m_{23}^{\nu}\right| + \left(c_{23}^{\nu}\right)^2\left|m_{33}^{\nu}\right|$$

3. Calculation of θ_{12}^{ν} **:**

$$\tan 2\theta_{12}^{\nu} = \frac{2\left|\tilde{m}_{12}^{\nu}\right|}{\left|\tilde{m}_{22}^{\nu}\right| - \left|\tilde{m}_{11}^{\nu}\right|}$$

where

$$\begin{pmatrix} \tilde{m}_{12}^{\nu} \\ \tilde{m}_{13}^{\nu} \end{pmatrix} = \left(R_{23}^{\nu}\right)^T \begin{pmatrix} \left|m_{12}^{\nu}\right| \\ \left|m_{13}^{\nu}\right| \end{pmatrix}$$

$$\tilde{m}_{22}^{\nu} = \left(c_{23}^{\nu}\right)^2\left|m_{22}^{\nu}\right| + 2 s_{23}^{\nu} c_{23}^{\nu}\left|m_{23}^{\nu}\right| + \left(s_{23}^{\nu}\right)^2\left|m_{33}^{\nu}\right|$$

$$R_{23}^{\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^{\nu} & s_{23}^{\nu} \\ 0 & -s_{23}^{\nu} & c_{23}^{\nu} \end{pmatrix}$$

and

$$\tilde{m}_{11}^{\nu} = m_{11}^{\nu} - \frac{(\tilde{m}_{13}^{\nu})^2}{\tilde{m}'_3}$$

Calculation of charged lepton part (U_l) :

We use the following relations : ***(PRD 80, 053013 (2009))***

$$\sin\theta_{13}^{l} = \lambda \qquad \sin\theta_{23}^{l} = \lambda^2 A \qquad \sin\theta_{12}^{l} = \lambda^3 B$$

- **By using charged leptonic part and neutrino part one can construct the U_{PMNS}:**

$$U_{PMNS} = U_l^{+} U_\nu$$

From this U_{PMNS} one can extract the $\theta_{13}, \theta_{12}, \theta_{23}$ values which are consistent with present oscillation data.

Bi-maximal

- Normal hierarchical

$$m_{LL} = \begin{pmatrix} -\eta^4 & \eta & \eta^3 \\ -\eta & 1-\eta & -1 \\ -\eta^3 & -1 & 1-\eta^3 \end{pmatrix} m_0$$

With zero order

$$m_{LL}{}^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} m_0$$

With $m_0 = 0.03$ and $\eta = 0.3$

Result

- Tri-bi-maximal mixing

Ranges	**Inverted hierarchical**	**Normal Hierarchical**
Range of *A* and *B*	0.25-5	0.25-5
Range of λ	0.104-0.247	0.104-0.247
λ value used	0.2	0.18
A and *B* value used	0.3	0.3
$\|Ue3\|^2$	0.064	0.049
$sin^2\theta_{12}$	0.27	0.29
$sin^2\theta_{23}$	0.49	0.51

Cont..

• Bi-maximal & Hexagonal mixing

Ranges	Hexagonal (Model Independent)	Bimaximal Normal Hierarchical
Range of A and B	0.25-5	0.25-5
Range of λ	0.104-0.247	0.104-0.247
λ value used	0.16	0.18
A and B value used	0.3	0.25
$\|Ue3\|^2$	0.061	0.047
$sin^2\theta_{12}$	0.31	0.26
$sin^2\theta_{23}$	0.42	0.44

Recent neutrino oscillation data

(Ref.: PRD 79, 076006 (2009))

Parameters	Best fit values	3σ range
$\nabla^2 m_{21}^2 [10^{-5} eV^2]$	7.65	7.05-8.34
$\nabla^2 m_{23}^2 [10^{-3} eV^2]$	2.40	2.07-2.75
$\sin^2\theta_{12}$	0.304	0.25-0.37
$\sin^2\theta_{23}$	0.5	0.36-0.67
$\sin^2\theta_{13}$	0.01	≤0.067

Cont..

- By this idea of correction we find another mixing pattern that satisfy symmetric property of U i.e. *$U=U^T$*
- Which gives *(PLB 644 (2007) 147)*

$$|U_{e3}| = |U_{\tau 1}|, \quad |U_{e2}| = |U_{\mu 1}| \quad \textit{and} \quad |U_{\mu 3}| = |U_{\tau 2}|.$$

Then $$|U_{\mu 3}|^2 - |U_{\tau 2}|^2 = |U_{e2}|^2 - |U_{\mu 1}|^2 = |U_{\mu 3}|^2 - |U_{\tau 2}|^2$$

$$|U_{e3}| = \frac{\sin\theta_{12}\sin\theta_{23}}{\sqrt{1-\sin^2\delta\cos^2}\theta_{12}\cos^2\theta_{23} + \cos\delta\cos\theta_{12}\cos\theta_{23}}$$

= 0.26, using λ= 0.18 and A, B = 0.25

Conclusion

- There is a good scope for extension of this work with CP violating phases and also one can get the sizable values with running of renormalization group equation.
- Tri-bi-maximal mixing provides a very close description of neutrino mixing angles.
- Also one can get mixings based on the golden ratio angle:

$$\cot\theta_{12} = \varphi \Rightarrow \sin^2\theta_{12} = \frac{1}{1+\varphi^2} \simeq 0.276\,,$$

$$\cos\theta_{12} = \frac{\varphi}{2} \Rightarrow \sin^2\theta_{12} = \frac{1}{4}(3-\varphi) \simeq 0.345$$

- This work will be useful for building of neutrino mass model.

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Thank You