Non-Stationary evolution of Primordial Black Holes in Brans-Dicke theory

BY

BIBEKANANDA NAYAK

UTKAL UNIVERSITY, BHUBANESWAR-4

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PLAN OF THE TALK

- Introduction
- Non-stationarity and Accretion
- PBH dynamics in different eras
- Constraint formalism
- Conclusion

BRANS-DICKE THEORY

For a flat FRW universe filled with perfect fluid, the BD gravitational field equations are

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} - \frac{\omega}{6}\frac{\dot{\Phi}^2}{\Phi^2} = \frac{8\pi\rho}{3\Phi}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} + \frac{\omega}{2}\frac{\dot{\Phi}^2}{\Phi^2} + \frac{\ddot{\Phi}}{\Phi} = -\frac{8\pi p}{\Phi}$$

The equation for scalar field $\Phi(t)$ is

$$\frac{\ddot{\Phi}}{8\pi} + 3\frac{\dot{a}}{a}\frac{\dot{\Phi}}{8\pi} = \frac{\rho - 3p}{2\omega + 3}$$

The solutions for a and G for different eras

(Barrow and Carr, Phys. Rev. D 54, 3920 (1996))

$$a(t) \propto \begin{cases} t^{(1-\sqrt{n})/3} & (t < t_1) \\ t^{1/2} & (t_1 < t < t_e) \\ t^{(2-n)/3} & (t > t_e) \end{cases}$$

and

$$G(t) = \begin{cases} G_0 \left(\frac{t_1}{t}\right)^{\sqrt{n}} \left(\frac{t_0}{t_e}\right)^{n} & (t < t_1) \\ G_0 \left(\frac{t_0}{t_e}\right)^{n} & (t_1 < t < t_e) \\ G_0 \left(\frac{t_0}{t}\right)^{n} & (t > t_e) \end{cases}$$

where $n = \frac{2}{4+3\omega}$ $(n \leq 0.00007)$

PRIMORDIAL BLACK HOLE

- Formed in early Universe
- Small mass
- Could be evaporated completely due to Hawking radiation
- Could account for Baryogenesis
- Could act as seeds for Structure Formation
- Could form a significant component of Dark Matter

ACCRETION OF RADIATION

Accretion :- Growth by accumulation

PBHs' mass increases with the rate

$$\dot{M}_{acc} = 4\pi f R_{BH}^{2}\,\rho_R$$

where $R_{BH} = 2GM$, PBH radius

$\rho_R = \frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2$, radiation energy density surrounding the PBH

f is the accretion efficiency

The accreting mass in radiation-dominated era

$$M(t) = \left[M_i^{-1} + \tfrac{3}{2} f\eta G_0 (\tfrac{t_0}{t_e})^n (\tfrac{1}{t} - \tfrac{1}{t_i}) \right]^{-1}$$

where $M_i \approx \eta G^{-1}(t_i) t_i$ initial mass of PBH

with η is the fraction of horizon mass the black hole comprises capturing ***non-stationarity*** in the formation process

Horizon mass grows as $M_H \sim G^{-1} t$

For large time $M(t) = \dfrac{M_i}{1 - \frac{3}{2} f\eta}$

which gives $f\eta < \dfrac{2}{3}$

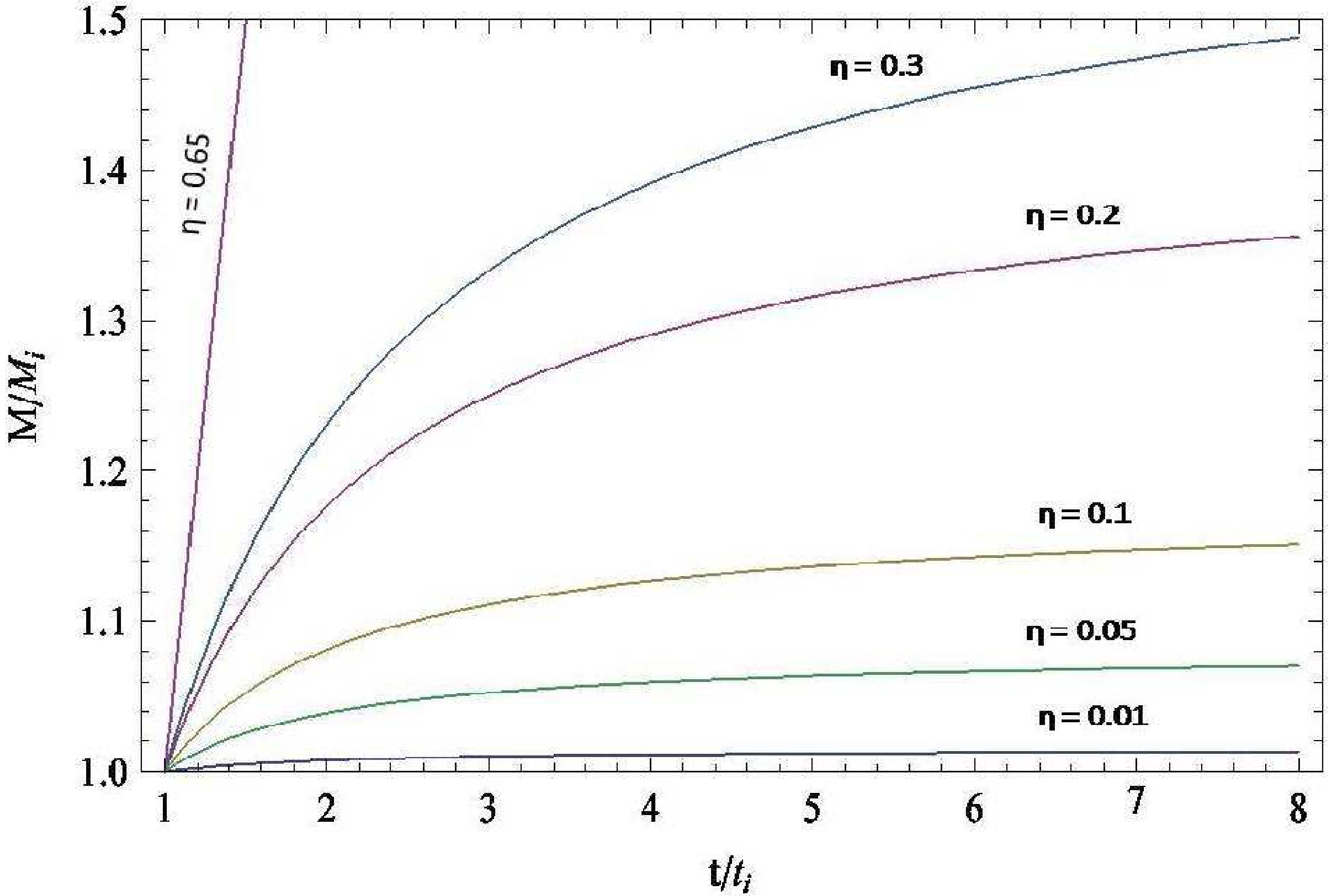

Fig 1 : Variation of accreting mass with different value of η = 0.65, 0.3, 0.2, 0.1, 0.05, 0.01 but with same accretion efficiency f=1

EVAPORATION+ACCRETION

Because of Hawking radiation, PBH mass decreases with the rate

$$\dot{M}_{evap} = -4\pi R_{BH}^2 a_H T_{BH}^4$$

where $T_{BH} = \frac{1}{8\pi GM}$, Hawking Temperature

Inclusion of accretion gives

$$\dot{M}_{BH} = 6fG\left(\frac{\dot{a}}{a}\right)^2 M^2 - \frac{a_H}{256\pi^3}\frac{1}{G^2M^2}$$

PBH DYNAMICS IN DIFFERENT ERAS

A. $t < t_1$

Hawking evaporation rate is

$$\dot{M}_{evap} = -\alpha \left(\frac{t_e}{t_0}\right)^{2n} \left(\frac{1}{t_1}\right)^{2\sqrt{n}} \left(\frac{t^{2\sqrt{n}}}{M^2}\right)$$

where $\alpha = \frac{a_H}{256\pi^3}$

=> Evaporation time

$$\tau = \left[(3\alpha)^{-1} (1 + 2\sqrt{n}) \left(\frac{t_0}{t_e}\right)^{2n} M_i^3 t_1^{2\sqrt{n}} + t_i^{1+2\sqrt{n}} \right]^{1/(1+2\sqrt{n}\}}$$

B. RADIATION-DOMINATED ERA $(t_1 < t < t_e)$

(a) PBH created before t_1

(b) PBH created after t_1

During radiation dominated era PBH follows the eq.

$$\dot{M}_{BH} = \frac{3}{2} f G_0 \left(\frac{t_0}{t_e}\right)^n \left(\frac{M^2}{t^2}\right) - \alpha \left(\frac{t_e}{t_0}\right)^{2n} \frac{1}{M^2}$$

PBHs formed before radiation-dominated era are completely evaporated during this era.

But for PBHs

(i) formed in the radiation-dominated era

and (ii) $M_i^2 > \frac{a_H G^{-1}}{384 f}$

accretion dominates over evaporation

At t=t_c , $$\frac{3}{2} f G_0 \left(\frac{t_0}{t_e}\right)^n \left(\frac{M_{\max}}{t^2}\right) = \alpha \left(\frac{t_e}{t_0}\right)^{2n} \left(\frac{1}{M_{\max}^2}\right)$$

But according to PBH accretion equation

$$M_{\max} = M_i \left[1 + \frac{3}{2} f\eta \left(\frac{t_i}{t_c} - 1\right)\right]^{-1}$$

These two equations give ⟹ $$M_{\max} = \frac{M_i}{1 - \frac{3}{2} f\eta}$$

Considering evaporation from t_c onwards

$$M = M_{\max}\left[1+3\alpha\left(\frac{t_e}{t_0}\right)^{2n}\left(\frac{t_c}{M_{\max}^3}\right)\left\{1-\left(\frac{t}{t_c}\right)\right\}\right]^{1/3}$$

$\Rightarrow$Evaporation time

$$t_{evap} = t_c\left[1+(3\alpha)\left(\frac{t_e}{t_0}\right)^{2n}\left(\frac{M_{\max}^3}{t_c}\right)\right]$$

B. Nayak, L. P. Singh and A. S. Majumdar, Phys. Rev. D 80, 023529 (2009)

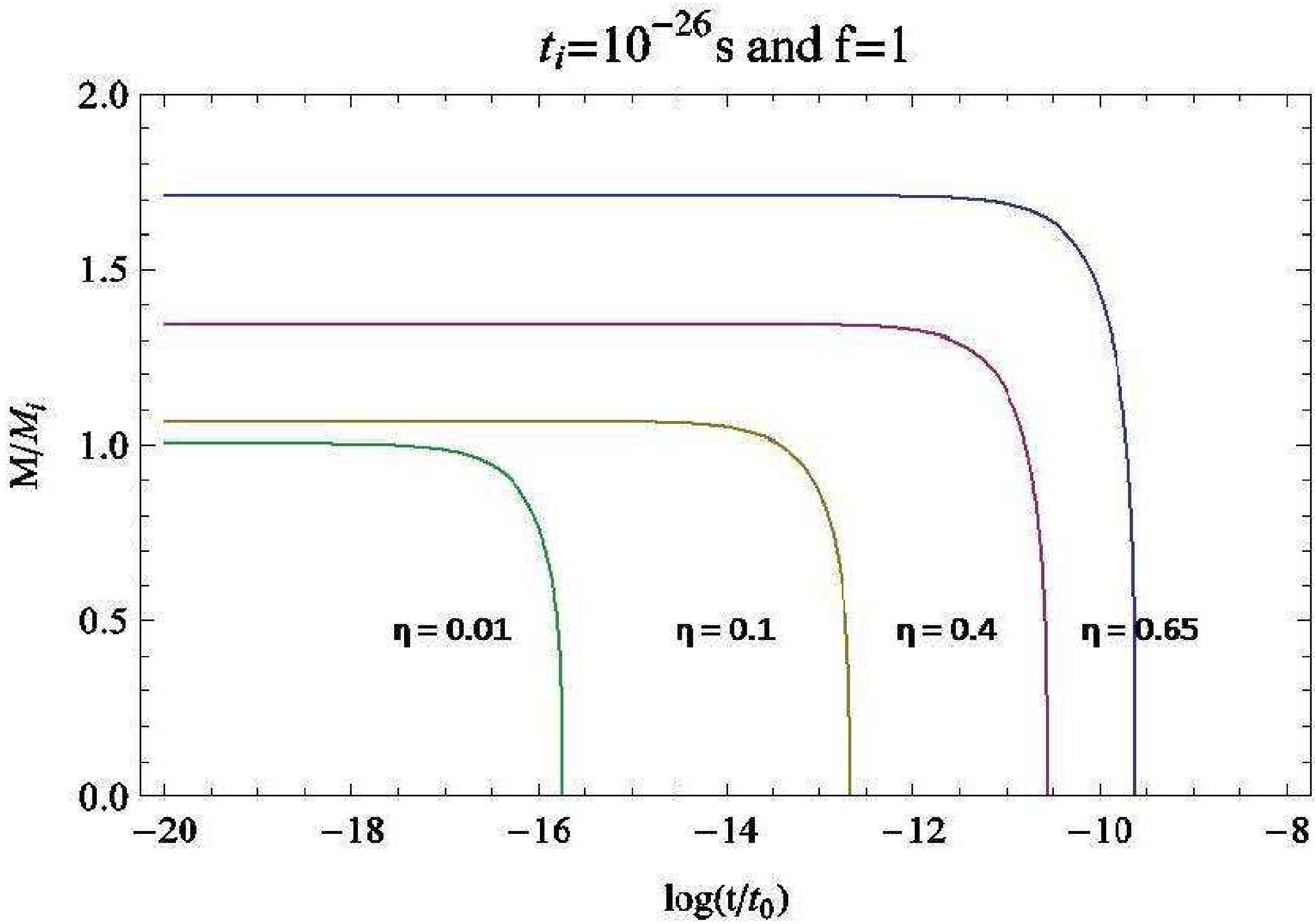

Fig 2 : Variation of evaporation time of PBH with formation time 10^{-26}s for different η = 0.65, 0.4, 0.1, 0.01, but with same accretion efficiency f=1

C. MATTER-DOMINATED ERA $(t > t_1)$

(i) PBHs can not form

(ii) Living PBHs are formed in r.d.e.

PBHs live in this era, evaporate accordingly

$$\dot{M}_{BH} = -\alpha \left(\frac{1}{t_0} \right)^{2n} \left(\frac{t^{2n}}{M^2} \right)$$

At t_e mass of PBH

$$M_e = M_i\left[1+\frac{3}{2} f G_0 M_i \left(\frac{t_0}{t_e}\right)^n \left(\frac{1}{t_i}-\frac{1}{t_e}\right)\right]$$

=> Formation time of evaporating PBHs

$$t_i \approx G_0 \frac{1}{\eta}\left\{1-\frac{3}{2} f\eta\right\}\left(\frac{t_0}{t_e}\right)^n \left[3\alpha\left(\frac{t_e}{t_0}\right)^{2n} t_e \times \left\{1+(2n+1)^{-1}\left(\frac{t_{evap}}{t_0}\right)^{2n+1}\right\}\right]^{1/3}$$

The initial masses of presently evaporating PBHs are shown in Table-1

B. Nayak and L. P. Singh, arXiv : 1005.1529 (Accepted in Phys. Rev. D)

	$f=0$		$f=1$	
η	$t_i \times 10^{-23}$s	$M_i \times 10^{15}$g	$t_i \times 10^{-23}$s	$M_i \times 10^{15}$g
$3/5$	3.949	2.366	0.395	0.237
$1/2$	4.739	2.366	1.185	0.592
$1/5$	11.847	2.366	8.293	1.657
$1/10$	23.693	2.366	20.148	2.013
$1/100$	236.934	2.366	233.384	2.331

Table 1 : The formation times and initial masses corresponding to two specific value of accretion efficiencies for $t_{evap}=t_0$ for different value of η

THE CONSTRAINT FORMALISM IN BRANS-DICKE THEORY

Constraints on abundance of PBHs => upper bounds on mass fraction $\alpha_t(M_i)$

where $$\alpha_t(M_i) = \frac{\rho_{PBH,M_i}(t)}{\rho_{rad}(t)} = \alpha_i \frac{M(t_c)}{M(t_i)} \frac{a(t_{evap})}{a(t_i)}$$

=> $$\alpha_{evap} = \alpha_i \left(\frac{1}{1 - \frac{3}{2} f\eta} \right) \left(\frac{a(t_{evap})}{a(t_i)} \right)$$

THE PRESENT MATTER DENSITY OF THE UNIVERSE

For any PBHs surviving to the present

$$\alpha_0(M) < \frac{0.3}{\Omega_{\gamma,0}}$$

The cosmic microwave background corresponds to a photon density of $\Omega_{\gamma,0}h^2 = 2.47\times10^{-5}$ (with $h=0.7$)

Thus, for PBHs that are about to evaporate today,

$$\alpha_{evap} < \frac{0.3}{\Omega_{\gamma,0}} \approx 6\times10^3$$

Which gives $\alpha_i < 3.43\times10^{-18}\times\left(1-\frac{3}{2}f\eta\right)^{3/2}$

THE PRESENT PHOTON SPECTRUM

The spectral brightness $I(E_0)$ is related to the integrated energy density $U_0(E_0)$

$$I(E_0) = \frac{c}{4\pi} \frac{U_0(E_0)}{E_0}$$

Observed spectral brightness is (at $E_0 = 100 Mev$)

$$I_{obs} = 1.11 \times 10^{-5} keV cm^{-2} s^{-1} sr^{-1} keV^{-1}$$

Our calculation gives

$$I = 2.08 \times 10^{20} (1 - \frac{3}{2} f\eta)^{-1} \alpha_i keV cm^{-2} s^{-1} sr^{-1} keV^{-1}$$

=> $$\alpha_i < 5.34 \times 10^{-26} \times (1 - \frac{3}{2} f\eta)$$

DISTORTION OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM

Hawking radiation emitted at $t \geq 4\times10^{-10} t_0$ can not fully thermalise and disturb CMB spectrum

$$\frac{\rho_{evap}(t)}{\rho_{rad}(t)} = 0.71\mu$$

ρ_{evap} : the energy density injected by evaporating pbhs.

Assuming a half of energy is emitted and $\mu < 9\times10^{-5}$

$$\alpha_{evap} < 1.28\times10^{-4}$$

Which gives $\alpha_i < 1.28\times10^{-21} \times (1-\frac{3}{2} f\eta)^{3/2}$

NUCLEOSYNTHESIS CONSTRAINTS

PBHs evaporated between 1 sec and 400 sec

- The Helium abundance constraint

$$\alpha_i < 3.82 \times 10^{-19} \times (1 - \frac{3}{2} f\eta)^{1/2}$$

- Deuterium photodisintegration constraint

$$\alpha_i \leq 5.1 \times 10^{-21} \times (1 - \frac{3}{2} f\eta)^{3/2}$$

Cause of the Constraint	$\eta = 0$	$\eta = 0.25$	$\eta = 0.45$	$\eta = 0.65$
Present Density	3.43×10^{-18}	1.69×10^{-18}	0.62×10^{-18}	0.01×10^{-18}
Photon Spectrum	5.34×10^{-26}	3.34×10^{-26}	1.73×10^{-26}	1.33×10^{-27}
Distortion of CMB	1.28×10^{-21}	0.63×10^{-21}	0.23×10^{-21}	0.05×10^{-22}
Helium abundance	3.82×10^{-19}	3.01×10^{-19}	2.18×10^{-19}	0.60×10^{-19}
Deuterium abundance	5.10×10^{-21}	2.52×10^{-21}	0.94×10^{-21}	0.02×10^{-21}

Table 2 : The variation of the upper bound of the initial PBH mass fraction with η at f=1 for different cases is shown in the Table.

CONCLUSION

- Lifetime of PBHs enhanced due to accretion
- Product of η and Accretion efficiency f can not exceed 2/3
- Growth of PBH increases with both f and η value
- Constraints on initial PBH mass fractions are tightened with both increase in f and η value
- Accretion becomes ineffective for $\eta \leq 0.01$


THANK YOU