The onset of the bipolar flavor conversion of supernova neutrinos

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Outline of the Talk

1. Introduction
2. Some Features
3. Collective oscillations with three phases
4. Onset of the bipolar oscillations
5. Conclusions

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$$\text{Production} \longrightarrow \text{Inside the SN}$$

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$$\text{Detection} \longrightarrow \text{Inside the earth}$$

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  Resonant matter effect and **Non-linear collective effect** (Flavor conversion)

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- Here the only relevant mixing angle is $\theta_{13}(= \Theta)$, governing the oscillation amplitude in the following channels.

$$\nu_e \to \nu_x \text{ and } \bar{\nu}_e \to \bar{\nu}_x \ (x = \mu \text{ or } \tau)$$

Outline of the talk

Collective oscillation

In the density matrix formalism EOM

$$i\partial_t \rho_{\mathbf{k}} = [\frac{M^2}{2k}, \rho_{\mathbf{k}}] + \sqrt{G_F}[L, \rho_{\mathbf{k}}] + \sqrt{G_F}\int \frac{d^3\mathbf{k}'}{(2\pi)^3}(1-\cos\theta_{\mathbf{kk}'})[(\rho_{\mathbf{k}'}-\bar{\rho}_{\mathbf{k}'}), \rho_{\mathbf{k}}]$$

$$i\partial_t \bar{\rho}_{\mathbf{k}} = -[\frac{M^2}{2k}, \bar{\rho}_{\mathbf{k}}] + \sqrt{G_F}[L, \bar{\rho}_{\mathbf{k}}] + \sqrt{G_F}\int \frac{d^3\mathbf{k}'}{(2\pi)^3}(1-\cos\theta_{\mathbf{kk}'})[(\rho_{\mathbf{k}'}-\bar{\rho}_{\mathbf{k}'}), \bar{\rho}_{\mathbf{k}}]$$

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In terms of Stodolsky equation

$$\partial_t \mathbf{P}_\omega = \omega(\mathbf{B} \times \mathbf{P}_\omega) + \lambda(\mathbf{z} \times \mathbf{P}_\omega) + \mu(\mathbf{D} \times \mathbf{P}_\omega)$$

$$\partial_t \overline{\mathbf{P}}_\omega = -\omega(\mathbf{B} \times \overline{\mathbf{P}}_\omega) + \lambda(\mathbf{z} \times \mathbf{P}_\omega) + \mu(\mathbf{D} \times \overline{\mathbf{P}}_\omega)$$

where, $\lambda = \sqrt{2}G_F n_e \quad \mu = \sqrt{2}G_F(n_\nu + \overline{n}_\nu) \longrightarrow$ self-interaction energy

Collective oscillation

- **Synchronized Oscillation**

  S.Samuel, Phys. Rev D, **48**, 1462 (1993)
  S. Pastor, G. G. Raffelt and D. V. Semikoz, *Phys. Rev. D* **65**, 053011 (2002)

- **Bipolar Oscillation**

  S. Hannestad, G. G. Raffelt, G. Sigl and Y. Y. Y. Wong, *Phys. Rev. D* **74**, 105010 (2006)
  G. L Fogli, E. Lissi, A. Marrone and A. Mirizzi,*JCAP* **12**, (2007)

- **Spectral Split**
  (Will not be discussed here)

  G. G. Raffelt and A. Y. Smirnov *Phys. Rev. D* **74**, 105010 (2006)
  B. Dasgupta, A. Dighe, G. G. Raffelt and A. Y. Smirnov *Phys. Rev. Lett.* **103**, 051105 (2009)

Synchronized Oscillation

$\mathbf{P}_j \longrightarrow$ Polarization vector of jth mode of neutrino
$\overline{\mathbf{P}}_k \longrightarrow$ Polarization vector of kth mode of antineutrino

The corresponding EOMs:

$$\partial_t \mathbf{P}_j = \omega_j (\mathbf{B} \times \mathbf{P}_j) + \mu (\mathbf{D} \times \mathbf{P}_j)$$
$$\partial_t \overline{\mathbf{P}}_k = -\omega_k (\mathbf{B} \times \overline{\mathbf{P}}_k) + \mu (\mathbf{D} \times \overline{\mathbf{P}}_k)$$

$\mathbf{D} = \mathbf{P} - \overline{\mathbf{P}} \longrightarrow$ Internal magnetic field $\quad \mathbf{P} = \sum \mathbf{P}_j, \quad \overline{\mathbf{P}} = \sum \overline{\mathbf{P}}_k$

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When, $(\mu D \gg \omega_j)$ All modes are coupled to each other by their strong 'internal magnetic field' $\mathbf{D}$ and as a result

$$\partial_t \mathbf{D} = \omega_{syn} (\mathbf{B} \times \mathbf{D})$$

$\omega_{syn} = \frac{1}{D}[\sum \omega_j (\mathbf{P}_j.\widehat{\mathbf{P}}) + \sum \omega_k (\overline{\mathbf{P}}_k.\widehat{\overline{\mathbf{P}}})] \longrightarrow$ synchronized frequency

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$\mathbf{B} \longrightarrow (\sin 2\Theta, 0, -\cos 2\Theta)$ $\quad \mathbf{P}(0) = (0, 0, 1)$ and $\overline{\mathbf{P}}(0) = \alpha(0, 0, 1)$

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The EOMs for $\mathbf{S}$ and $\overline{\mathbf{D}} \longrightarrow$

$$\partial_t \mathbf{S} = \omega(\mathbf{B} \times \mathbf{D}) + \mu(\mathbf{D} \times \mathbf{S})$$

$$\partial_t \mathbf{D} = \omega(\mathbf{B} \times \mathbf{S})$$

where, $\mathbf{S} = \mathbf{P} + \overline{\mathbf{P}}$ and $\mathbf{D} = \mathbf{P} - \overline{\mathbf{P}}$

Bipolar Oscillation

Let us construct

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where, $\sigma = \frac{\mathbf{D} \cdot \mathbf{Q}}{Q} \rightarrow$ lepton asymmetry $\quad \mathbf{q} = \frac{\mathbf{Q}}{Q}$

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In **Normal hierarchy**: No dip features develop

In **Inverted hierarchy** $(\Theta \rightarrow \widetilde{\Theta} = \frac{\pi}{2} - \Theta)$

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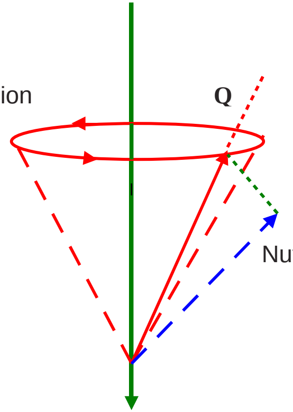
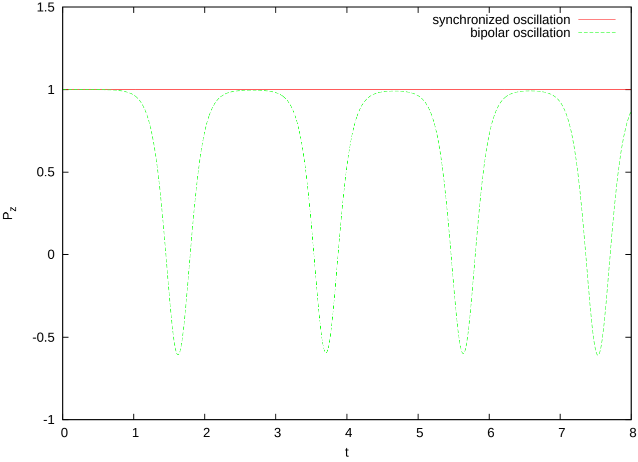
Thus for a small mixing angle the suitable mass hierarchy can cause a complete flavor conversions (**Bipolar Oscillation**)

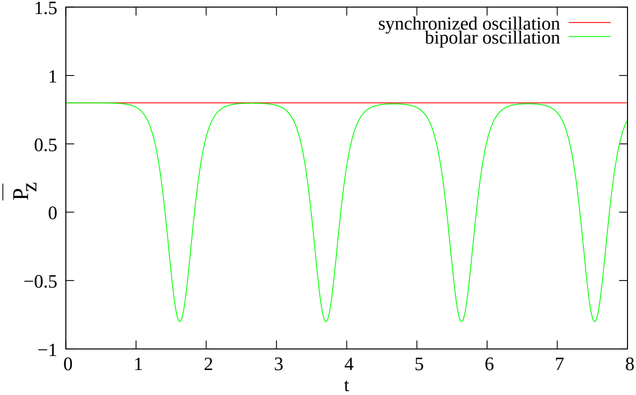
Precession and nutation



Synchronized ($\mu = 200$) and bipolar oscillations ($\mu = 10$) for neutrinos



Synchronized ($\mu = 200$) and bipolar oscillations ($\mu = 10$) for antineutrinos



Outline of the talk

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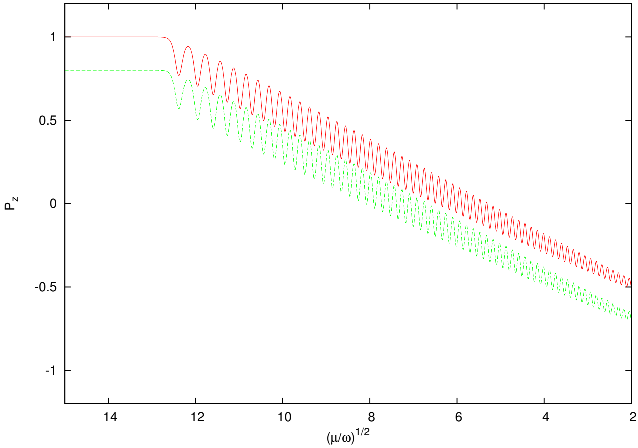
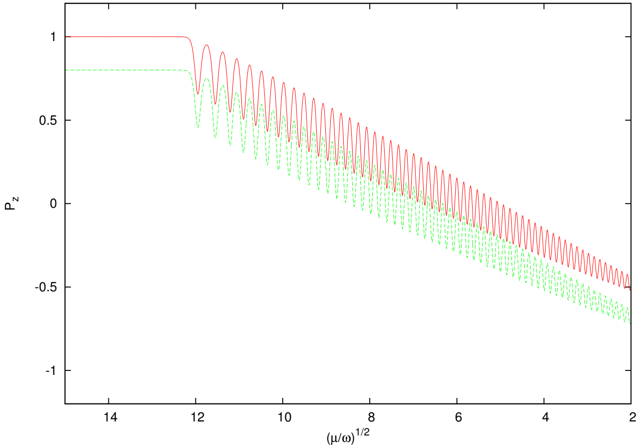
- How and when the nutation (bipolar effect) starts from a purely precession (synchronized effect)?
- Adiabatic change of μ plays a crucial role.
- The form of μ is taken as

$$\mu = \mu_I e^{-kt}$$

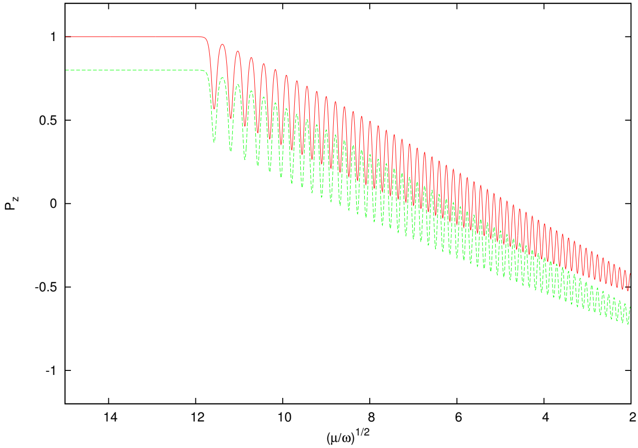
Evolution of P_z and $\bar{P}_z$ against $(\frac{\mu}{\omega})^{\frac{1}{2}}$ at $\theta_0 = 10^{-3}$



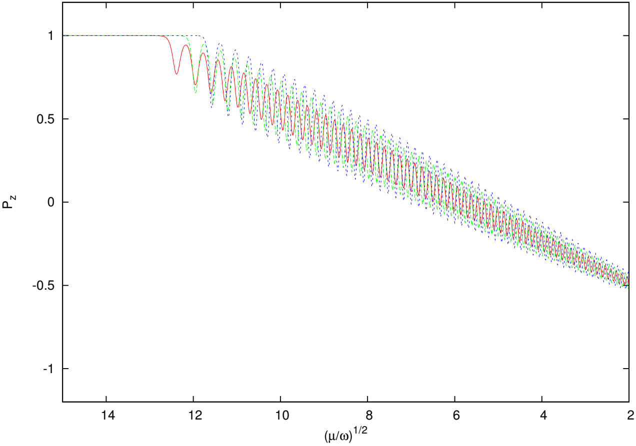
Evolution of P_z and $\bar{P}_z$ against $\left(\frac{\mu}{\omega}\right)^{\frac{1}{2}}$ at $\theta_0 = 10^{-5}$



Evolution of P_z and $\bar{P}_z$ against $\left(\frac{\mu}{\omega}\right)^{\frac{1}{2}}$ at $\theta_0 = 10^{-7}$



Evolution of P_z against $\left(\frac{\mu}{\omega}\right)^{\frac{1}{2}}$ at different $\theta_0 = 10^{-x}$



Observations

- The evolution of P_z and $\bar{P}_z$ are plotted against $(\frac{\mu}{\omega})^{\frac{1}{2}}$ with three different vacuum mixing angles.

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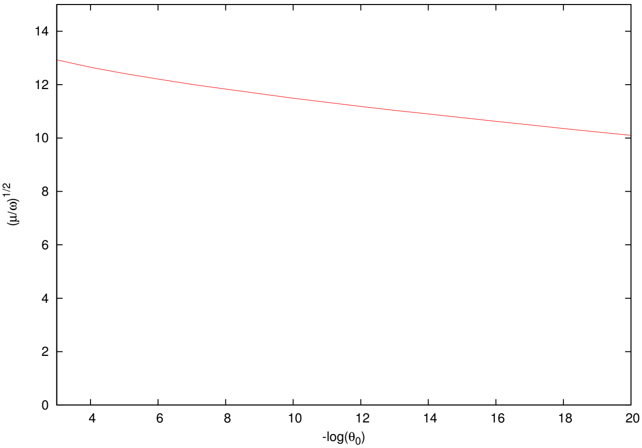
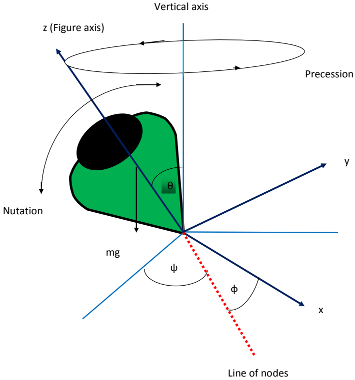
- The evolution of P_z and $\bar{P}_z$ are plotted against $(\frac{\mu}{\omega})^{\frac{1}{2}}$ with three different vacuum mixing angles.
- A sharp deviation of P_z from its initial value denotes the onset.
- It is observed that the onset decreases with decreasing the mixing angle. That indicates the longer time is required to acheive the onset with smaller mixing angle.
- It was also observed that the onset value of $(\frac{\mu}{\omega})^{\frac{1}{2}}$ would decrease logarithmically with decreasing small mixing angle. We shall examine it.

The onset values of $\left(\frac{\mu}{\omega}\right)^{\frac{1}{2}}$ (obtained numerically) are plotted against $-\log\theta_0$



Spinning top



Spinning top

The Lagrangian of a top is given by

$$L = \frac{I_1}{2}(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{I_3}{2}(\dot{\psi} + \dot{\phi}\cos\theta)^2 - mgl\cos\theta$$

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There is no torque either along the spin axis or along the veritical axis as both of them are perpendicular to the line of nodes.

Spinning top

$$p_\psi = \frac{\partial L}{\partial \dot{\psi}} = I_3(\dot{\psi} + \dot{\phi}\cos\theta) = constant = I_1 a$$

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Using the above two equations we can get the expression of the energy in terms of θ as $E_c = \frac{I_1\dot{\theta}^2}{2} + \frac{I_1}{2}\frac{(b-a\cos\theta)^2}{\sin^2\theta} + mgl\cos\theta$

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Using the above two equations we can get the expression of the energy in terms of θ as $\quad E_c = \frac{I_1\dot{\theta}^2}{2} + \frac{I_1}{2}\frac{(b-a\cos\theta)^2}{\sin^2\theta} + mgl\cos\theta$

Taking $\beta = \frac{2mgl}{I_1}$ it can be shown that the top undergoes

$$\frac{a^2}{2\beta} \geq \cos\theta_0 \quad \text{(precession)}$$

$$\frac{a^2}{2\beta} < \cos\theta_0 \quad \text{(precession and nutation)}$$

θ_0 is the initial inclination during pure precession mode

Spinning top

$$p_\psi = \frac{\partial L}{\partial \dot{\psi}} = I_3(\dot{\psi} + \dot{\phi}\cos\theta) = constant = I_1 a$$

$$p_\phi = \frac{\partial L}{\partial \dot{\psi}} = (I_1 \sin^2\theta + I_3\cos^2\theta)\dot{\phi} + I_3\dot{\psi}\cos\theta = constant = I_1 b$$

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It starts to wobble at $\frac{a^2}{2\beta} = \cos\theta_0 \approx 1$ (onset of nutation)

Comparison: Spinning top and Collective Oscillation

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Spinning top

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Let us now compare the various terms

Comparison: Spinning top and Collective Oscilation

$\mathbf{D} = \frac{1}{\mu}(\mathbf{q} \times \dot{\mathbf{q}}) + \sigma \mathbf{q} \rightarrow$ angular momentum

$q = l = 1; \qquad ml^2 = I_1 = I_3 = \mu^{-1}$

$\mathbf{B} \rightarrow$ positive vertical axis; $\qquad \mathbf{q} \rightarrow$ positive spin axis

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Thus the onset condition becomes

$$\frac{\mu}{\omega} \approx 4\frac{Q}{\sigma^2}$$

for the small θ_0.

The same onset condition was observed analytically by Hannested et al. [Phys. Rev. D **74**, 105010 (2006)]

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- What to be fixed then?
- Let us consider the situation just after the onset, where

$$\mu = \mu_0 e^{-kt} \approx \mu_0(1 - kt)$$

$$\mu_0 \approx 4\omega \frac{Q}{\sigma^2}$$

Variation of onset with θ_0

$$L = \frac{1}{2\mu}(\dot{\theta}^2 + \dot{\phi}^2\theta^2) + \frac{1}{2\mu}(\dot{\psi} + \dot{\phi})^2 - \omega Q\mu$$

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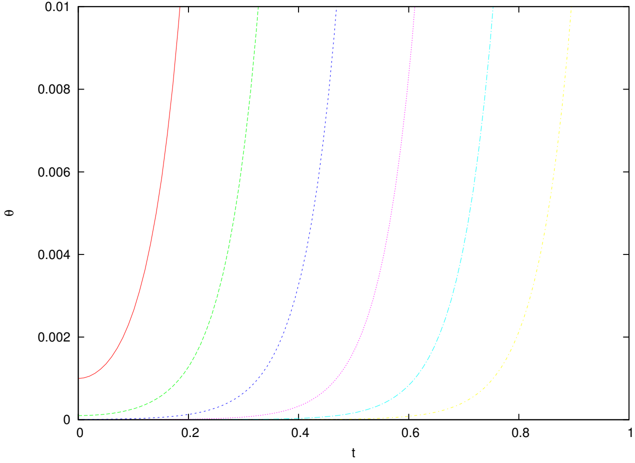
From which we get

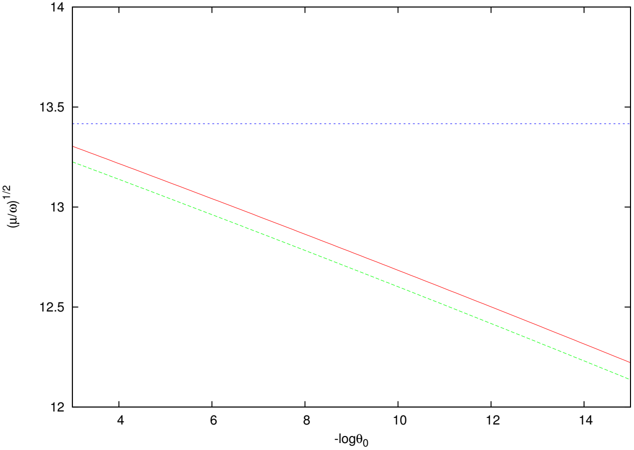
$$x = [x_0 - \frac{2k}{\omega\sigma}\cosh^{-1}(\frac{\theta}{\theta_0})]^{\frac{1}{2}}$$

where, $x = (\frac{\mu}{\omega})^{\frac{1}{2}}$

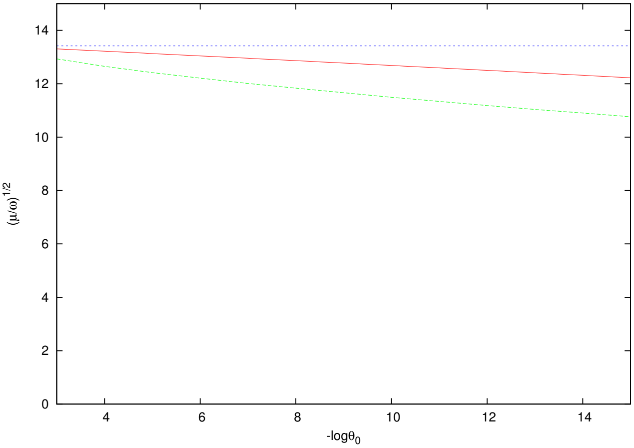
The change of θ with time at θ_0=10^{-3} (red), 10^{-4} (green), 10^{-4} (blue), 10^{-5} (pink), 10^{-6} (light blue) and 10^{-7} (yellow).



The change of $(\frac{\mu}{\omega})^{1/2}$ with θ_0 at $\theta = 0.01$ (red) and 0.08 (green). Here blue line indicates $\frac{\mu}{\omega} = 180$, true onset value.



Similar to the previous figure; here the theoretical result at $\theta = 0.01$ (red) is compared to the numerically obtained value (green). Scale is different



Outline of the talk

Conclusions

- If we watch an exponential curve in a large scale we find that at the beginning the curve seems to be a constant. But if the curve is observed in a small scale it can be clearly verified that the initial portion, which have been observed as constant in the larger scale, would actually have the exponential nature.

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Thank you.