Ultra High Energy neutrino-nucleon interaction cross-section

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Plan of the talk

(1) Deep Inelastic UHE neutrino-nucleon interaction.

(2) Sources of UHE neutrino and detector.

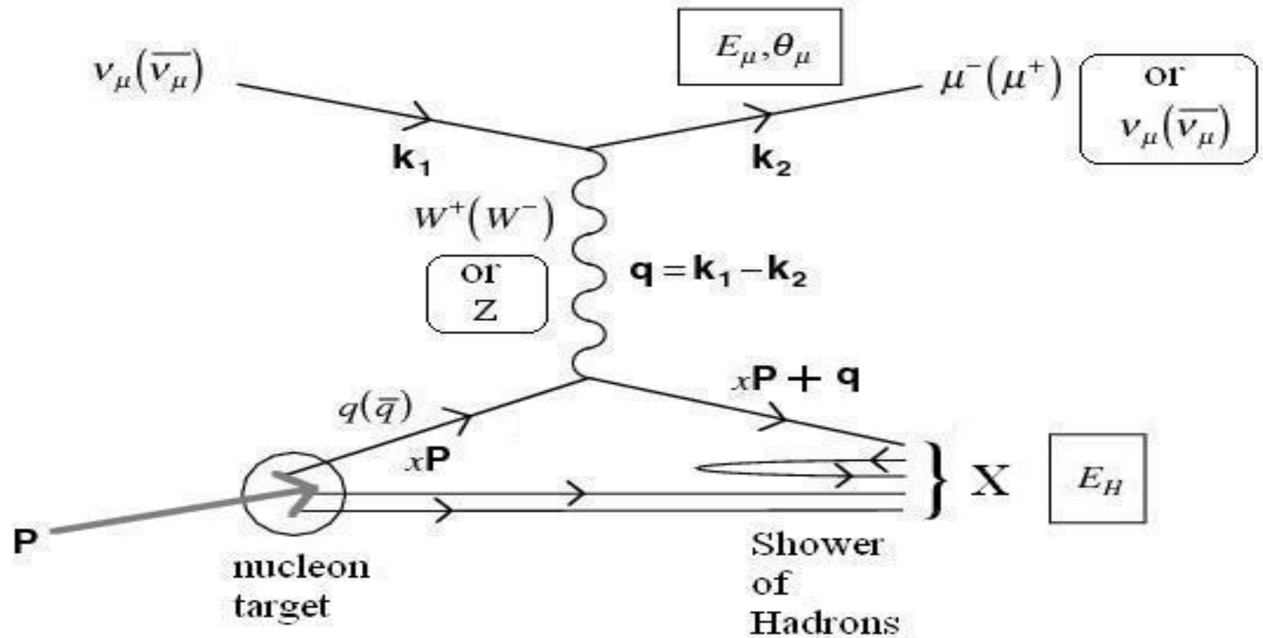
(2) Development of formalism for UHE neutrino-nucleon interaction cross-section.

(3) Results.

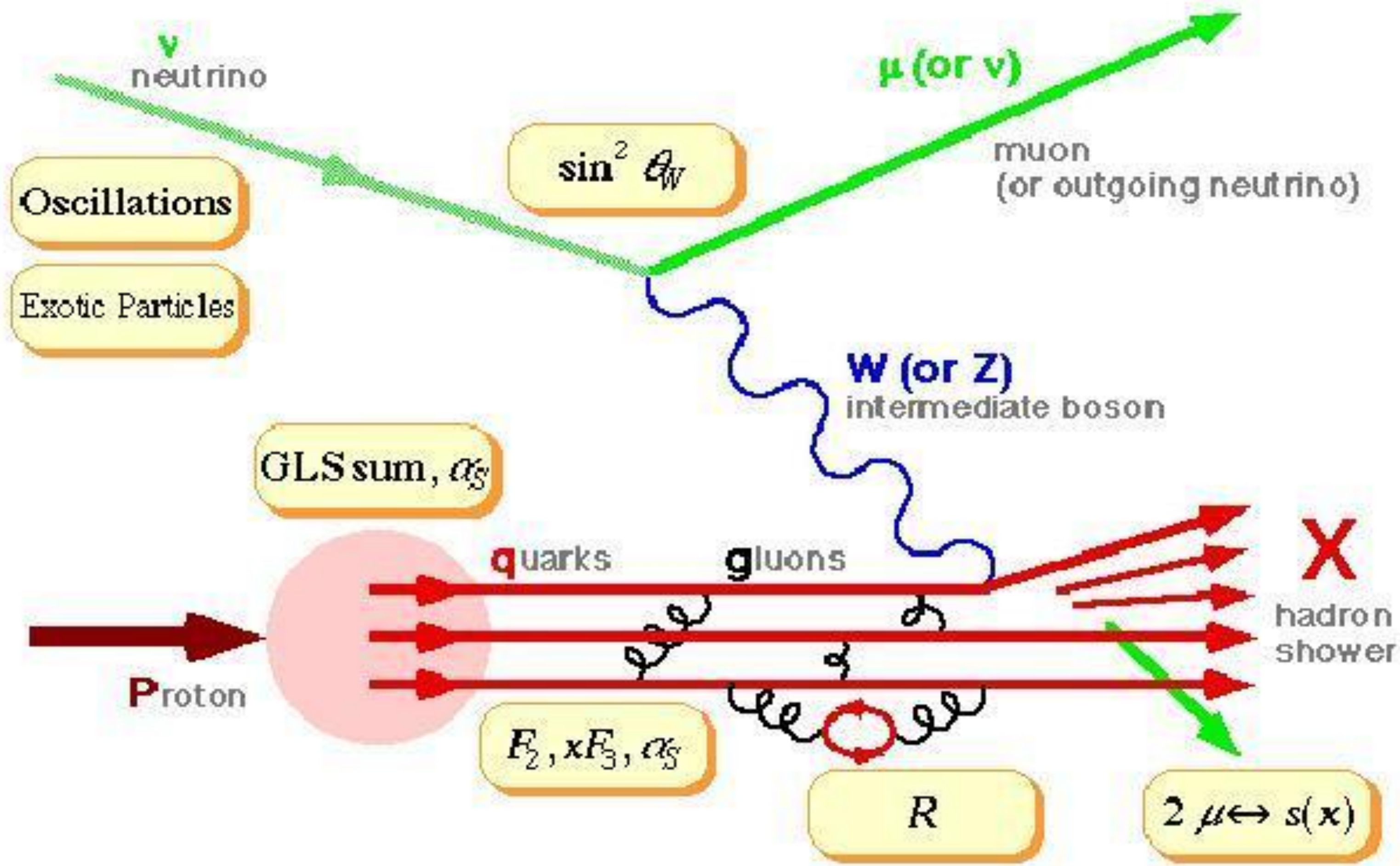
(4) Limitations.

(5) Summary and conclusion.

(5) Future Scope.



E_μ, θ_μ
$\nu_\mu(\overline{\nu_\mu})$
$\mu^-(\mu^+)$
or
$\nu_\mu(\overline{\nu_\mu})$
$\mathbf{k_1}$
$\mathbf{k_2}$
$W^+(W^-)$
or
Z
$\mathbf{q} = \mathbf{k_1} - \mathbf{k_2}$
$x\mathbf{P} + \mathbf{q}$
$q(\overline{q})$
$x\mathbf{P}$
X
E_H
P
nucleon
target
Shower
of
Hadrons



ν
neutrino
μ (or ν)
muon
(or outgoing neutrino)
$\sin^2 \theta_W$
Oscillations
Exotic Particles
W (or Z)
intermediate boson
GLS sum, α_S
quarks
gluons
X
hadron
shower
Proton
F_2, xF_3, α_S
R
$2\,\mu \leftrightarrow s(x)$

Deep inelastic Interaction of ultra-high energy $(E_{\nu(\overline{\nu})} \geq 10^3$ GeV) neutrinos

Charged current interactions-> $\nu_\mu N \rightarrow \mu^- X, \quad \overline{\nu_\mu} N \rightarrow \mu^+ X$

Neutral current interactions-> $\nu_\mu N \rightarrow \nu_\mu X, \; \overline{\nu}_\mu N \rightarrow \overline{\nu}_\mu X$

Weak interactions are mediated by $W^\pm$ boson and Z^0 boson respectively.

W boson mass=80.136 $\pm$ 0.084 GeV

Z boson mass=91.1876 $\pm$ 0.0021 GeV

Sources of UHE neutrino

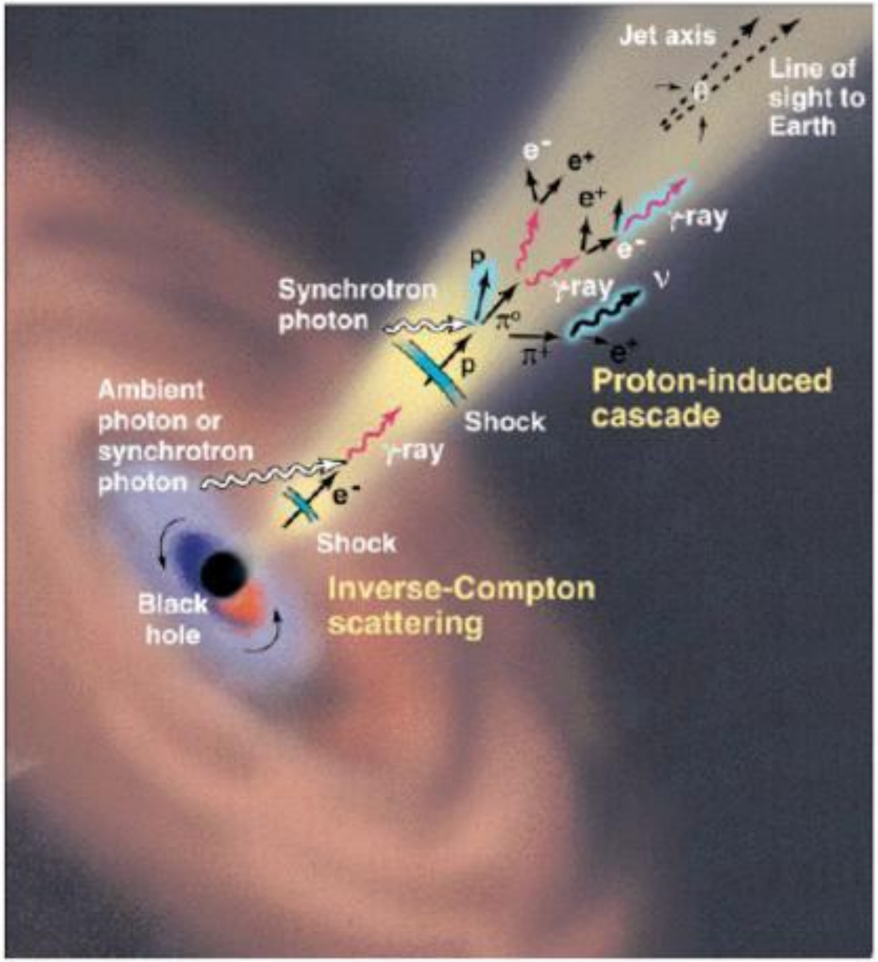
1. **Interaction of UHE Cosmic Rays with Cosmic Microwave background (CMB).** UHE neutrino may be produced from the decay of charged pions, which are produced when protons with energy greater than 5×10^{10} GeV interact with omnipresent photons of Cosmic Microwave Background leading to the generation of pions. This is represented as $p+\gamma_{CMB}\rightarrow\pi^{\pm}$.

   Pions consecutively break down into muons along with UHE neutrinos as follows: $\pi\rightarrow\mu+\nu_{\mu},\ \mu\rightarrow e+\nu_{\mu}+\nu_{e}$

2. Interaction of Galactic Cosmic Ray with intersteller matter.

3. Gamma Ray Bursts (GRB) and Active Galactic Nuclei (AGN).

4. Decay of Topological defects like Monopoles, Cosmic Strings, Superheavy dark matter and Mirror matter.

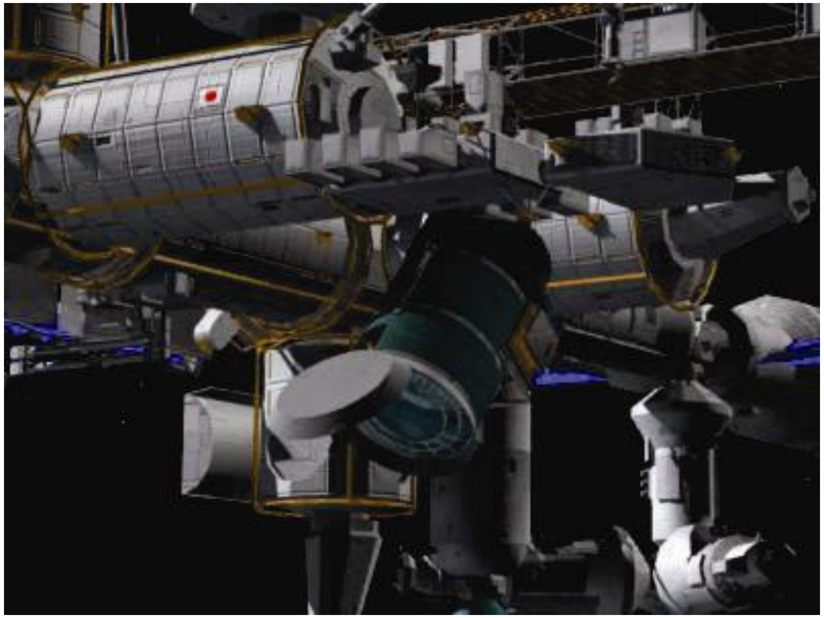


Active galactic nuclei

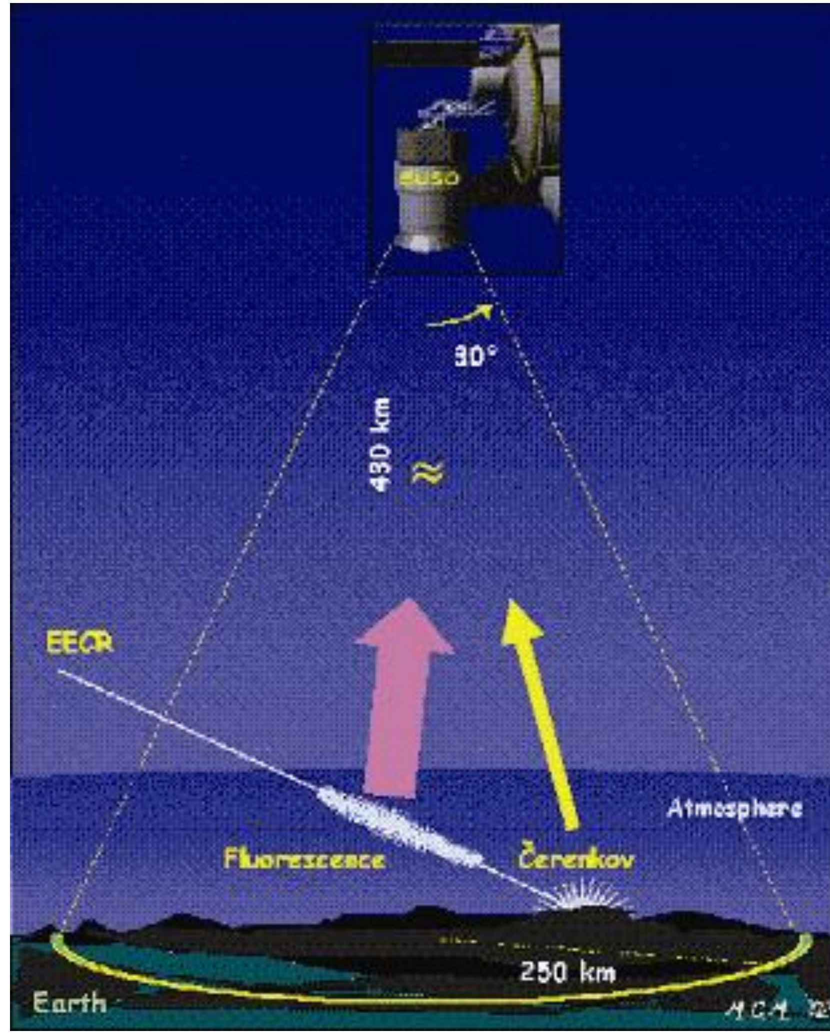
- Current paradigm:
  - **Synchrotron Self Compton**
  - External Compton
  - Proton Induced Cascades
  - Proton Synchrotron
- Energetics, mechanism for jet formation and collimation, nature of the plasma, and particle acceleration mechanisms are still poorly understood.

TeV γ-rays have been seen from AGN, however no *direct* evidence so far that protons are accelerated in such objects

… renewed interest triggered by possible correlations with UHECRs - e.g. 2 Auger events within 3^0 of Cen A



Jem-Euso detector



UHE neutrino-nucleon interaction cross-section

The differential cross-section for UHE neutrino-nucleon interaction is given by

$$\frac{d^2\sigma_{CC}^{\nu(\bar{\nu})N}}{dx\,dy} = \frac{G_F^{\,2}}{2\pi}\left(1+\frac{Q^2}{M_W^{\,2}}\right)^{-2}\left[(1-y)F_2^{\nu(\bar{\nu})}(x,Q^2) + y^2 x F_1^{\nu(\bar{\nu})}(x,Q^2) \pm y\left(1-\frac{y}{2}\right)xF_3^{\nu(\bar{\nu})}(x,Q^2)\right]$$

where Fermi coupling constant $G_F = 1.1603\times10^{-5}\ GeV^{2}$

Total cross-section is $\sigma_{CC}^{\nu(\bar{\nu})N} = \int_0^1 dx \int_0^1 dy \frac{d^2\sigma_{CC}^{\nu(\bar{\nu})N}}{dx\,dy}$

Non-singlet DGLAP equation is given by

$$\frac{\partial F^{NS}(x,t)}{\partial t} = \frac{A_f}{t}\left[\left(3+4\log(1-x)\right)F^{NS}(x,t) + 2\int_x^1 \frac{dz}{1-z}\left\{(1+z^2)F^{NS}\left(\frac{x}{z},t\right) - 2F^{NS}(x,t)\right\}\right]$$

where $A_f = \frac{4}{33-2n_f}$, n_f being the number of quarks

Singlet DGLAP equation is given by

$$\frac{\partial F_2^{S}(x,t)}{\partial t} = \frac{A_f}{t}\left[\left(3+4\log(1-x)\right)F_2^{S}(x,t) + 2\int_x^1 \frac{dz}{(1-z)}\left\{(1+z^2)F_2^{S}\left(\frac{x}{z},t\right) - 2F_2^{S}(x,t)\right\} + \frac{3}{2}n_f\int_x^1 dz\left(z^2+(1-z)^2\right)G\left(\frac{x}{z},t\right)\right]$$

Approximate Solution of DGLAP equations for non-singlet and singlet structure functions are

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^{n(x,t)}$$

$$F_2^{S}(x,t) = F_2^{S}(x,t_0)\,\frac{t^{k(x,t)}}{t^{k(x,t_0)}}\,\frac{[X^{S}(x)]^{k(x,t)}}{[X^{S}(x)]^{k(x,t_0)}}$$

where we have from CCFR experiment,

$$n(x,t) = 0.380[-5.796x + 0.996(1-x)],$$

$$K(x,t) = 1.284[-4.64x + 1.02(1-x)](1.52 - 1.16s - 0.06s^2)$$

LO expressions for CC interactions is

$$F_1^{\nu,light} = \frac{1}{2}(\bar{u}+\bar{d}) + \frac{1}{2}(d+u)|V_{ud}|^2 + s|V_{us}|^2$$

$$F_2^{\nu,light} = 2x F_1^{\nu,light}$$

$$F_3^{\nu,light} = -(\bar{u}+\bar{d}) + (d+u)|V_{ud}|^2 + 2s|V_{us}|^2$$

where

$$x = \frac{Q^2}{2M\nu},\ \nu = \frac{p.q}{M}$$

Using all the above equations, we get analytical expression for CC cross-section as

$$\sigma_{CC}^{\nu(\bar{\nu})N} = \frac{G_F^{\,2}}{2\pi}\int_{Q_0^2}^{s} dQ^2 \left(1+\frac{Q^2}{M_W^{\,2}}\right)^{-2} \int_{Q^2/s}^{1}\frac{dx}{x}\left[\left(1-\frac{Q^2}{xs}+\frac{Q^4}{2x^2s^2}\right)\times 2x\left\{\frac{1}{2}\left[u(x,t_0)+\bar{d}(x,t_0)\right] \pm \frac{1}{2}\left[d(x,t_0)+u(x,t_0)\right]|V_{ud}|^2 + s\,|V_{us}|^2\right\} \times \frac{t^{k(x,t)}}{t_0^{\,k(x,t_0)}}\frac{\left[X^S(x)\right]^{\bar{k}(x,t)}}{\left[X^S(x)\right]^{\bar{k}(x,t_0)}} \pm \frac{Q^2}{xs}\left(1-\frac{Q^2}{2xs}\right)x\left\{\left[u(x,t_0)+\bar{d}(x,t_0)\right] \pm \left[d(x,t_0)+u(x,t_0)\right]|V_{ud}|^2 + 2s\,|V_{us}|^2\right\}\left(\frac{t}{t_0}\right)^{n(x,t)}\right]$$

We can also evaluate CC cross-section numerically from the following expression

$$\sigma_{CC}^{\nu(\bar{\nu})N} = \frac{G_F^{\,2}}{2\pi}\int_{Q_0^2}^{s} dQ^2 \left(1+\frac{Q^2}{M_W^{\,2}}\right)^{-2} \int_{Q^2/s}^{1}\frac{dx}{x}\left[\left(1-\frac{Q^2}{xs}+\frac{Q^4}{2x^2s^2}\right)\times 2x\left\{\frac{1}{2}\left[u(x,t)+\bar{d}(x,t)\right] \pm \frac{1}{2}\left[d(x,t)+u(x,t)\right]|V_{ud}|^2 + s\,|V_{us}|^2\right\} \pm \frac{Q^2}{xs}\left(1-\frac{Q^2}{2xs}\right)x\left\{\left[u(x,t)+\bar{d}(x,t)\right] \pm \left[d(x,t)+u(x,t)\right]|V_{ud}|^2 + 2s\,|V_{us}|^2\right\}\right]$$

Similarly, we can get analytical expression for NC cross-section as

$$\sigma_{NC}^{\nu(\bar{\nu})N} = \frac{G_F^{\,2}}{2\pi}\int_{Q_0^2}^{s} dQ^2\left(1+\frac{Q^2}{M_Z^{\,2}}\right)^{-2}\int_{\frac{Q^2}{s}}^{1}\frac{dx}{x}\left[\left(1-\frac{Q^2}{xs}+\frac{Q^4}{2x^2s^2}\right)\left\{\begin{array}{l}0.1432x\times\left[u_v(x,t_0)+d_v(x,t_0))+2(\bar{u}(x,t_0)+\bar{v}(x,t_0))\right]+\\ 0.1849x\times\left[u_v(x,t_0)+d_v(x,t_0))+2(\bar{u}(x,t_0)+\bar{v}(x,t_0)+4s)\right]\end{array}\right\} \times\frac{t^{k(x,t)}}{t_0^{\,k(x,t_0)}}\frac{\left[X^S(x)\right]^{\bar{k}(x,t)}}{\left[X^S(x)\right]^{\bar{k}(x,t_0)}} \pm\frac{Q^2}{xs}\left(1-\frac{Q^2}{2xs}\right)\left\{0.268x\left[u_v(x,t_0)+d_v(x,t_0)\right]\right\}\left(\frac{t}{t_0}\right)^{n(x,t)}\right]$$

We can evaluate NC cross-section numerically from the following expression

$$\sigma_{NC}^{\nu(\bar{\nu})N} = \frac{G_F^{\,2}}{2\pi}\int_{Q_0^2}^{s} dQ^2\left(1+\frac{Q^2}{M_Z^{\,2}}\right)^{-2}\int_{\frac{Q^2}{s}}^{1}\frac{dx}{x}\left[\left(1-\frac{Q^2}{xs}+\frac{Q^4}{2x^2s^2}\right)\left\{\begin{array}{l}0.1432x\times\left[u_v(x,t)+d_v(x,t))+2(\bar{u}(x,t)+\bar{v}(x,t))\right]+\\ 0.1849x\times\left[u_v(x,t)+d_v(x,t))+2(\bar{u}(x,t)+\bar{v}(x,t)+4s)\right]\end{array}\right\} \pm\frac{Q^2}{xs}\left(1-\frac{Q^2}{2xs}\right)\left\{0.268x\left[u_v(x,t)+d_v(x,t)\right]\right\}\right]$$

LO expressions for NC interactions is

$$2F_1^{light} = \frac{1}{2}\left(u+\bar{u}+d+\bar{d}\right)\left(V_u^{\,2}+A_u^{\,2}\right)+\frac{1}{2}\left(u+\bar{u}+d+\bar{d}+4s\right)\left(V_d^{\,2}+A_d^{\,2}\right)$$

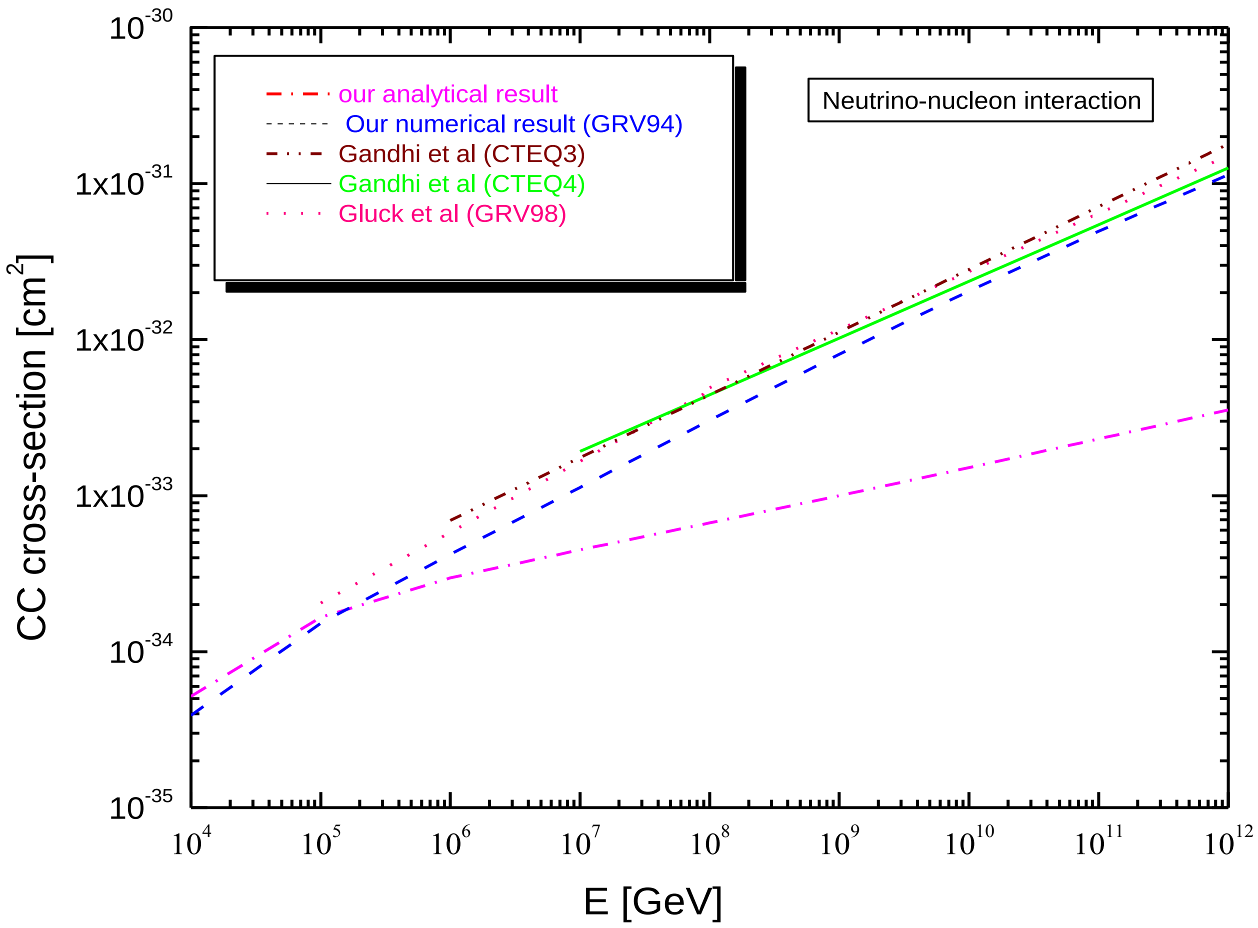
$$F_2^{light} = 2xF_1^{light}$$

$$2F_3^{\nu,light} = 2\left(u_v+d_v\right)\left(V_uA_u+V_dA_d\right)$$

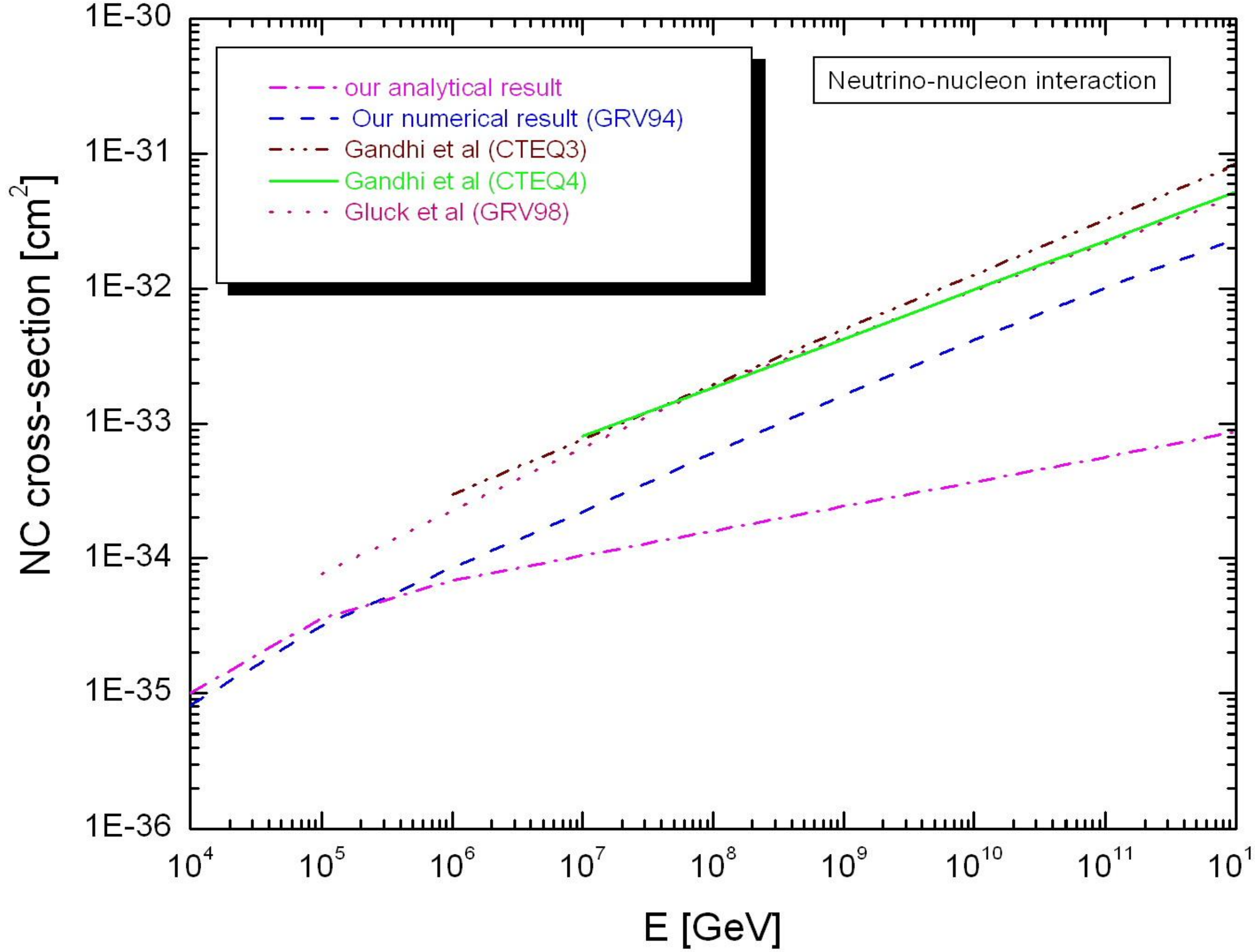
where

$$V_u = \frac{1}{2}-\frac{4}{3}\sin^2\theta_w,\; A_u = -A_d = \frac{1}{2}$$

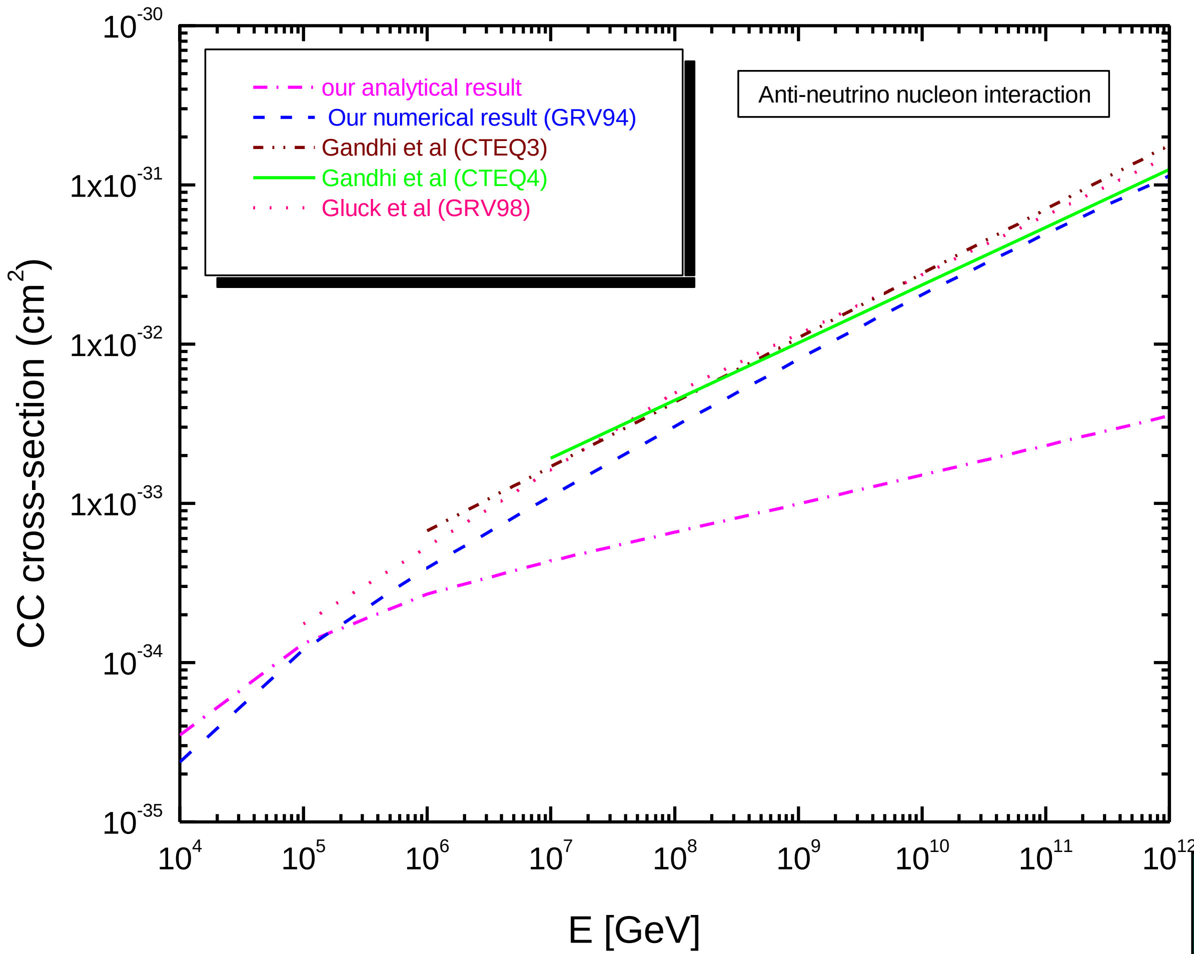
$$\sin^2\theta_w = 0.232$$



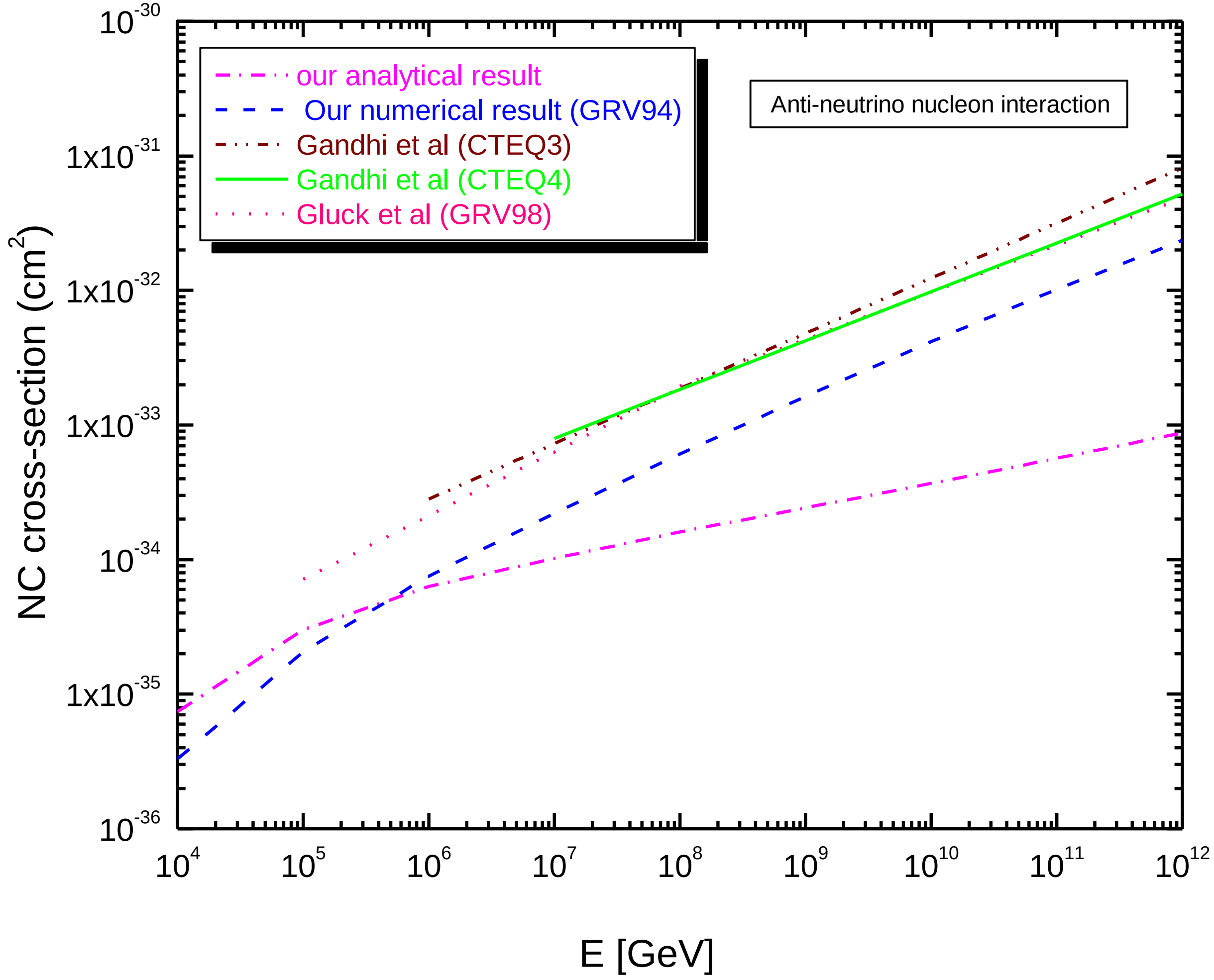
UHE neutrino nucleon interaction cross-section versus neutrino energy from our analytical and numerical solution and compared with the results of various authors like Gandhi et al, Gluck et al, Fiore et al, Rahul et al etc.



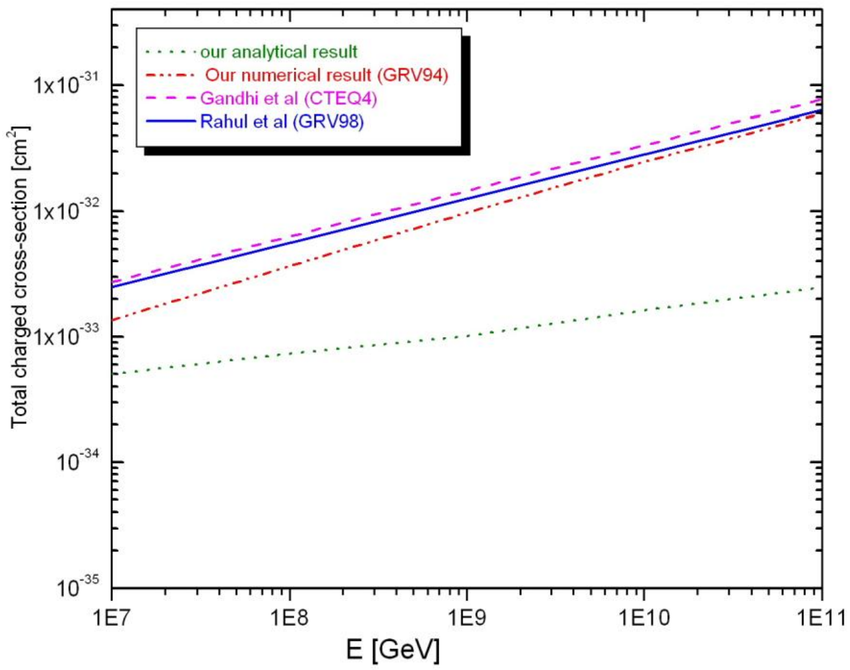
our analytical result
Our numerical result (GRV94)
Gandhi et al (CTEQ3)
Gandhi et al (CTEQ4)
Gluck et al (GRV98)
Neutrino-nucleon interaction
NC cross-section [cm^2]
1E-30
1E-31
1E-32
1E-33
1E-34
1E-35
1E-36
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
E [GeV]

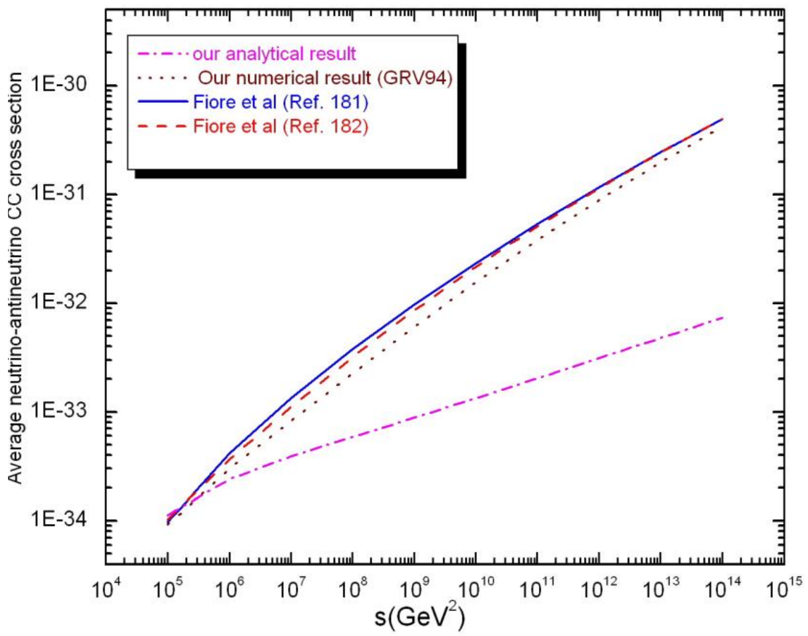


UHE antineutrino nucleon interaction cross-section versus neutrino energy from our analytical and numerical solution and compared with the results of various authors like Gandhi et al, Gluck et al, Fiore et al, Rahul et al etc.



our analytical result
Our numerical result (GRV94)
Gandhi et al (CTEQ3)
Gandhi et al (CTEQ4)
Gluck et al (GRV98)
Anti-neutrino nucleon interaction
NC cross-section (cm2)
E [GeV]
10-30
1x10-31
1x10-32
1x10-33
10-34
1x10-35
10-36
104
105
106
107
108
109
1010
1011
1012





where

$$\overline{\sigma}_{CC}{}^{\nu N} = \frac{\sigma_{CC}{}^{\nu N} + \sigma_{CC}{}^{\bar{\nu} N}}{2}$$

Result

1. Our analytical result indictes a sharp rise for $E_{\nu(\bar{\nu})} \leq 10^3$ GeV. This is due to the absence of propagator effect in low energy regime.

2. For $E_{\nu(\bar{\nu})} \geq 10^3$ GeV, the dampening due to propagator effect takes place. We find that our numerically determined result is in sufficient agreement with the result of various authors. As far as our analytical result is concerned, we find it to match better with other results at the lower end of the energy spectrum of the UHE neutrino rather than the higher end .

3. We have used MRST 2004 distribution for our analytical calculation and GRV 94 distribution for our numerical calculation.

Limitations:

(1) We have not extrapolated the MRST distribution for values of x below 10^{-5} .

(2) We have considered solutions at tree level. Result might come better at next-to-leading order.

(3) We have neglected the finite x corrections coming from the higher derivatives of $\frac{\partial F^{NS}(x,t)}{\partial x}, \frac{\partial F_2^{S}(x,t)}{\partial x}, \frac{\partial G(x,t)}{\partial x}$ in Taylor approximations of DGLAP equation.

(4) We have neglected heavy quarks like b and t quarks. At the highest energies of neutrino, this might account for about 10% of cross-section.

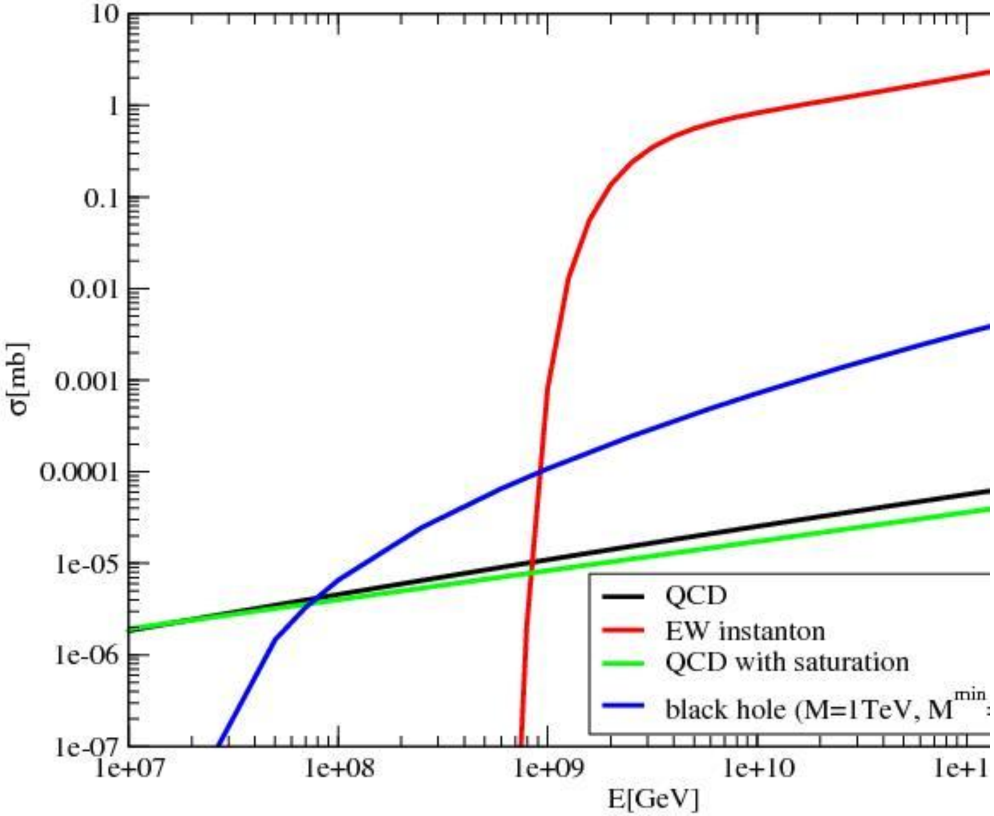
Summary and Conclusions

1. We have shown that our analytical evaluation of neutrino (antineutrino) cross-section tallies well with the selected part of the neutrino-antineutrino energy spectrum.
2. On the other hand, our LO numerical result matches quite well with the NLO results of other authors.
3. We have neglected the contribution of heavy quarks and hence ignored the transition channels $W^+s \to c, W^+\underline{d} \to c$ and the fusion subprocess channels $W^+g \to cs, W^+g \to t\bar{b}.$
4. At ultra high energy, there is a power suppression from the gauge-boson propagator, while on the other hand there is a logarithmic growth of the PDFs. In overall, the propagator dominates and it leads to the generation of dampness of cross-section.

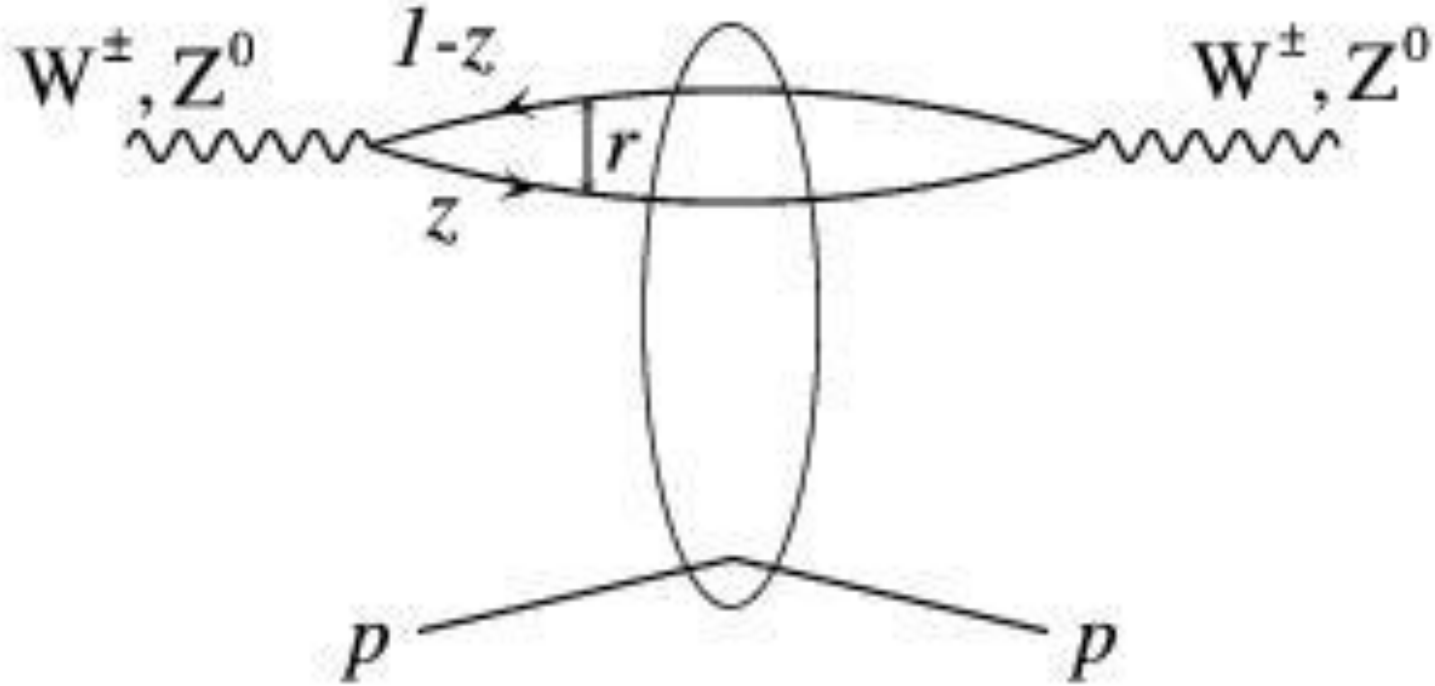
References:

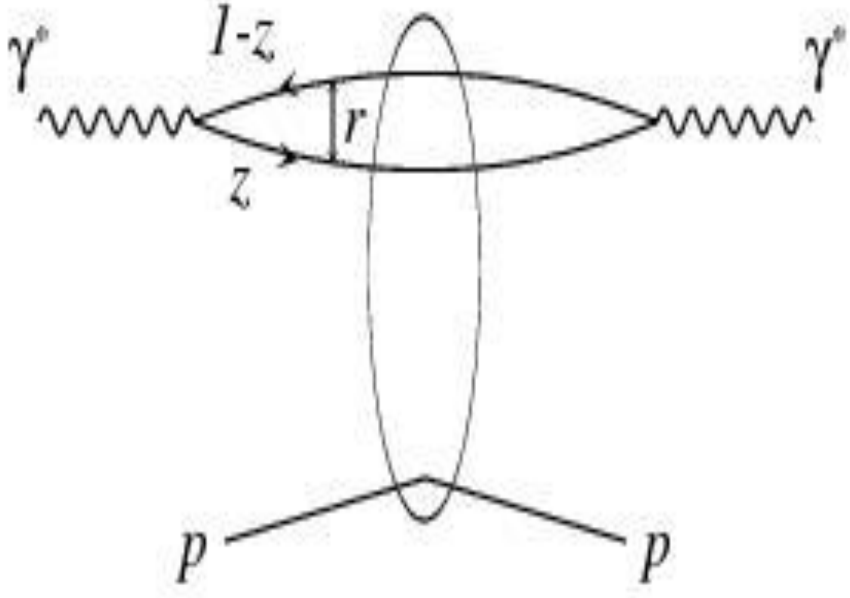
1. R. Gandhi et al, Astropart Phys.,5:81, (1996),arXiv:hep-ph/9512364, Phys. Rev. D58:093009, (1998), arXiv:hep-ph/9807264
2. Gluck et al, Astropart Phys., 11:327, (1999), arXiv:astro-ph/9809273
   Rahul Basu et al, JHEP, 0210: 012, (2002), arXiv:hep-ph/0208125
3. Fiore et al, Phys. Rev., D68:093010, (2003), arXiv:hep-ph/0302251, Phys. Rev. D71: 033002, (2005), arXiv:hep-ph/0412003
4. Gluck, Jimenez-Delgado and E.Reya, Phys. Rev. D81, 097501 (2010).
5. For online display of parton distributions, please see World Wide Web, http://durpdg.dur.ac.uk/hepdata/pdf3.html

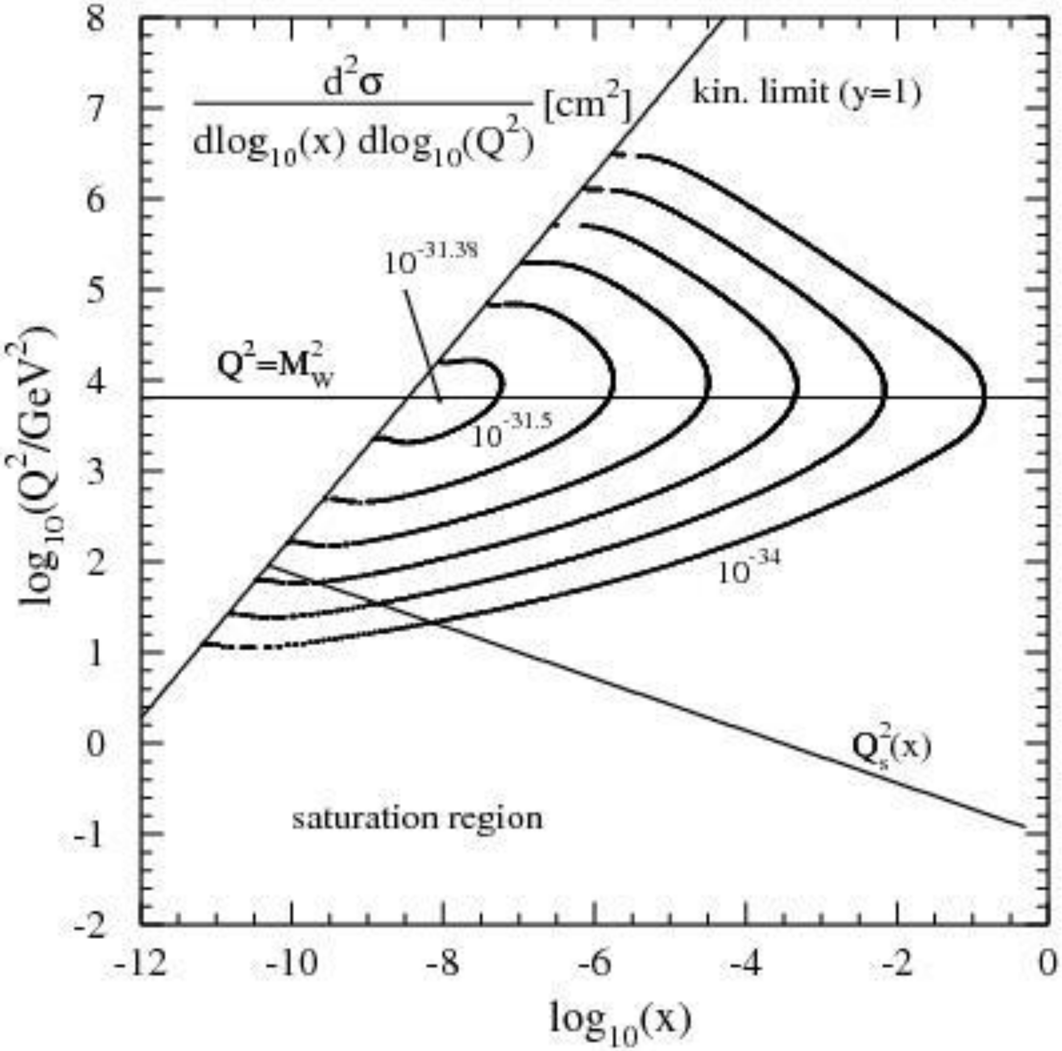
Thank you



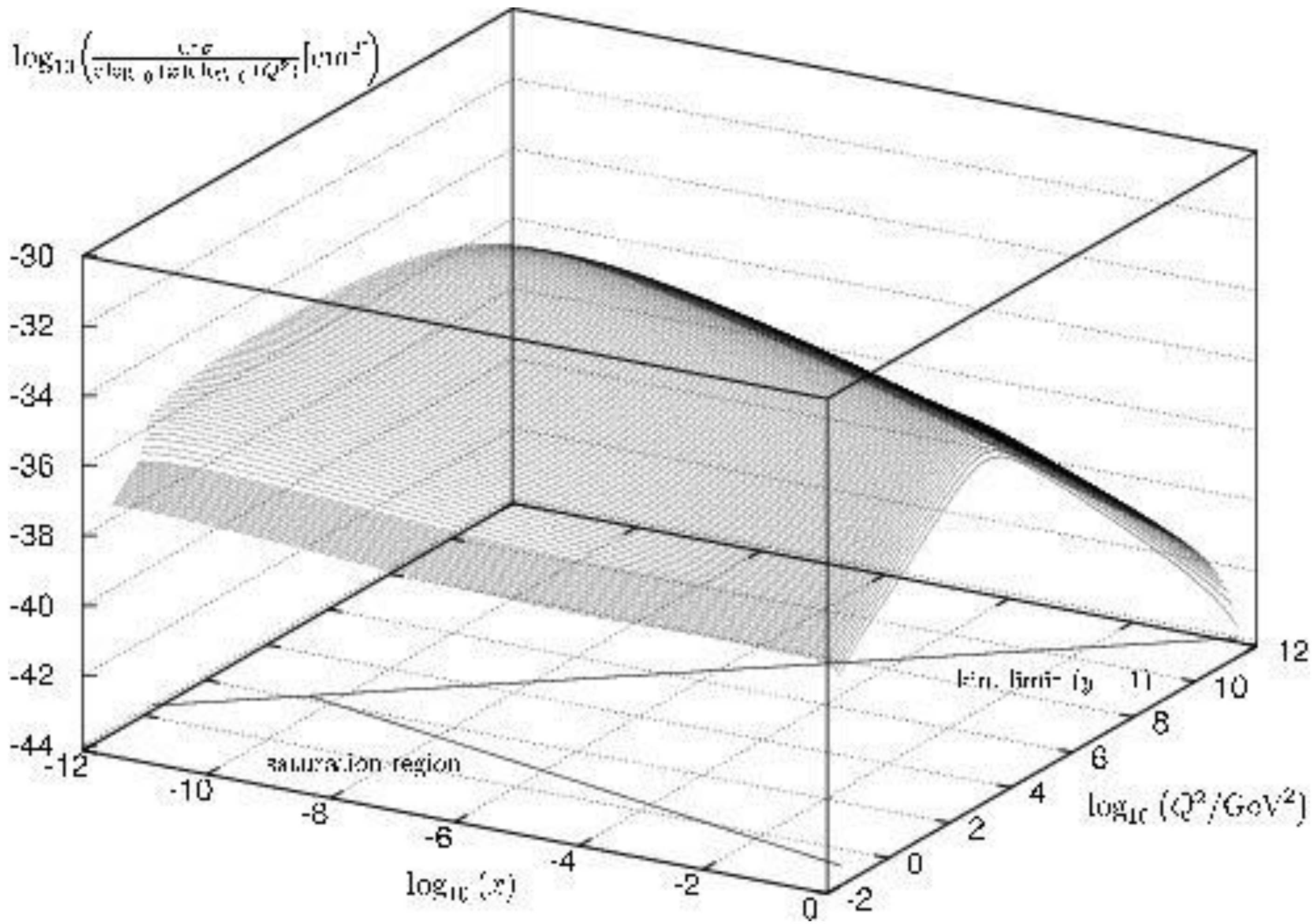
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
σ[mb]
1e+07
1e+08
1e+09
1e+10
1e+1
E[GeV]
QCD
EW instanton
QCD with saturation
black hole (M=1TeV, M
min


W±, Z0
1-z
r
z
W±, Z0
p
p


γ*
1-z
r
z
γ*
p
p



$\frac{d^2\sigma}{d\log_{10}(x)\, d\log_{10}(Q^2)}$ [cm^2]
kin. limit (y=1)
$10^{-31.38}$
$Q^2=M_W^2$
$10^{-31.5}$
10^{-34}
$Q_s^2(x)$
saturation region
$\log_{10}(Q^2/\mathrm{GeV}^2)$
$\log_{10}(x)$
8
7
6
5
4
3
2
1
0
-1
-2
-12
-10
-8
-6
-4
-2
0


-30
-32
-34
-36
-38
-40
-42
-44
-12
-10
-8
-6
-4
-2
0
log10 (x)
-2
0
2
4
6
8
10
12
log10 (Q^2/GeV^2)
saturation region
kin. limit

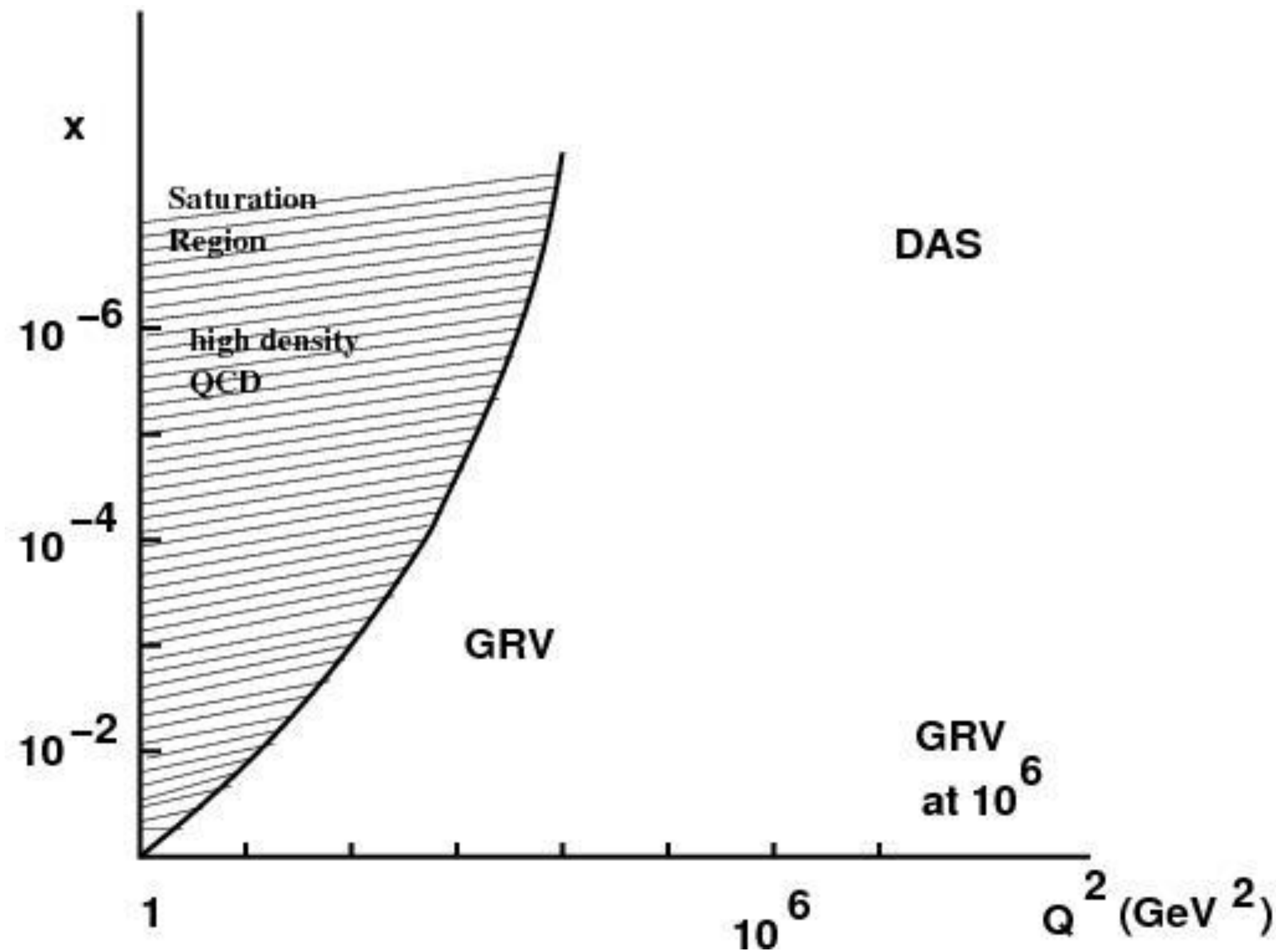
$$Q_S{}^2 = (1GeV)^2 \left(\frac{x_0}{x} \right)^{\lambda}$$

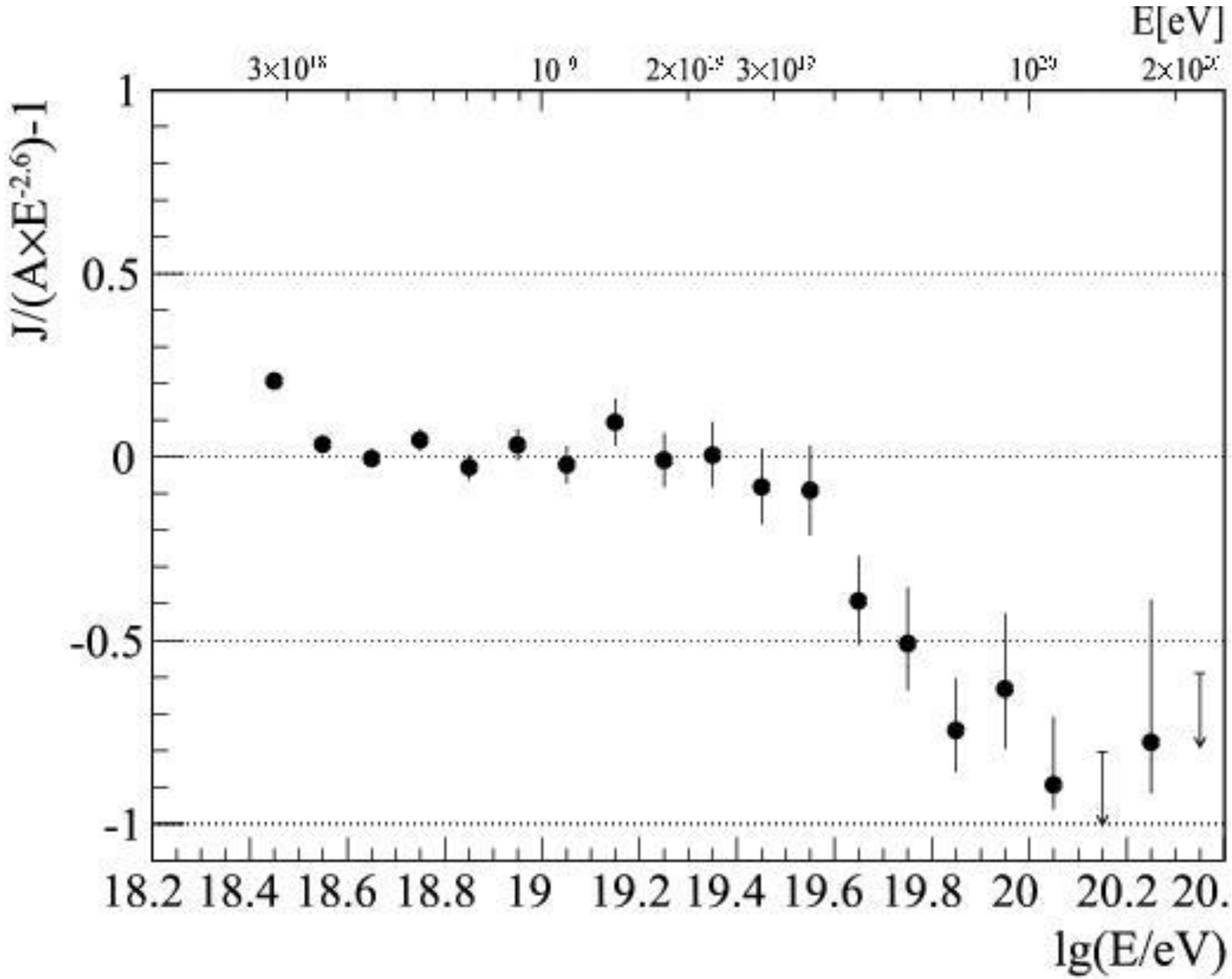
$$\lambda = 0.288$$

$$x_0 = 3.04 \times 10^{-4}$$

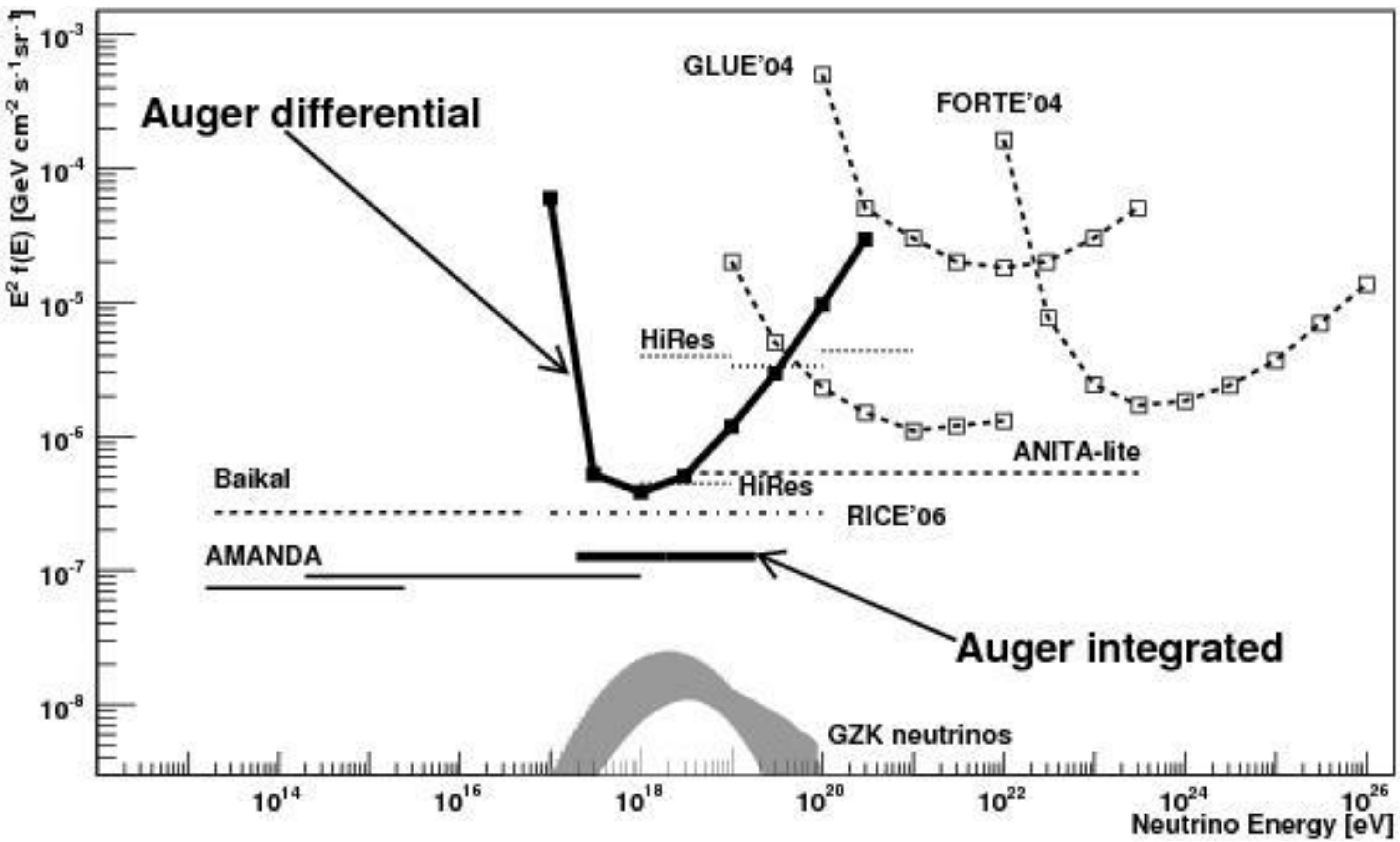
In the highest sensitive UHE neutrino interaction domain $x \simeq 10^{-8}$ saturation scale is around $Q_S{}^2 = 20\ GeV^2$. It is evident from the figure that the total cross-section is dominated by the scale $Q \approx M_{w,z}$. The region of $Q^2 < 1\ GeV^2$ and consequently the saturation region at $E = 10^{12}$ GeV contribute very little to the cross-section.

$Q \approx M_{w,z}$


x
Saturation
Region
high density
QCD
DAS
GRV
GRV
at 10^6
10^{-6}
10^{-4}
10^{-2}
1
10^6
Q^2 (GeV^2)



E[eV]
3×10^18
2×10^19 3×10^19
J/(A×E^-2.6)-1
1
0.5
0
-0.5
-1
18.2 18.4 18.6 18.8 19 19.2 19.4 19.6 19.8 20 20.2 20.
lg(E/eV)



E^2 f(E) [GeV cm^{-2} s^{-1}sr^{-1}]
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
GLUE'04
FORTE'04
Auger differential
HiRes
ANITA-lite
Baikal
HiRes
RICE'06
AMANDA
Auger integrated
GZK neutrinos
10^{14}
10^{16}
10^{18}
10^{20}
10^{22}
10^{24}
10^{26}
Neutrino Energy [eV]