Neutrino-nuclei scattering and xF_3 structure function from DGLAP evolution equation upto next-next-to-leading order at small-x

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Outline

- Abstract
- Introduction
  - Deep Inelastic Scattering
  - Neutrino-Nucleon Interaction
  - xF_3 (x, t) structure function
- Theory
- Result and Discussion
- Conclusion
- References

Abstract

Neutrino-nuclei scattering which produces the parity violating term $xF_3(x, t)$, the non-singlet structure function in their weak interaction has significant contribution towards the understanding of valence quark distribution at small-x region. In this paper the $xF_3(x, t)$ structure function have been obtained by solving DGLAP evolution equation upto next-next-to-leading order (NNLO) by using a method to get unique solution. The method of characteristics is also used to get the solution and both the solutions are compared. Moreover, the results are compared with recent experimental data and parametrizations and finally the nuclear effects in $xF_3(x, t)$ quark distribution for small-x have been studied.

INTRODUCTION

Deep Inelastic Scattering

Deep inelastic lepton-hadron scattering is recognised as an important testing ground for understanding the structure of matter. **DIS** investigates the electromagnetic, weak and strong interaction, dynamics of partons in nucleon and searches new physics in the lepton-quark sector.

The cross-sections for lepton hadron scattering is given by

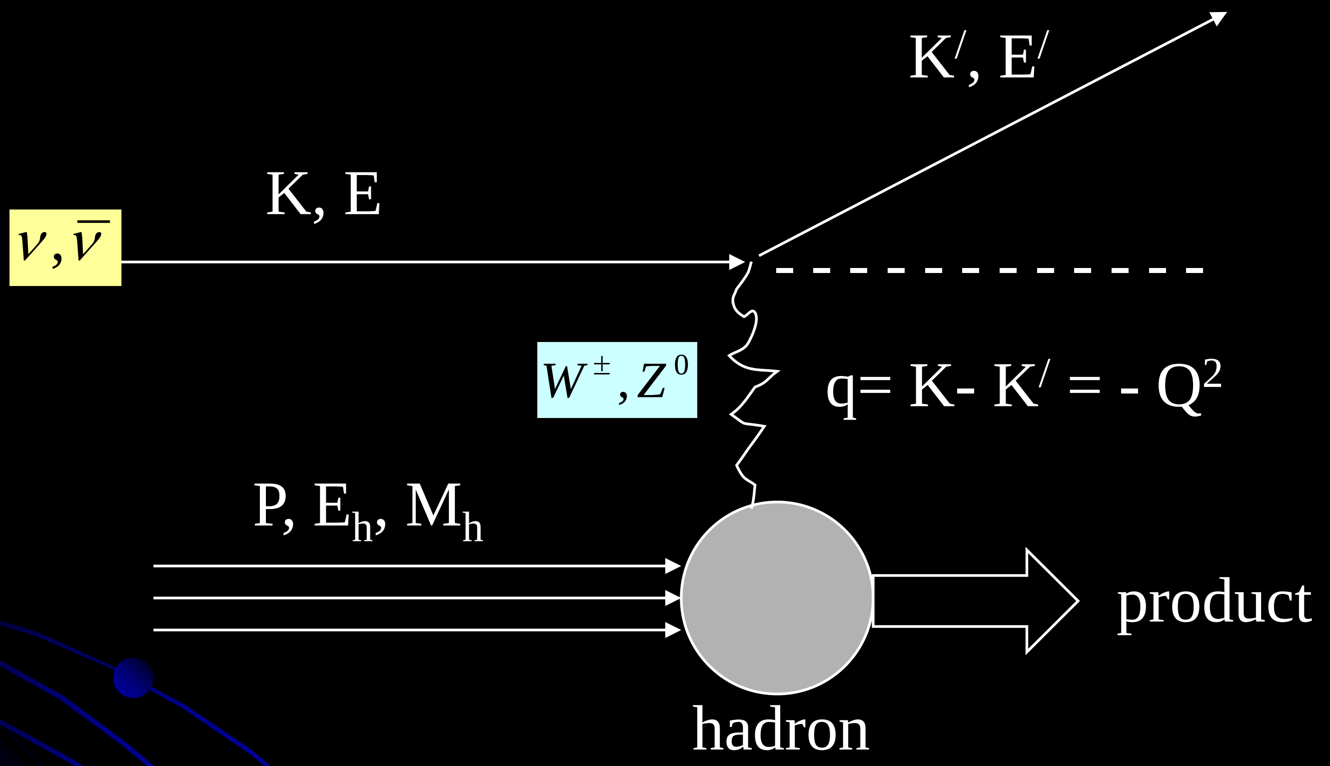
$$\frac{d^2\sigma}{dE.d\Omega} \propto L_{\alpha\beta} W^{\alpha\beta}$$

$L_{\alpha\beta}$ ⟶ Lepton tensor

$W^{\alpha\beta}$ ⟶ Hadron tensor

Leptons used in deep inelastic processes are either charged leptons (electron or muon) or neutrinos which scatter off the target nucleons via the electromagnetic or weak interactions respectively.

Neutrino-nucleon interaction



$x = Q^2/2p.q$ = Fraction of hadrons momentum carried by quark

In neutrino-nucleon scattering

- the lepton tensor is given by

$$L_{\alpha\beta}{}^{\nu,\bar{\nu}} = L_{\alpha\beta}{}^{S} \pm i L_{\alpha\beta}{}^{A}$$

- the hadron tensor is given by

$$W^{\alpha\beta} = -g^{\alpha\beta} W_1 + \frac{p^{\alpha} p^{\beta}}{M^2} W_2 - \frac{i\varepsilon^{\alpha\beta\gamma\delta} p_{\gamma} q_{\delta}}{2M^2} W_3 + \frac{q^{\alpha} q^{\beta}}{M^2} W_4 +$$

$$\frac{p^{\alpha} q^{\beta} + p^{\beta} q^{\alpha}}{M^2} W_5 + i \frac{p^{\alpha} q^{\beta} - p^{\beta} q^{\alpha}}{2M^2} W_6$$

If current is considered to be conserved (which eliminates W_6) and neglect lepton mass term (W_4 and W_5) $W^{\alpha\beta}$

will contain the remaining terms W_1, W_2 and W_3.

In the deep-inelastic scattering regime, the scattering cross-section for neutrino-nucleon DIS is given by

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F{}^2 E M_N}{\pi\left(1+\frac{Q^2}{M_W{}^2}\right)}\left[\frac{y^2}{2}2xF_1(x,Q^2) + (1-y-\frac{M_N xy}{2E})F_2(x,Q^2) \pm y(1-\frac{y}{2})xF_3(x,Q^2)\right]$$

$$x = \frac{Q^2}{2p.q}$$

$$y = \frac{p.q}{p.k}$$

E = neutrino beam energy

M_N= neutrino beam energy

xF_3 (x, t) structure function

- The function xF_3 (x, t) appears only in the cross section for the weak interaction because it originated from the parity violating term in the product of leptonic and hadronic tensors.
- xF_3(x, t) is restricted to neutrino-nucleon scattering.
- xF_3(x, t) receives contribution from flavor-nonsinglet part of interaction and the valence quark densities.
- xF_3(x, t) is not marred by the presence of the sea quark and gluon densities about which we have very poor information in particular in the small-x region.

THEORY

Theory for Leading Order

The DGLAP evolution equation for the non-singlet $xF_3(x, t)$ structure function in standard form in LO is given by

$$\frac{\partial[xF_3(x,Q^2)]}{\partial \ln Q^2} = P(x,Q^2) \otimes xF_3(x,Q^2)$$

$$\Rightarrow \frac{\partial[xF_3(x,Q^2)]}{\partial \ln Q^2} = \frac{\alpha(Q^2)}{2\pi}\int \frac{d\omega}{\omega} xF_3(\frac{x}{\omega},Q^2)P(\omega) \longrightarrow \quad (1)$$

The splitting function

$$P(\omega) = C_F(\frac{1+\omega^2}{1-\omega})_+ = C_F[\frac{1+\omega^2}{(1-\omega)_+} + \frac{3}{2}\delta(\omega-1)]$$

Substitution of the splitting function in equation (1) gives

$$\frac{\partial[xF_3(x,Q^2)]}{\partial \ln Q^2} = \frac{\alpha(Q^2)}{3\pi}\{[3+4\ln(1-x)].xF_3(x,Q^2)+2\int\frac{d\omega}{(1-\omega)}[\frac{(1+\omega^2)}{\omega}xF_3(\frac{x}{\omega},Q^2)-2xF_3(x,Q^2)]\}$$

This equation can be rewritten in terms of a more convenient variable t defined by $t = \ln(\frac{Q^2}{\Lambda^2})$ instead of the variable Q^2 as

$$\frac{\partial[xF_3(x,t)]}{\partial t} = \frac{\alpha(t)}{3\pi}\{[3+4\ln(1-x)]xF_3(x,t)+2\int\frac{d\omega}{(1-\omega)}[\frac{(1+\omega^2)}{\omega}xF_3(\frac{x}{\omega},t)-2xF_3(x,t)]\}$$

→ (2)

Let us introduce the variable $u = 1 - \omega$ which gives

$$\frac{x}{\omega} = \frac{x}{1-u} = x\sum_{k=0}^{\infty} u^k$$

Since $x \langle \omega \langle 1$ so $0 \langle u \langle 1 - x$ and hence this series is convergent for

Thus $\frac{x}{\omega} = \frac{x}{1-u} = x + \frac{xu}{1-u}$

This leads to the Taylor's expansion of $xF_3(\frac{x}{\omega}, t)$ as

$$xF_3(\frac{x}{\omega}, t) = xF_3(x + \frac{xu}{1-u}, t)$$

$$= xF_3(x) + \frac{xu}{1-u}\frac{\partial xF_3(x,t)}{\partial x} + \frac{1}{2}(\frac{xu}{1-u})^2 \frac{\partial^2 xF_3(x,t)}{\partial x^2} + \text{--------}$$

For smaller values of x the higher order terms in Taylor expansion can be neglected. Hence we can write

$$xF_3(\frac{x}{\omega},t) \approx xF_3(x) + \frac{xu}{1-u}\frac{\partial xF_3(x,t)}{\partial x}$$

Substituting this expansion in equation (2) and performing u-integration we get gives

$$\frac{\partial[xF_3(x,t)]}{\partial t} - \frac{A_f}{t}[P(x)xF_3(x,t) + Q(x)\frac{\partial xF_3(x,t)}{\partial x}] = 0 \longrightarrow (3)$$

Where

$$A_f = \frac{4}{33-2N_f}$$

$N_f =$ The number of flavor

$$P(x) = 1 + 2x + 4\ln(1-x) - 2\ln x$$

$$Q(x) = 2(1-x^2)$$

The general solution of equation (3) is $F(U,V)=0$, where $F(U,V)$ is an arbitrary function. Here $U(x,t,xF_3)=C_1$ and $V(x,t,xF_3)=C_2$ are two independent solutions of equation

$$\frac{dx}{A_f Q(x)} = \frac{dt}{-t} = \frac{dxF_3(x,t)}{-A_f P(x) xF_3(x,t)} \qquad (4)$$

Solving equation (4) we obtain

$$U(x,t,xF_3) = t.\exp[\frac{1}{A_f}\int \frac{1}{Q(x)}dx]$$

$$V(x,t,xF_3) = xF_3(x,t).\exp[\int \frac{P(x)}{Q(x)}dx]$$

If U, V are two independent solution of equation (4) and α, β are two arbitrary constant then

$$V = \alpha U + \beta \qquad (5)$$

may taken as complete solution of equation (4) which becomes

$$xF_3(x,t).\exp[\int \frac{P(x)}{Q(x)} dx] = \alpha.t.\exp[\frac{1}{A_f} \int \frac{1}{Q(x)} dx] + \beta \qquad (6)$$

From the partial conservation of vector and axial current we obtain the condition that the transverse structure function F_T vanishes in the limit $Q^2 \to 0$

Thus at $Q^2 \approx 0,$ i.e., $Q^2 \approx \Lambda^2,$

$$t = \ln(\frac{Q^2}{\Lambda^2}) = 0$$

and this condition gives $xF_3(x,t) = 0$

The equation (6) at $t = 0$ gives $\beta = 0$

Thus the substitution of $\beta = 0$ in equation (6) we obtain

$$xF_3(x,t) = \alpha.t.\exp[\int(\frac{1}{A_f}\frac{1}{Q(x)} - \frac{P(x)}{Q(x)})dx] \longrightarrow (7)$$

t- evolution

At $t = t_0$ for any lower value of $Q = Q_0$ let us define

$$xF_3(x,t_0) = \alpha . t_0 . \exp\left[\int \left(\frac{1}{A_f}\frac{1}{Q(x)} - \frac{P(x)}{Q(x)}\right)dx\right]$$

Equation (7) along with this expression gives

$$xF_3(x,t) = xF_3(x,t_0)\left(\frac{t}{t_0}\right)$$

This gives the t-evolution of $xF_3(x,t)$ structure function in leading order.

x- evolution

At a smaller value of x = x_0 we can define

$$xF_3(x,t)\Big|_{x=x_0} = \alpha.t.\exp[\int(\frac{1}{A_f}\frac{1}{Q(x)}-\frac{P(x)}{Q(x)})dx]\Big|_{x=x_0}$$

And substituting this expression in equation (7) we get

$$xF_3(x,t) = xF_3(x,t)\Big|_{x=x_0}\exp[\int_{x_0}^{x}(\frac{1}{A_f}\frac{1}{Q(x)}-\frac{P(x)}{Q(x)})dx]$$

This gives the x-evolution of $xF_3(x,t)$ structure function in leading order.

Theory for Next-to-Leading Order

The DGLAP evolution equation for the non-singlet $xF_3(x, t)$ structure function in standard form in NLO is given by

$$\frac{\partial[xF_3(x,Q^2)]}{\partial \ln Q^2} = \frac{\alpha(Q^2)}{2\pi}\int_x^1 \frac{d\omega}{\omega} xF_3(\frac{x}{\omega},Q^2).[\frac{\alpha_s(Q^2)}{2\pi}P^0(\omega)+(\frac{\alpha(Q^2)}{2\pi})^2 P^1(\omega)]$$

Where the NLO splitting function

$$P^1(\omega) = f(\omega) + \delta(1-\omega)K$$

And

$$P^1(\omega) = C_F{}^2\{P_F(\omega) - P_A(\omega)\} + \frac{1}{2}C_F C_A\{P_G(\omega) + P_A(\omega)\} + C_F T_R N_F P_{N_F}(\omega)$$

$$K = C_F{}^2\{\frac{3}{8} - \frac{1}{2}\pi^2 + \varsigma(3) - 8\tilde{S}(\infty)\} + \frac{1}{2}C_F C_A\{\frac{17}{12} + \frac{11}{9}\pi^2 - \varsigma(3) + 8\tilde{S}(\infty)\} - C_F T_R N_F\{\frac{1}{6} + \frac{2}{9}\pi^2\}$$

t- evolution

$$xF_3(x,t) = xF_3(x,t_0)\left[\frac{t^{(\frac{b}{t}+1)}}{t_0^{(\frac{b}{t_0}+1)}}\right].\exp\left[b\left(\frac{1}{t}-\frac{1}{t_0}\right)\right]$$

x- evolution

$$xF_3(x,t) = xF_3(x,t)\Big|_{x=x_0}\exp\left[\int_{x_0}^{x}\left(\frac{1}{a}\frac{1}{[Q(x)+T_0S(x)]}-\frac{[P(x)+T_0(R(x)+K)]}{[Q(x)+T_0S(x)]}\right)dx\right]$$

Where

$$R(x) = \int_x^1 f(\omega)\frac{d\omega}{\omega}$$

$$S(x) = \int_x^1 (\omega-1)f(\omega)\frac{d\omega}{\omega}$$

Theory for Next-Next-to-Leading order

The DGLAP evolution equation for the non-singlet $xF_3(x, t)$ structure function in standard form in NNLO is given by

$$\frac{\partial[xF_3(x,Q^2)]}{\partial \ln Q^2} = \frac{\alpha(Q^2)}{2\pi}\int_x^1 \frac{d\omega}{\omega} xF_3(\frac{x}{\omega},Q^2).\left(\frac{\alpha_s(Q^2)}{2\pi}P^0(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^2 P^1(\omega) + \left(\frac{\alpha(Q^2)}{2\pi}\right)^3 P^2(\omega)\right)$$

Where the NNLO splitting function

$$P^2(\omega) = n_f\left[\begin{array}{l}\{L_1(-163.9\omega^{-1} - 7.208\omega) + 151.49 + 44.51\omega - 43.12\omega^2 + 4.82\omega^3\}(1-\omega) + L_0 L_1 \\ [-173.1 + 46.18L_0] + 178.04L_0 + 6.892L_0{}^2 + \frac{40}{27}\left(L_0{}^4 - 2L_0{}^3\right)\end{array}\right]$$

And

$$L_0 = \ln(\omega) \qquad L_1 = \ln(1-\omega)$$

t- evolution

$$xF_3(x,t) = xF_3(x,t_0)\left[\frac{t^{(1+\frac{b}{t}-\frac{b^2}{t})}}{t_0^{(1+\frac{b}{t_0}-\frac{b^2}{t_0})}}.\exp\left\{(b-c)\left(\frac{1}{t}-\frac{1}{t_0}\right)-\frac{b^2\ln^2 t}{t}+\frac{b^2\ln^2 t_0}{t_0}\right\}\right]$$

x- evolution

$$xF_3(x,t)= xF_3(x,t)\Big|_{x=x_0}\exp\left[\int_{x_0}^{x}\left\{\frac{1}{a}\frac{1}{[Q(x)+T_0S(x)+T_1L(x)]}-\frac{[P(x)+T_0(R(x)+K)+T_1M(x)]}{[Q(x)+T_0S(x)+T_1L(x)]}\right\}dx\right]$$

Where

$$M(x)=\int_x^1 P^2(\omega)\frac{d\omega}{\omega}$$

$$L(x)=\int_x^1(\omega-1)P^2(\omega)\frac{d\omega}{\omega}$$

Comparison of $T^2(t)$ and $T_0.T(t)$ as well as $T^3(t)$ and T_1. T(t) values

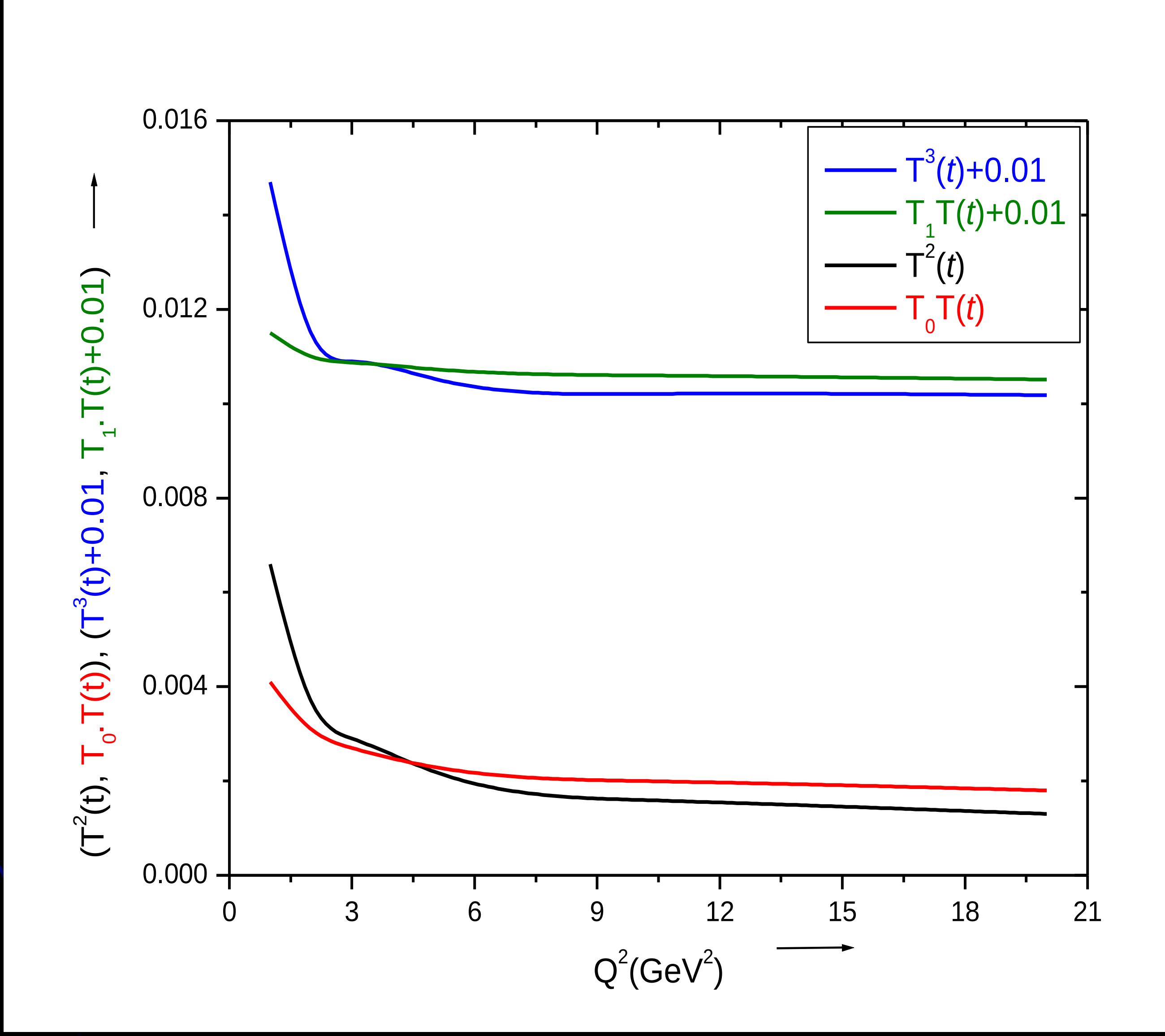
Where,

$T^2(t) = T_0 . T(t)$

$T^3(t) = T_1 . T(t)$

and

$$T(t) = \frac{\alpha(t)}{2\pi}$$



RESULT AND DISCUSSION

We compared our results of t-evolutions and x- evolutions of the non-singlet structure function $xF_3(x,t)$ with the experimental data taken from CCFR [5] and NUTEV [6]. In Fig. 1(a,b) the structure function $xF_3(x,t)$ is plotted against Q^2 for different values of x. Here for clarity, data are scaled up by adding 0.5i (i = 0,1,2,3,…) with all the values of $xF_3(x,t)$

In Fig. 2(a,b) $xF_3(x,t)$ is plotted against x for different values of Q^2

Here the vertical error bars are both statistical and systematic for the NUTEV data but for CCFR data the errors are statistical only and the values of Λ, the QCD cut off parameter in NUTEV and CCFR are considered to be 583 ± 17 MeV and 337 ± 28 MeV respectively.

In all graphs lowest –t and highest-x points are taken as inputs for t- and x-evolution respectively.

Comparison of t- evolution results in LO and NLO for CCFR data

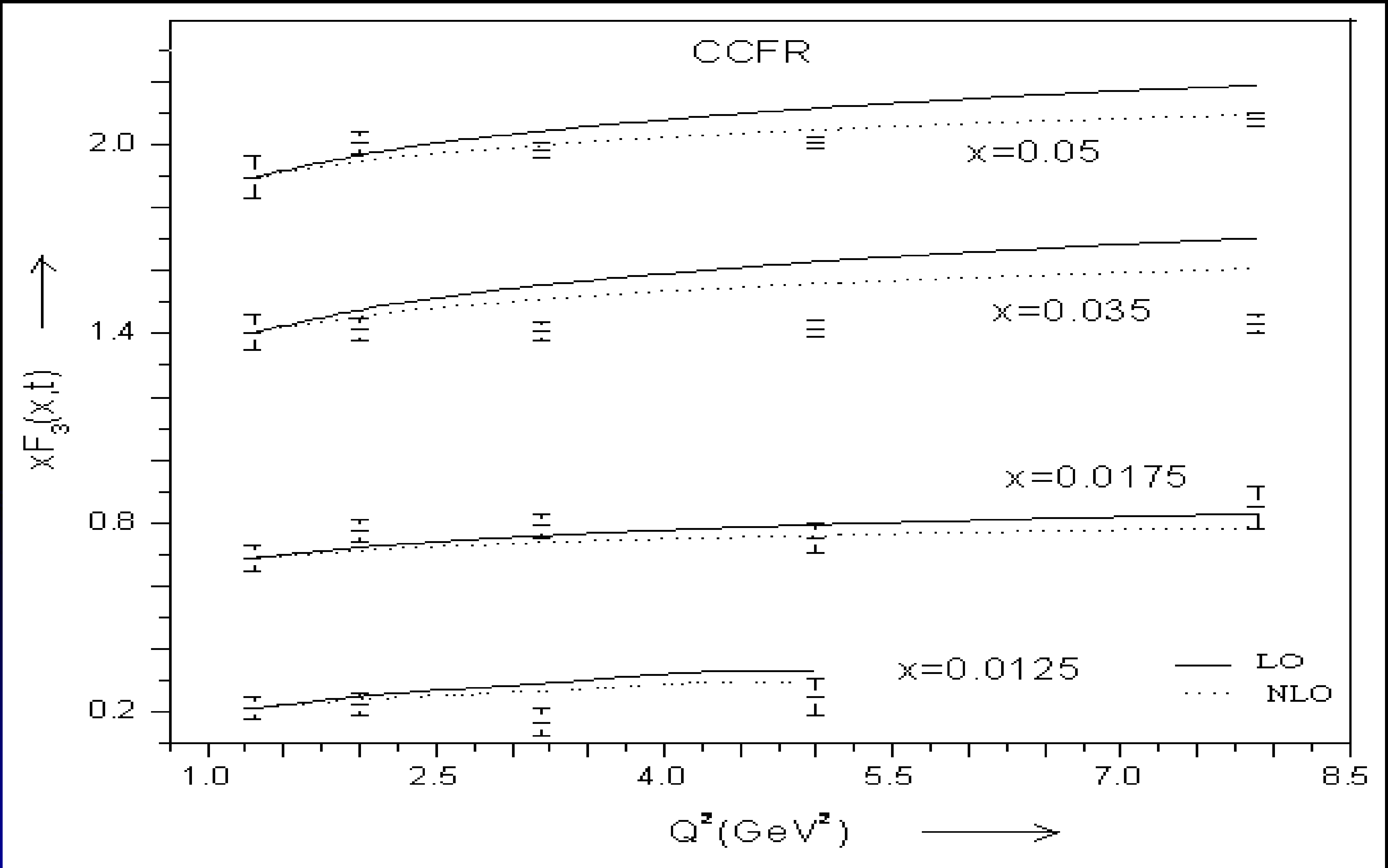


Fig. 1a

Comparison of t- evolution results in LO and NLO for NuTeV data

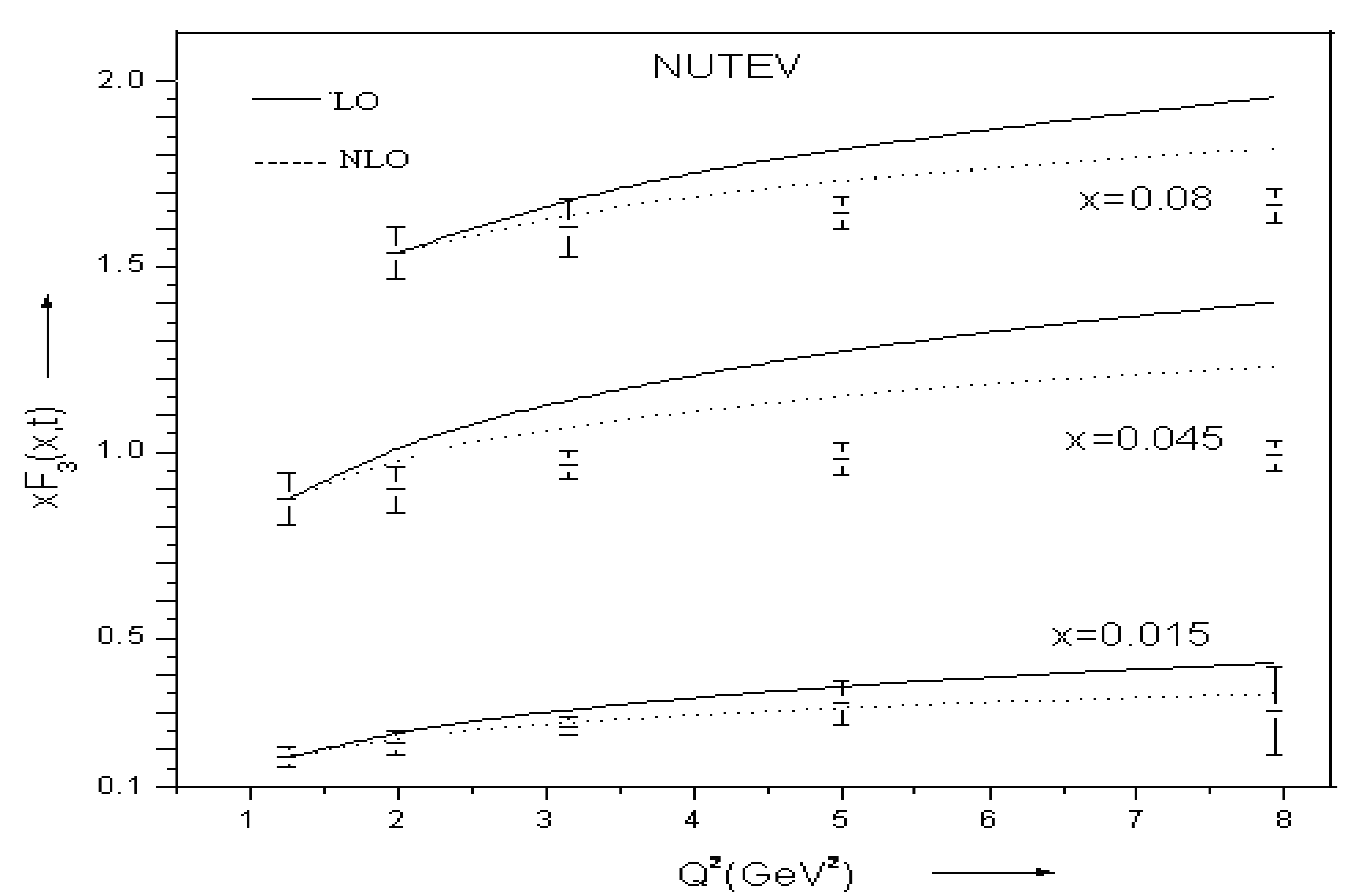


Fig. 1b

Comparison of x- evolution results in LO, NLO and NNLO for CCFR data

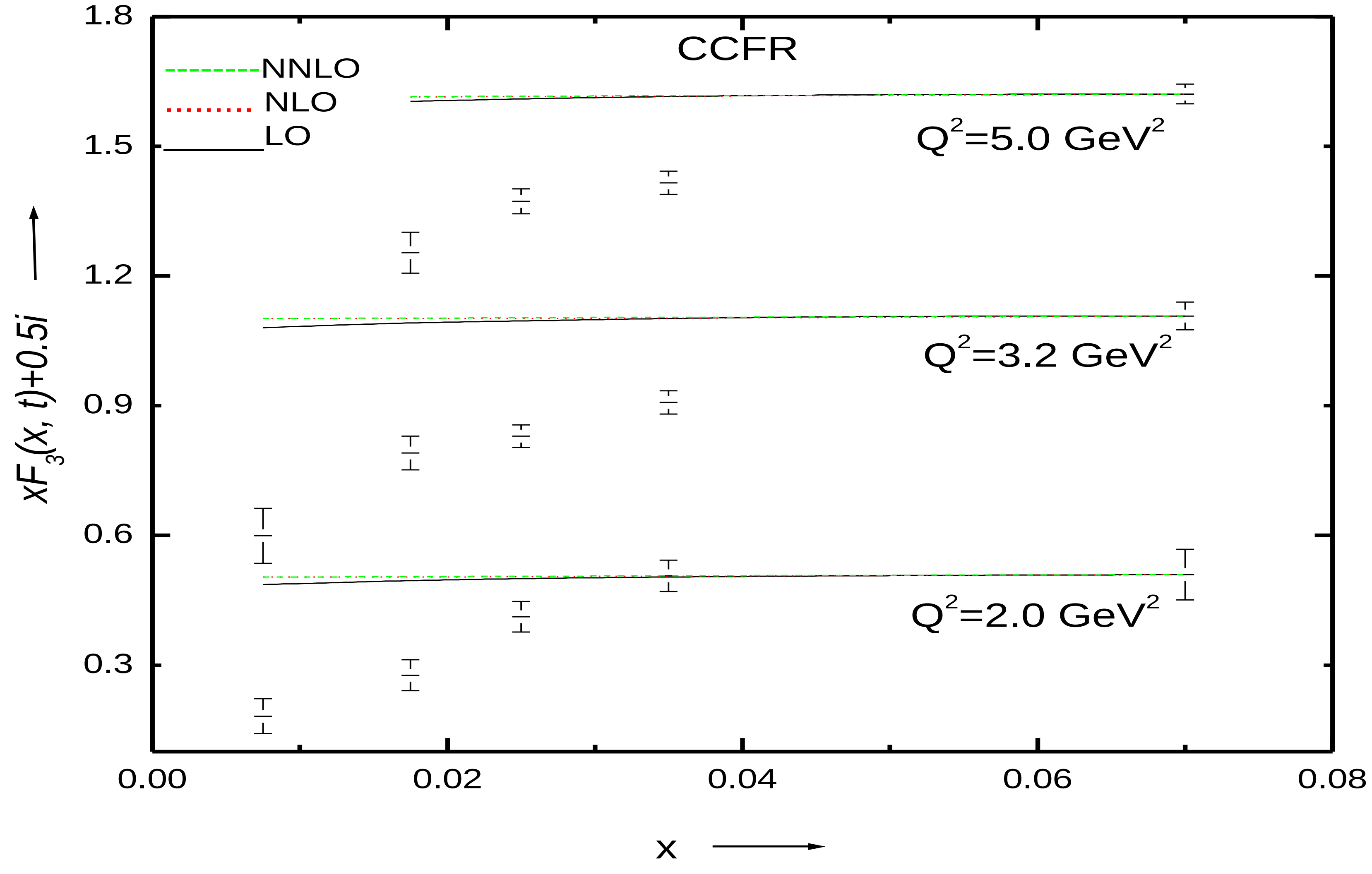


Fig. 2a

Comparison of x- evolution results in LO, NLO and NNLO for NUTEV data

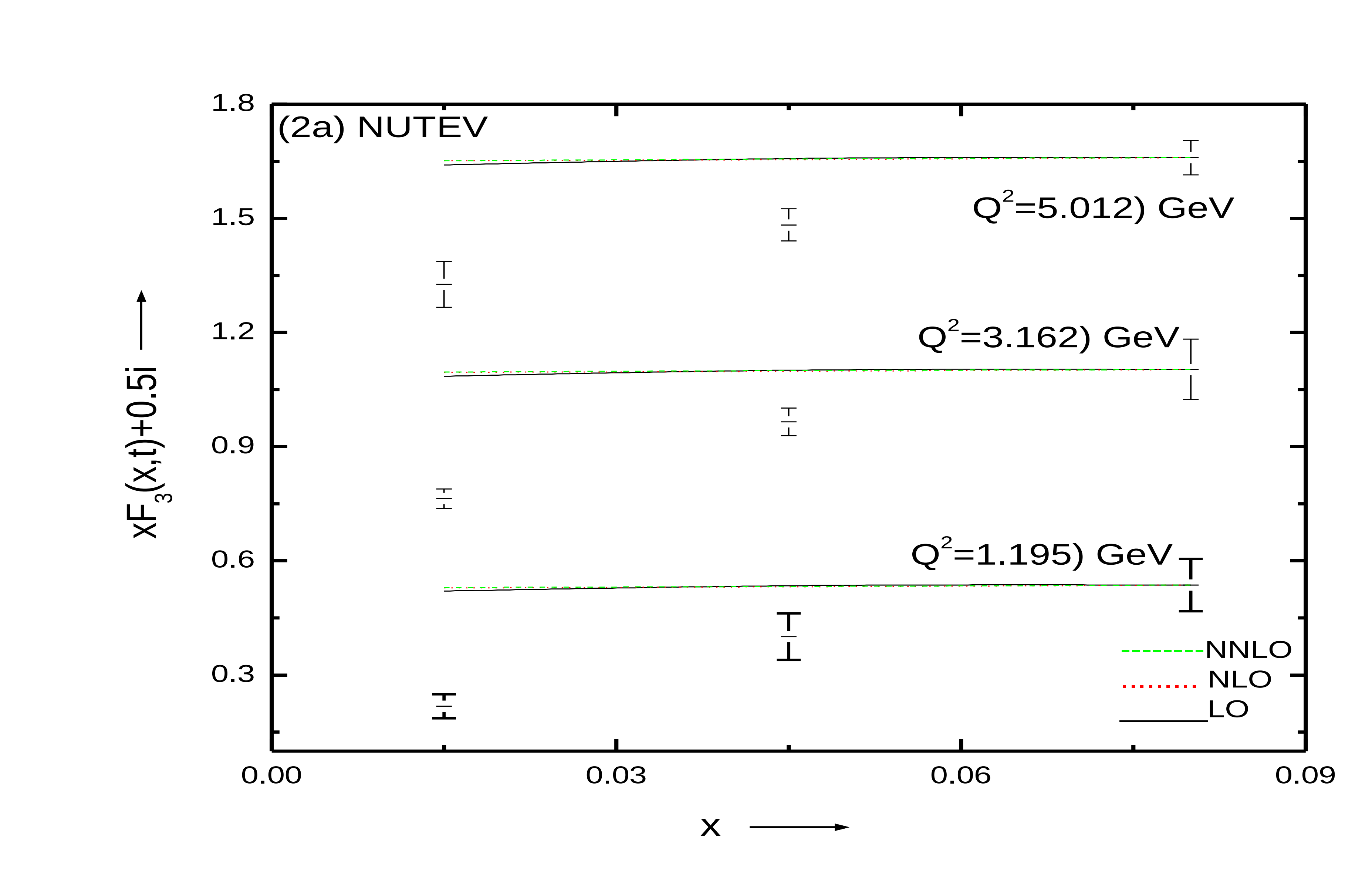


Fig. 2b

It is observed that our results do not agree excellently with experimental data. The probable reason for this is nuclear effects. The experimental data contain nuclear effects and our data are free from it. Here we have not considered the nuclear effect.

Nuclear effects

The ratio $R = \frac{F^{A}}{F^{N}}$ which gives the structure function per nucleon depends on different kinematical region and the atomic number of the nucleus used as target.

European Muon Collaboration (EMC) first observed that the structure functions of bound nucleons are different from those of free nucleon.

Nuclear effects

- Shadowing $x < 0.1$
- Anti-shadowing $0.1 < x < 0.3$
- EMC $0.3 < x < 0.1$
- Fermi motion $x > 0.7$

Nuclear effects in F_2 structure function

Nuclear effects are well investigated theoretically and experimentally in the structure function F_2 in different kinematical region.

At small-x the shadowing effect in F2 structure function is explained by two model:

1. Vector-meson-dominance-type model

   The central constituents are "shadowed" due to existance of nuclear surface constituents.

2. Parton-recombination-type-model

   This model explains the shadowing by interactions of partons from different nucleons in a nucleus, and the interactions are called parton recombination.

Two different model produce similar shadowing results in F_2, it is difficult to Distinguish among the models in comparison with (inaccurate) experimental data.

Nuclear effects in xF_3 structure function

Use of xF_3 structure function is better way for testing the models for shadowing.

- xF_3 structure function reflects only the valence quark distribution

- xF_3 structure function is free from sea quark and gluon densities about which we have poor information in particular in the small-x region. So its QCD analysis is less complex and hence easier to discuss than that for F_2.

CONCLUSION

- In this paper the xF_3 *(x, t)* structure function have been obtained by solving DGLAP evolution equation upto next-next-to-leading order (NNLO) by using a method to get unique solution.

- It is observed that our results do not agree excellently with experimental data. The probable reason for this is nuclear effects.

- Only a little attention is given to nuclear effects in neutrino DIS as well as xF_3 structure function. Thus F_3 shadowing is an undeveloped research area.

APPENDIX

$$P_F(\omega) = -2\frac{1+\omega^2}{1-\omega}\ln\omega.\ln(1-\omega) - (\frac{3}{1-\omega} + 2\omega)\ln\omega - \frac{1}{2}(1+\omega)\ln^2\omega - 5(1-\omega)$$

$$P_G(\omega) = \frac{1+\omega^2}{1-\omega}[\ln^2\omega + \frac{11}{3}\ln\omega + \frac{67}{9} - \frac{1}{3}\pi^2] + 2(1+\omega)\ln\omega + \frac{40}{3}(1-\omega)$$

$$P_{N_F}(\omega) = \frac{2}{3}[\frac{1+\omega^2}{1-\omega}[-\ln\omega - \frac{5}{3}] - 2(1-\omega)]$$

$$P_A(\omega) = 2[\frac{1+\omega^2}{1-\omega}]\int_{\frac{\omega}{1+\omega}}^{\frac{1}{\omega}} \frac{dz}{z}\ln\frac{1-z}{z} + 2(1+\omega)\ln\omega + 4(1-\omega)$$

$$\beta_0 = 11 - \frac{4}{3}T_R N_F$$

$$\beta_1 = \frac{34}{3}C_G{}^2 - \frac{10}{3}C_G N_F - 2C_F N_F$$

$$C_G = C_A = N_C = 3$$

$$\beta_1 = \frac{2857}{54}N_C{}^3 + 2C_F{}^2 T_f - \frac{205}{9}C_F N_C T_f + \frac{44}{9}C_F T_f{}^2 + \frac{44}{9}N_C T_f{}^2$$

$$C_F = \frac{4}{3}$$

$$a = \frac{2}{\beta_0} \qquad b = \frac{\beta_1}{\beta_0{}^2} \qquad c = \frac{\beta_2}{\beta_0{}^3} \qquad N_F = 4 \qquad T_R = \frac{1}{2} \qquad T_f = \frac{1}{2}N_F$$

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THANK YOU